# Multiple attenuation through independent component analysis: a case study

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## Abstract

**This paper discusses applications of Independent Component Analysis (ICA) to the removal of multiple reflections from seismic data. In the special case of surface-related multiples, the adaptive Surface Related Multiple Elimination (SRME) algorithm has shown to be an effective tool for such endeavor. Nonetheless, it uses an energy-minimizing criterion to remove the predicted multiples, which can incur in loss of primary events. This paper explores the use of another criterion, based on the statistical independence of the primary and multiple events to execute the subtraction. This criterion is expressed through the use of ICA. It is shown through two simple illustrative examples that the proposed method has the potential of improving the removal of the multiples, while maintaining primaries.**


## Introduction

In the method of seismic reflection, besides desired primaries, undesired events, such multiple reflections, refractions, diffractions ar also recorded making further processing and interpretation more difficult. Primary reflections contain pointwise information about subsurface interfaces and are thus widely used in seismic processing. Many popular algorithms in seismic processing, such as various migration and inversion schemes, require input seismic data that has primaries only. However, field-measured data contain non-primary interfering events, that may severely limit the application and performance of such algorithms. Elimination or, at least, attenuation of undesired events becomes, thus, a very important task in seismic processing. In that framework, multiple elimination, in particular those related to the water surface in marine data (referred to as surface-related multiples), is of paramount importance. Verschuur (2006) describes methods of multiple elimination based on moveout discrimination to separate primaries and multiples in different domains by applying suitable transforms. Most important examples of such methods include the parabolic Radon transform, the $f$–$k$ transform and the linear Radon transform ($\tau$–$p$ transform). In the transformed domains, it may be possible to mute regions associated with multiples and the transform the modified data back to the original domain, thus preserving the primaries and eliminating the multiples. However, it was readily recognized that multiples still leak to the unmuted regions, so that additional filtering is necessary to improve results with these methods.

A second class of methods is based on a two-fold procedure composed of prediction and extraction of multiples. Predictive deconvolution and wave-field extrapolation methods fall into this category, with a particular role in surface-related multiple elimination (SRME) schemes. In the present work, this method will be employed and extended.

Prediction of surface-related multiples using the recorded data was first described in Berkhout (1985) and was developed into a multiple elimination scheme in Verschuur (1991). However, for the subtraction of the multiples, that method needed a deconvolution of each shot, process, which was considerably time consuming. As a consequence, adaptive algorithms have been designed to automate the multiple removal part of the method (see Verschuur et al., 1992; Berkhout and Verschuur, 1997; Verschuur and Berkhout, 1997). The idea is to adaptively subtract the multiples from the data by minimizing the energy between the multiple prediction and the recorded data. The approach showed itself to be a viable option for performing surface-related multiple elimination.

A more recent approach of multiple elimination has been introduced by Lu (2006) and developed by Kaplan and Innanen (2008) and Donno (2011), proposing the use of independent component analysis (ICA) for multiple elimination. ICA concerns itself with viewing a signal statistically and trying to represent it as a (commonly) linear combination of mutually independent components. As this method does not rely on a minimum energy criteria, it has the possibility of preserving overlapping data more accurately. Examples reported in (see, for instance, Kaplan and Innanen, 2008; Donno, 2011), have confirmed the good properties of the ICA approach, indicating its good potential for multiple elimination, motivating the present study.

In the following, we use the ICA technique for further improvement after a conventional multiple attenuation scheme has been applied. He, our methodology consists of an SRME scheme followed by event separation using ICA. We apply the proposed approach to a simple synthetic and a field datasets, with encouraging results, also discussed in the paper.

## Surface-related multiple elimination

In this section the SRME algorithm is described in two dimensions. A detailed overview of the technique for one, two and three dimensions is given by (e.g. Dragoset et al., 2010; Verschuur, 2006). As stated in the introduction, SRME fits in the predict-subtract schemes of multiple removal techniques. The prediction part will be addressed first: Berkhout (1985) demonstrates that the primary reflections in the data, $\mathbf{P}_0$, can be expressed in terms of the recorded data, $\mathbf{P}$, and the source wavelet, $\mathbf{S}$. In the frequency domain, this is expressed in the following equation.

$$\mathbf{P}_0 = \mathbf{P}\left[\mathbf{I} - \mathbf{S}^{-1}\mathbf{P}\right]^{-1} \tag{1}$$

The above methodology has the disadvantage that the source wavelet must be known for each shot.

From the previous equation, an iteration to adaptively remove the multiples can be derived (Verschuur et al., 1992), being described by the following algorithm.

