Velocity and density estimation from nonlinear amplitude inversion of prestack multicomponent shallow seismic data: A numerical model study

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Abstract

This paper shows a numerical study aiming at predicting seismic velocities and densities from nonlinear AVO inversion of prestack multicomponent shallow seismic data. In this approach we propose to use PP, PSv, SvP and SvSv amplitudes at pre- and post-critical angle**.** Amplitudes are evaluated by the exact Zoeppritz equations. In order to optimize the least-squares fit of observed and computed amplitudes we used parallel Multi-Start run for Controlled Random Search algorithm (CRS) and kernel density estimation. We present a new approach for multidimensional objective function analysis, with numerous implications for accuracy and efficiency of nonlinear inversion.

Introduction

Acquisition feasibility of multicomponent seismic data for shallow scale with a good signal-to-noise ratio has been demonstrated in recent studies, as an example we mention Dasi *et al.* (1999), Guy (2006), Pugin (2008) and Pugin *et al.* (2009).

Knowledge of P-wave (α) and S-wave (β) velocities and density (ρ) of subsurface media allows direct access to information about mechanical and hydraulic behavior of rock and soil masses (compressibility, shear rigidity, porosity, permeability, Poisson's ratio etc.). Among the possible applications of this information in shallow investigation range, there are: quantitative detection of changes in properties of subsurface media during tunnel design (Kneib, G. *et al.*, 2000); characterization of aquifers (Giustiniani *et al.*, 1999); identification of lithology and porosity changes (Domenico, 1984); understanding effects produced by low-velocity layer in seismic imaging for hydrocarbon exploration (Guevara, 2001).

Dependence of reflectivity with angle of incidence allows the estimation of elastic media parameters by AVO or AVA (reflected signal amplitude variation, respectively, with offset, or angle of incidence) inversions, which are frequently being used in petroleum industry. However, few studies are conducted in shallow range.

Reflection coefficients of incident seismic waves on interface that separates two distinct media are provided by Zoeppritz equations, described in terms of six elastic parameters (P-wave (α) and S-wave (β) velocities and density (ρ), all above and below reflecting interface). Due to mathematical complexity and nonlinear character of these expressions it is habitual the use of linearization for seismic amplitudes, which are valid approximations only for angles of incidence below critical angle (Aki, & Richards, 1980; Castagna, 1993; de Nicolao, 1993), and in general, below 30 degrees. In most cases, these approximations are acceptable in the scale of hydrocarbon exploration. However, in near surface, where media have high contrasts of speed and due to interference of coherent noises hindering the observation of reflections at short offsets, it is common to record reflections with angles of incidence above the critical angle, in such a way making difficult the use of approximations for reflection coefficients.

Several authors assert use Zoeppritz equations is too difficult for estimating media parameters based on inversion of amplitudes (Castagna, 1993; Rabben *et al.*, 2008). However, some ones agree there are several ways to stabilize the inverse problem. One possibility is to include converted waves (PSv and/or SvP) and SvSv reflected wave, besides usual P-wave reflectivity (de Haas & Berkhout, 1990; Demirbag and Çoruh, 1988). Another way is to use a wide offset range, before and after critical angle (Ostrander, 1984; de Haas & Berkhout, 1990).

One of the most controversial aspects, as regards elastic amplitude inversion, is the appropriate choice of model parameter vector. Debski and Tarantola (1995) support the best choices are: {density, P-wave impedance, and Poisson's ratio} or {density, P-wave impedance, and S-wave impedance}. Ursin and Tjaland (1996) affirm it is possible to estimate reasons with Zoeppritz equations: {$\alpha 1/\alpha 2$; $\alpha 1/\beta 1$; $\alpha 1/\beta 2$; $\rho 1/\rho 2$}. However, Larsen (1999) showed that in a numerical study by using reflected P-wave and PS converted amplitudes it is possible to estimate seismic velocities and density ratio: {$\alpha 1$; $\alpha 2$; $\beta 1$; $\beta 2$; $\rho 1/\rho 2$}.

In this paper we present a strategy for nonlinear inversion of reflection coefficient of reflected PP, PSv, SvP and SvSv seismic waves, calculated from exact Zoeppritz equations, to obtain seismic velocities and densities: {$\alpha 1$, $\alpha 2$, $\beta 1$, $\beta 2$; $\rho 1$; $\rho 2$}. Evaluating feasibility of this parameterization for being the most convenient parameters for characterization of rock and soil masses was choice.

It was suggested a strategy for reflection coefficient inversion, which ensure reliability of parameter estimates in the face of an inverse problem known as extremely ill-posed. Methodology used to solve optimization problem allowed evaluating multidimensional characteristics of objective functions and search algorithm performance.

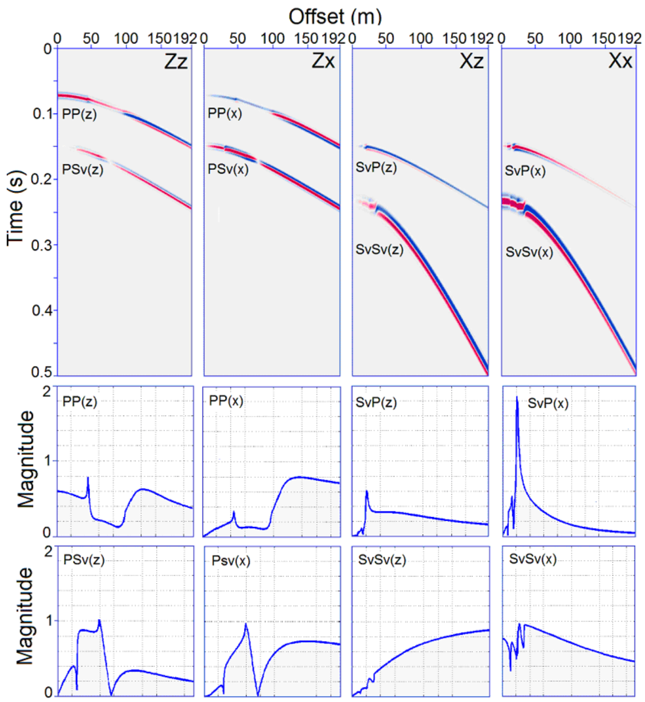

Figure 1: Synthetic seismograms of reflections and their curves of reflection coefficients, illustrating PP, PSv, SvP, SvSv seismic waves decomposed into vertical (z) and radial (x) plans.

Layer number	α (m/s)	β (m/s)	ρ (kg/m^3)	h (m)
1	1500	452	1530	50
2	3750	2165	2430	-

Table 1**.** Parameters of numerical model cited in Pullan & Hunter (1985).

Inversion Methodology

Estimation of elastic subsurface parameters formulated by inversion of exact reflection coefficients of reflected PP, PSv, SvP and SvSv waves is a nonlinear problem, to find the parameter vector m = {α1, α2, β1, β2; ρ1; ρ2} so that an objective function f(*m*) is minimized. Inversion problem of reflection coefficient is known as extremely ill-posed. In this case, diverse models can equally well represent data, because there is not a single solution, and convergence of a global optimization algorithm to an extreme objective function point cannot guarantee the correct inverse problem solution. A careful analysis of the characteristics of nonlinear direct and inverse problems is indispensable for elaboration of an appropriate strategy to successfully achieve the estimation of parameters.

Forward problem

Mathematical formulation of forward problem solved within inversion process corresponded to exact calculation of reflection coefficient: R_{PP}, R_{PSv}, R_{SvP} and R_{SvSv} through Zoeppritz equations formulated according to Ikelle and Amundsen (2005).

Figure 1 shows curves of reflection coefficients for a test model (Table 1), and their synthetic seismograms illustrating PP, PSv, SvP and SvSv waves decomposed in vertical (z) and radial (x) components (2D example). We analyzed a wide window of source-receiver offsets: from 1 m to 192 m, with 1 m of interval. To generating synthetic seismograms, reflection times were calculated by implemented ray method in package Seis88 (Cervený & Psencik, 1988).