**Algorithm 1** Iterative 2D SRME

Choose $\mathbf{P}_0^{(0)} = \mathbf{P}$.
**for** $0 \le i < \infty$ **do**
Find $\mathbf{A}^{(i+1)}$ that minimizes $\|\mathbf{P} - \mathbf{P}_0^{(i)}\mathbf{A}^{(i+1)}\mathbf{P}\|$

$$\mathbf{P}_0^{(i+1)} \leftarrow \mathbf{P} - \mathbf{P}_0^{(i)}\mathbf{A}^{(i+1)}\mathbf{P}$$

**end for**

When the estimates for $\mathbf{A}^{(i)}$ are correct for all $i$, it is straightforward to note that the algorithm converges to $\mathbf{P}_0$. However, the minimum energy condition does not guarantee the convergence.

### Independent component analysis

The method of ICA has been historically motivated by the desire to separate mixtures of independent signals (Herault and Jutten, 1987). Mathematically, the problem can be described as follows: Given a vector of random variables $\mathbf{x}$ such that it satisfies

$$\mathbf{x} = \mathbf{A}\mathbf{s}, \tag{2}$$

for a certain unknown "mixing" matrix $\mathbf{A}$, recover the vector $\mathbf{s}$ of "sources" assuming that each of its coordinates is statistically independent from the others. Imposing a few assumptions on the matrix $\mathbf{A}$ and on the vector $\mathbf{s}$, it is possible to prove that this problem has a unique solution (Comon, 1994; Cao and Liu, 1996; Taleb and Jutten, 1999).

While there are many algorithms that aim to solve in practice the problem stated above, the one used throughout this work is the FastICA (Hyvärinen, 1999). The version of the FastICA considered in this work is based on the maximization of nongaussianity, which in turn can be shown to minimize the mutual information between the sources (Hyvärinen et al., 2001). Below, we provide a brief overview of how FastICA estimates the extracting vector $\mathbf{w}_m$, which provides the $m$-th source — in our problem, there are only two sources to be retrieved, being one of them associated with the primaries and the other one with the multiples. Therefore, after performing the FastICA, the second source is found by suppressing the contribution of the first estimated source to the mixtures (see Hyvärinen, 1999, for details).

### Improving adaptive multiple attenuation using ICA

In order to use ICA, one must first define a model that fits the ICA framework. In this work, the linear framework is being used, and thus a system of linear equations must be derived.

The first equation results from the fact that the total data $\mathbf{P}$ is a superposition of primary events, $\mathbf{P}_0$, and multiples

**Algorithm 2** FastICA

Whiten $\mathbf{x}$ to become $\mathbf{z}$. Choose vector $\mathbf{w}_0$ randomly.
**while** $\nabla\mathbf{w}_n \neq 0$ **do**

$$\mathbf{w}^+ \leftarrow \mathrm{E}[\mathbf{z}({\mathbf{w}_n}^T\mathbf{z})^3] - 3\,\mathrm{E}[({\mathbf{w}_n}^T\mathbf{z})^2]\mathbf{w}_n$$

$$\mathbf{w}_{n+1} \leftarrow \frac{\mathbf{w}^+}{\|\mathbf{w}^+\|}$$

**end while**

reflections, $\mathbf{M}$.

$$\mathbf{P} = a_{11}\mathbf{P}_0 + a_{12}\mathbf{M} \tag{3}$$

As stated previously, the adaptive filtering may remove primary reflections from the data. Thus, if we denote $\tilde{\mathbf{M}}$ as the estimated, filtered multiples obtained from applying Algorithm 1 one may state that

$$\tilde{\mathbf{M}} = a_{21}\mathbf{P}_0 + a_{22}\mathbf{M}. \tag{4}$$

This equation can be interpreted in the following manner. Assuming that the adaptive filtering removed primaries, these primaries must have been contained in $\tilde{\mathbf{M}}$. Given this, one can ascertain that the filtered multiples contain not only the true multiples, but also a residual of the primaries.

Equations 3 and 4 can be expressed succinctly in the following model

$$\begin{bmatrix}\mathbf{P}\\ \tilde{\mathbf{M}}\end{bmatrix} = \mathbf{A}\begin{bmatrix}\mathbf{P}_0\\ \mathbf{M}\end{bmatrix}, \tag{5}$$

where $(\mathbf{A})_{ij} = a_{ij}$.

The previous equation describes a relationship between known data, $\mathbf{P}$ and $\tilde{\mathbf{M}}$ and the sought-after quantity $\mathbf{P}_0$ in such a way that is tractable by the ICA framework.

This linear model is similar to the one employed by Donno (2011), but it is not the only possible one. Other authors such as Lu (2006) and Kaplan and Innanen (2008) consider different mixtures.

### Results

This section shows results that are obtained by applying Algorithm 2 to different datasets. The datasets used will be a synthetic and a real dataset.

It consists of two flat, horizontal, reflectors positioned at 300 m and 1000 m, respectively. The first set is a synthetic model generated with a 2D finite difference modeling tool from the *Madagascar* package.