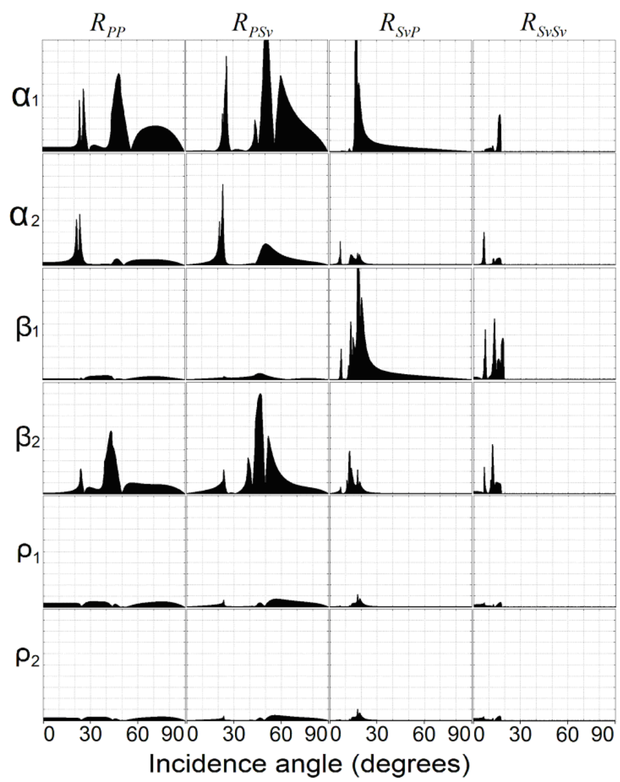

Figure 2: Sensitivity analysis: modulus of difference between values of reflection coefficients, calculated for all correct parameters, and with 10% perturbation in indicated parameter.

Parameter sensitivity analysis

The objective of sensitivity analysis is to study the sensitivity of reflection coefficients with parameter variation. Greater change in reflection coefficients when one media parameters is changed, better determination of relevant parameter, it means, lower the ambiguity in this parameter estimate will be. All reflection coefficients (R_{PP}, R_{PSv}, R_{SvP}, *and* R_{SvSv}) were calculated by introducing a 10% perturbation in a parameter while leaving correct the other five parameters. Figure 2 shows, in a 4x6 matrix, the differences between the reflection coefficients calculated with the original and perturbed parameters. Each column represents a reflection coefficient and each line a parameter.

Briefly, the main conclusions of sensitivity study are: (1) densities are parameters that have a lower sensitivity in all coefficients, becoming more difficult (but not impossible) to be estimated; (2) a good option for estimating the 4 velocities is to use SvSv and SvP amplitudes, since reflections can be identified in seismic records in short offsets, for small angles of incidence (<30 degrees), and (3) PP and PSv reflection coefficients present sensitive in a wide offset range.

Objective function

The least squares criterion was used to quantify the similarity between observed (A^{obs}) and calculated (A^{calk}) amplitudes, respectively by Zoeppritz equations for reflected PP, PSv, SvP, and SvSv waves. Thus, we defined four objective functions (equations 1 to 4), and in order to evaluate advantages in exploring potential redundancies or additional information simultaneous objective functions were also analyzed, it means, all possible combinations of equations 1-4.

$$f_{Rpp} = \left[\sum_j \left(A^{obs}_{pp_j} - A^{calk}_{pp_j} \right)^2 \right]^{1/2} \qquad (1)$$

$$f_{Rpsv} = \left[\sum_j \left(A^{obs}_{psv_j} - A^{calk}_{psv_j} \right)^2 \right]^{1/2} \qquad (2)$$

$$f_{Rpsv} = \left[\sum_j \left(A^{obs}_{svp_j} - A^{calk}_{svp_j} \right)^2 \right]^{1/2} \qquad (3)$$

$$f_{Rpsv} = \left[\sum_j \left(A^{obs}_{svsv_j} - A^{calk}_{svsv_j} \right)^2 \right]^{1/2} \qquad (4)$$

Where j index refers to the number of seismic trace.

The analysis of objective function characteristics is essential to validate the choice of optimization method suitable for solving the inverse problem. Observed topography in contour maps of objective function (Larsen, 1999; Kurt, 2007) is the traditional way of evaluating optimization problem characteristics, such as linearity, local minima, ambiguities, and convergence, allowing defining the degree of complexity, and thus guiding search algorithm choice. This procedure works well if the inverse problem has two unknowns. However, for multidimensional problems, as investigated with six unknowns, the analysis provides erroneous information about function topography, and does not allow observing its global behavior, since function shares (cross sections) are observed, it means, there is only a variation of two parameters while other parameters are fixed at their correct values.

In this study, we adopted a new methodology to evaluate topography of multidimensional objective function consisting of dispersion analysis of the set of points obtained by multiple runs of a stochastic optimization algorithm. Figure 3 shows an example of dispersion map obtained with Nelder-Mead SIMPLEX (Nelder and Mead, 1965) search algorithm, override to residual function map of f_{Rpp} function share (Eq. 1) for the pair of parameters: α_1 and α_2. The observed ambiguity in the residual function map of f_{Rpp} (contour plot on Figure 3) is not the same as that revealed by dispersion map (dots on Figure 3). Analyzing the dispersion of optimization solutions is easy to differentiate the overall ambiguity of local minima, and color differentiation of different ranges of objective function values allows visualizing function topography.

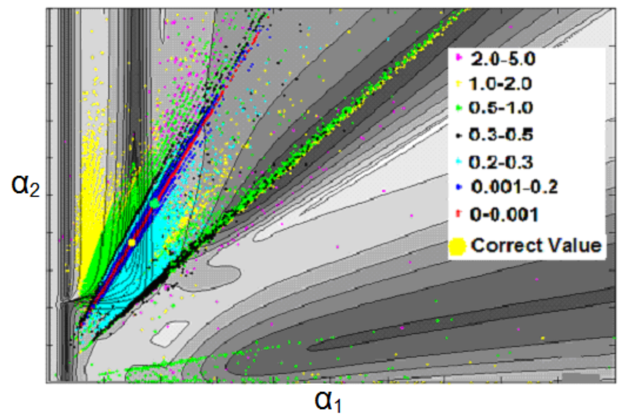

Figure 3: Dispersion map of points obtained by multiple realizations of optimization algorithm override to contour map of f_{Rpp} share for the pair of parameters (α_1 - α_2). The color scale marks the range of function values found by repeating optimization procedure.

Optimization

The global minimum objective function search was performed with a Multi-Start procedure implementation by using the stochastic global CRS (Controlled Random Search) optimization algorithm (Price, 1979; Price 1983; Eligius *et al.*, 2001) connected with Nelder-Mead simplex algorithm to improve local search ability. When executed more than once, stochastic algorithms rarely converge to the same point. Many times, the search procedure produces a set of solutions that form a distribution. When this distribution is normal, arithmetic mean value is equal to maximum likelihood distribution, such as reflection coefficient inversion using the least squares criterion, it is acceptable to use the mean or median for final parameter estimation. Usually, median is considered more robust (Isaaks, 1989).

Dispersion maps proposed for multidimensional analysis of objective function also proved to be very useful to evaluate various aspects of optimization procedure, allowing validating and/or refining the defined search strategy. Figure 4 shows one examples of this type of analysis applied to evaluate performance of search procedure on two different objective functions.

Inversion results

Without noise, it was possible to estimate all proposed parameters using each reflection coefficients (PP, PSv, SvP and SvSv) individually. In general, densities were worse determined, consistent with conclusions obtained from sensitivity analysis. Error in estimate was less than 0.3% for all parameters (Figure 5) using any objective functions (single or joint).

Adding white Gaussian noise to synthetic data (Figure 6) the error increases. In this case, the use of joint objective functions is more relevant. The best joint objective function was $f_{Rpp,Rpsv,Rsvp,Rsvsv}$, because all parameters had an error less than 0.5%. The best individual objective function was f_{Rpp} with error less then 2% (Figure 7).