A common shot panel of is displayed in Figure 1. The muted direct wave has been removed in accordance with the needs of SRME. The white dashed arrow shows a region where the multiple reflection overlaps the primary.

Figure 2 shows the common shot panels with attenuated multiples resulting from applying Algorithm 1 to the data. The arrows indicate regions where this is considerable difference between the SRME panel and the ICA panel.

The black arrow shows that the first multiple event from the first reflector was more attenuated after the application of the ICA algorithm. One notices that the event that persists unto the zero offset in the SRME section is eliminated fully

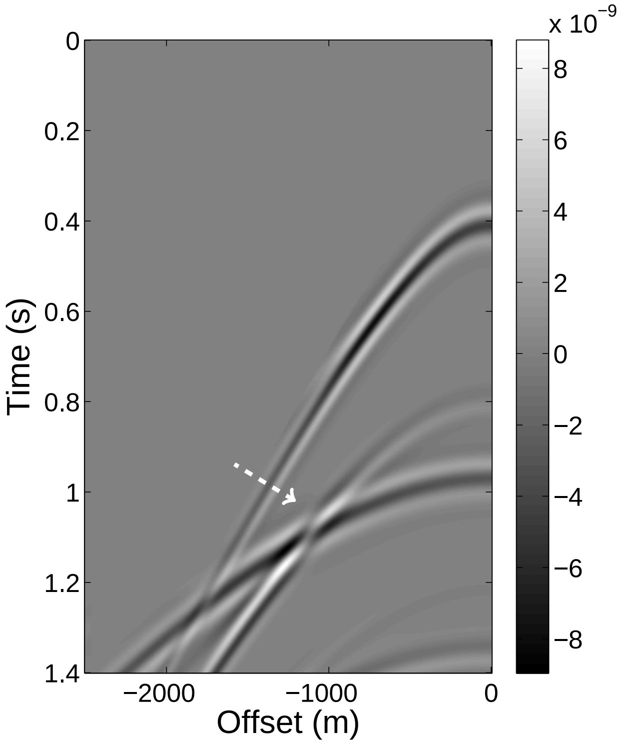


Figure 1: Synthetic common shot with muted direct wave.

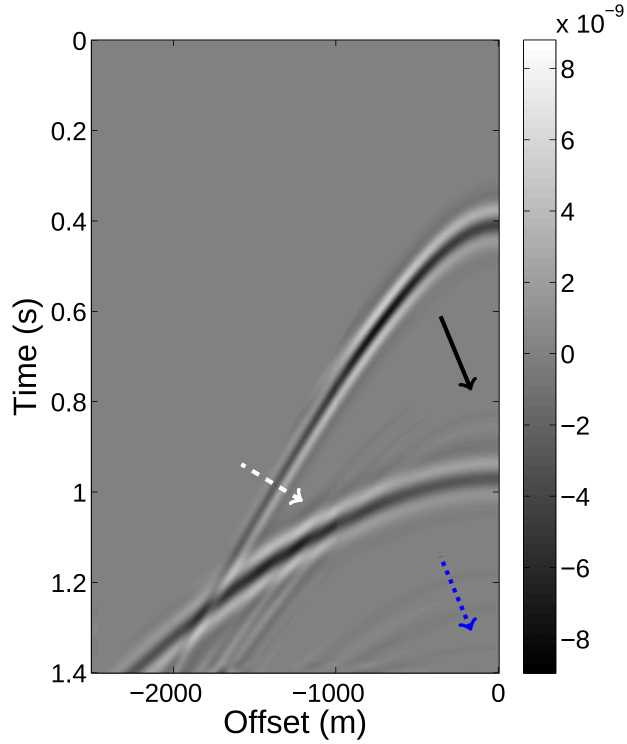


Figure 3: Primaries recovered using ICA.

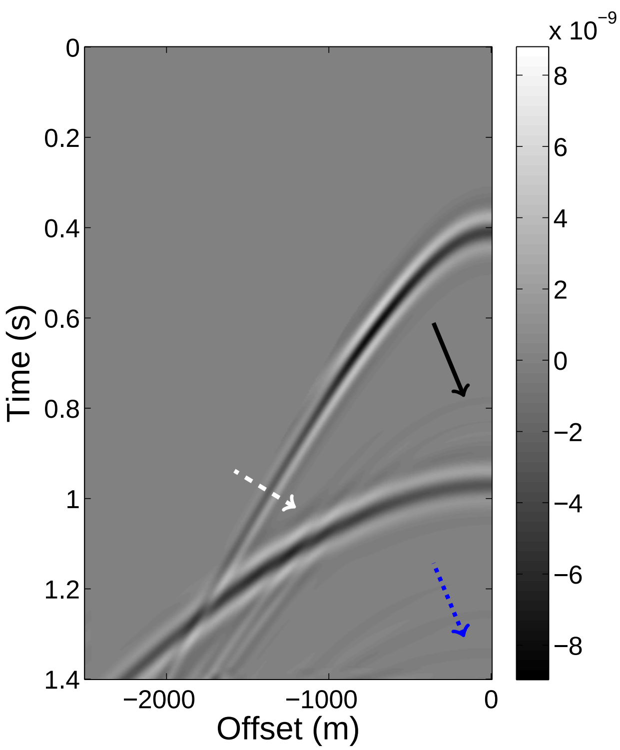


Figure 2: Primaries resulting from adaptive SRME.