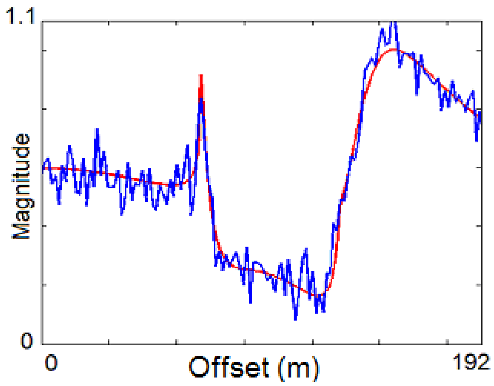

Figure 6: P-wave reflection coefficient curve with Additive White Gaussian Noise.

Discussions and Conclusions

Results obtained from tested model indicate it is possible to use the exact Zoeppritz equations in order to individually determine the six model parameters. It was achieved through implementation of a Multi-Start procedure for a stochastic algorithm for global optimization followed by a statistical analysis of distributions around global minimum. Idealized optimization strategy does not require *a priori* information about model parameters.

Another important advantage of Multi-Start procedure is the possibility and easiness in using parallel computing.

Dispersion maps were very useful to analyze topography of multidimensional objective functions, and optimization results. This procedure may have several applications but mainly it is an excellent tool for analyzing nonlinear inversion problems in more than two dimensions dealing with multimodal objective functions, allowing that global optimization strategy is more safely designed.

Taking into consideration an ideal noiseless situation, the use of any reflection coefficient is adequate to estimate parameters. In noise presence, it is desirable to use joint objective functions.

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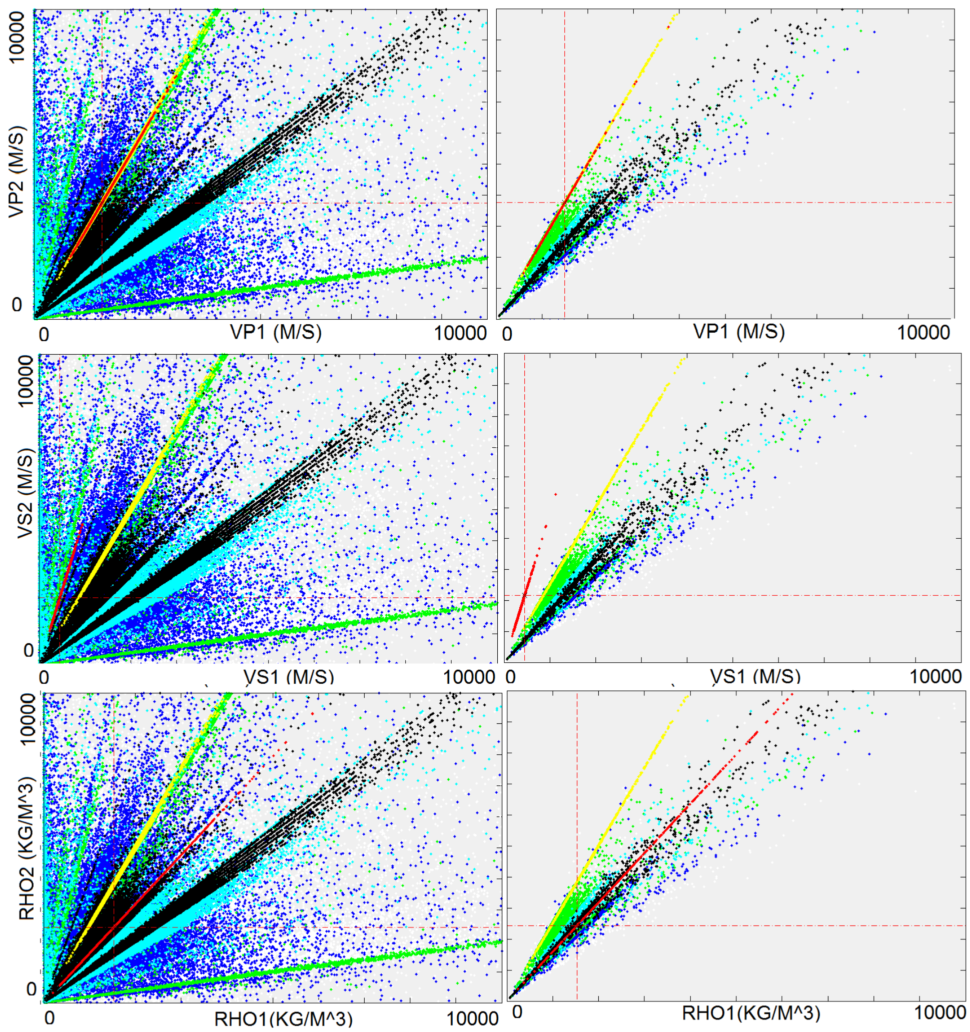

Figure 4: Examples of dispersion maps for objective function $f_{Rpp,}$ (left) versus joint objective function $f_{Rpp,Rpsv,Rsvp,Rsvsv}$ (right).

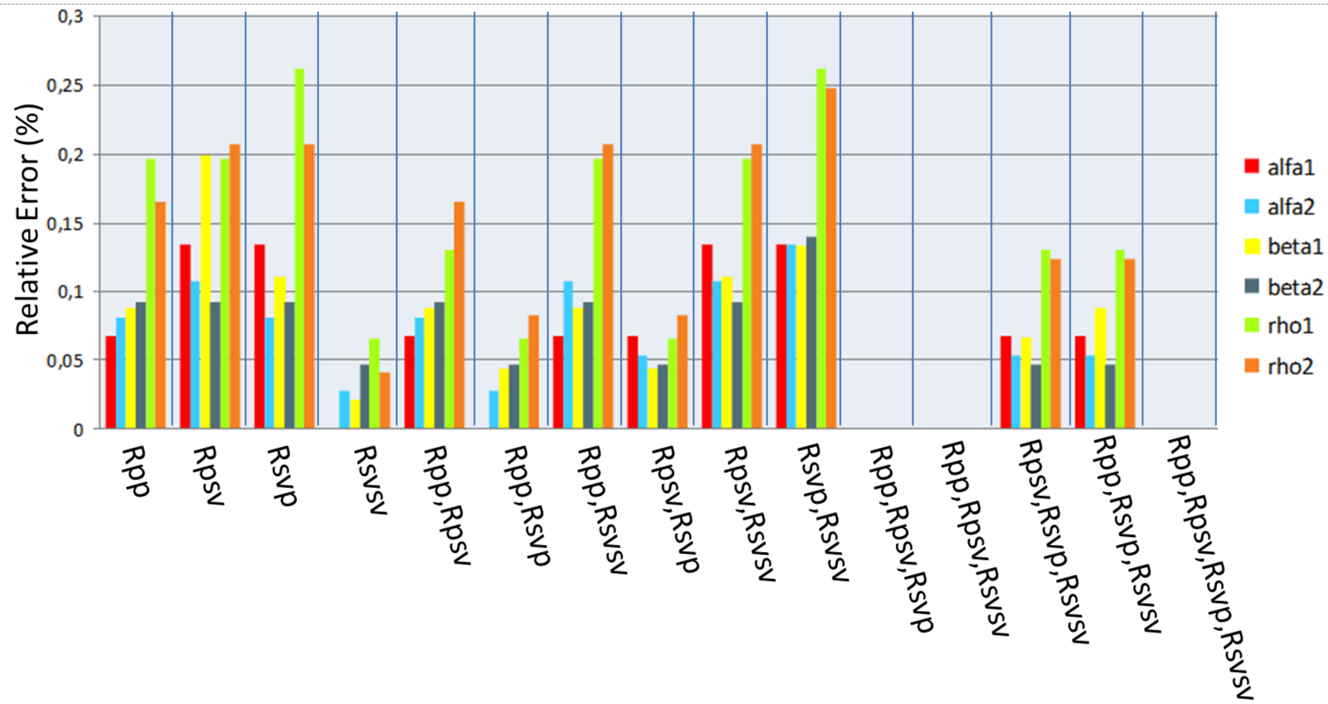

Figure 5: Relative error for each parameter when performing individual and joint inversions using data without noise.