approximately up to the 500 m offset. This is substantial a substantial improvement, specially given that the reflector is positioned at 300 m. The blue arrow shows a similar result for the first multiple of the second reflector. The event in the ICA panel is considerably thinner, and thus more attenuated, than that in the SRME panel. Finally, the white arrow indicates the overlapping section of the panels. One observes that the amplitudes in the ICA panel are higher than that of the SRME. This means that ICA has managed to preserve more primary events than the adaptive filtering. Following the synthetic data, a real dataset was used. The data was acquired over the Jequitinhonha Basin, located offshore Brazil. It features a regular geometry with spacing between consecutive receivers and sources of 25 m. The direct wave has been muted and the offsets up to the zero offset have been completed, necessary operations to apply SRME.

Figure 4 shows a windowed difference panel between the ICA-extracted and SRME primaries. The window focuses on the first multiple events, arriving at around 3.4 s. Thus above the event indicated by the white arrow, there are only primary waves. Since the amplitudes above this event are near zero, one concludes that the ICA and SRME sections have very similar amplitudes for primary events, that is, they agree where no multiple events exist.

The event that the white arrow indicates is the first multiple of the seafloor. Its amplitude in the difference section is negative. This shows that the ICA-extracted section has less amplitude than the conventional SRME section over that event. This in turn signifies that ICA has attenuated this multiple better. A similar analysis can be made about other multiple events on the panel. Thus, one can affirm that ICA preserved the same primaries as SRME but attenuated the multiples more.

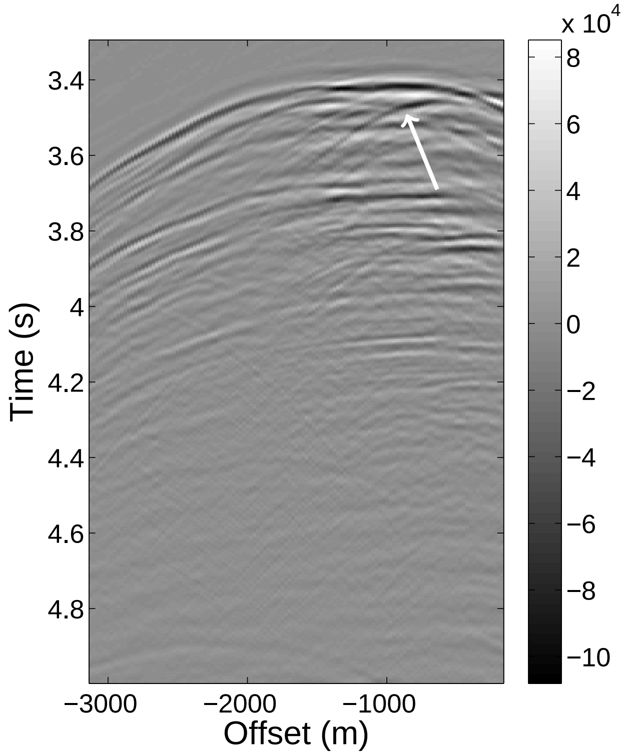


Figure 4: Windowed difference panel between ICA-extracted and SRME primaries.

### Conclusion

Surface-related multiple elimination has proved to be exceptional at predicting multiples in a data-driven fashion, but lacks the same outstanding performance when removing them. The most common form of performing this subtraction is by using adaptive filters to attenuate multiple reflections in the data. This has the advantage that the process becomes, in theory, entirely automated, but has the disadvantage that it may harm primary reflections.

Using a linear model to describe the relationship between primaries, multiples, SRME predicted multiples and the total data, the problem of recovering primary reflections becomes tractable by the ICA technique. By using an independence criterion to distinguish between primaries and multiples, ICA can recover the primaries with less harm than adaptive subtraction. The results displayed above positively show this. Both in the synthetic data and in the real data, it is observed that the ICA extraction of the primaries have more attenuated multiples. The synthetic data also shows that in an intersection between a primary and a multiple, the amplitude of the primary was preserved.

Being a relatively young addition to the field of geophysics, further work must be carried out in order to perfect the technique. Nevertheless, these and other preliminary results already attest to the usefulness of ICA in the problem of multiple attenuation.

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