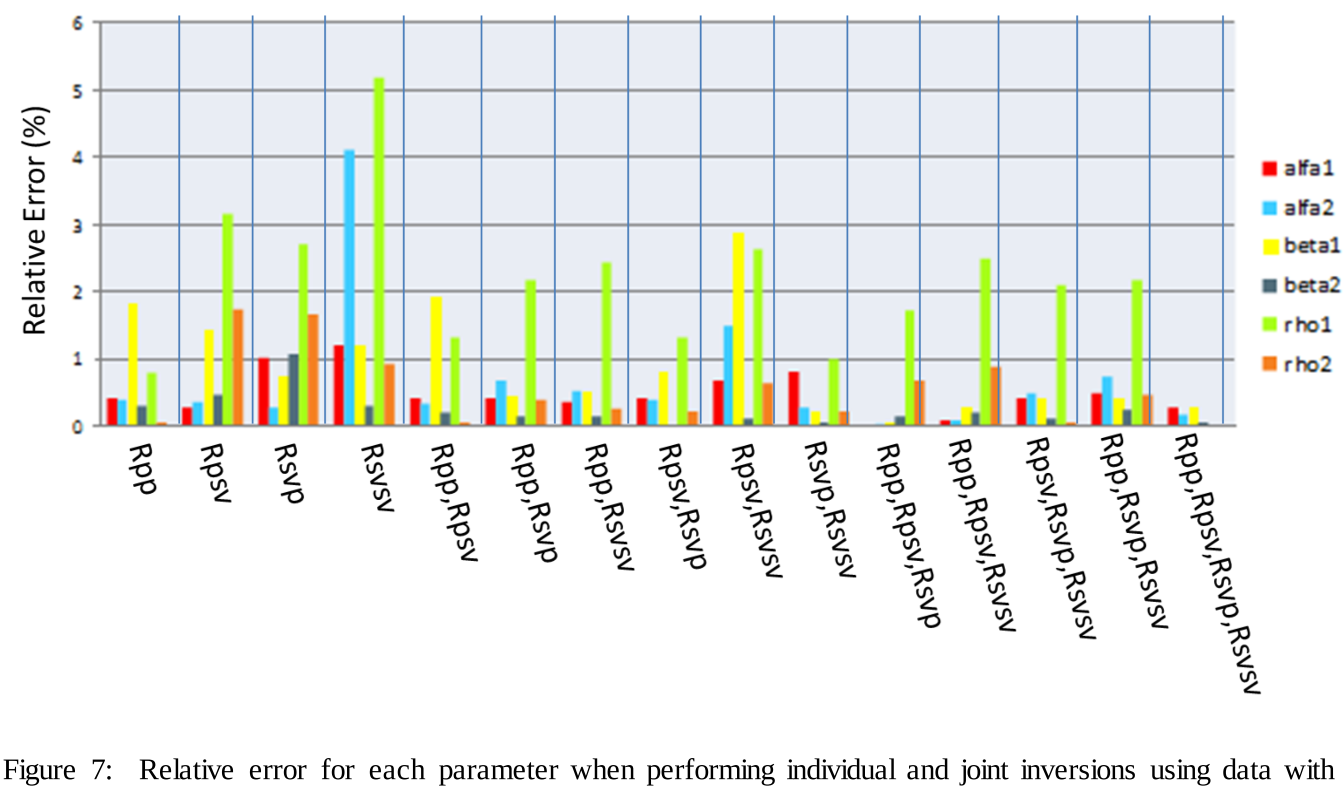

Figure 7: Relative error for each parameter when performing individual and joint inversions using data with noise.