Comparing 1D and 2.5D AEM inversions in 3D geological mapping using a new adaptive inversion solver

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Abstract

The advantages of 2.5D airborne electromagnetic inversion in 3D geological mapping applications compared to the more commonly used CDI transforms or simple 1D inversions are described using an example from the Bryah Basin in Western Australia.

We demonstrate this using a substantially rewritten version of ArjunAir (Wilson et al., 2006), a product of the CSIRO/AMIRA consortia (project P223F).

The ArjunAir inversion solver has been replaced with a new GSVD (Paige, C. C. et al, 1981) solver, with adaptive regularisation which also incorporates a misfit to the reference model and a model smoothness function.

The ArjunAir forward modelling code has been revised to fix two errors which manifest at late times around high resistivity discontinuities and in steep topography.

The software has been parallelised using Intel MPI.

We allow the use of a starting or reference geology/ resistivity model to influence the inversion.

The software is implemented in a 3D geological modelling package (McInerney et al, 2005) with an intelligent graphical user interface for inversion setup, for introduction of geological reference models and for visualising results. Apparent Resistivity, 2.5D Forward and 1D and 2.5D Inversion methods are integrated in one 3D geological and potential field gravity and magnetics inversion environment.

Introduction

ArjunAir (Wilson et al., 2006), is a computer program for modelling and interpretation of geophysical airborne electromagnetic (AEM) data from a single profile using a two-dimensional (2D) model of electrical resistivity and susceptibility. ArjunAir was originally developed by Drs. Glenn Wilson, Art Raiche and Fred Sugeng for the CSIRO/AMIRA consortia (project P223F). It became public domain software http://p223suite.sourceforge.net/ in 2010.

Airborne EM (AEM) data can be both forward modelled and inverted using 2.5D modelling provided that the geoelectrical cross-section is relatively constant along a strike length that exceeds the AEM system footprint.

The technology is realised using a numerical approximation afforded by the 2D finite-element method. This enables the accurate simulation of 3D source excitation for full domain models inclusive of topography, non-conforming boundaries and very high resistivity contrasts.

This work has been the subject of much review and use by the community since these times. It has fallen out of favour as some of the claimed capability could not be realised. This becomes evident in areas where the geology exhibits high lateral resistivity contrasts.

Also, the original inversion scheme was not always stable.

We report on efforts to overcome these ArjunAir weaknesses and also describe the addition of a new inversion solver which is now integrated into a full 3D structural geology modelling/mapping environment.

We choose to show a case study from Western Australia, based upon a SPECTREM reconnaissance survey (Munday et al, 2013).

Both frequency and time domain AEM survey systems from the major commercial providers have been shown to produce significant geological detail, including the ability to indicate not just planar dipping features, but also near surface synclines and anticlinal features.

Methods

EM Modelling

2.5D modelling is based on a full wave solution to Maxwell's equations using a frequency-domain, spatial Fourier domain finite element method (Sugeng et al, 1992). In the spatial Fourier domain, Maxwell's frequency-domain equations reduce to two coupled partial differential equations for the along strike components of the secondary electric and magnetic fields. These coupled equations are solved using an isoparametric finite-element method with quadratic basis and test functions.

This allows the mesh to conform to topography and heterogeneous geoelectrical regions with curved or sloping boundaries. The implicit continuity of the along strike components across discontinuous resistivity boundaries in the conventional 2D finite-element scheme ensures numerical stability and accuracy when modelling problems with extremely high resistivity contrasts.

Homogeneous Dirichlet boundary conditions are implemented to preserve the sparse, small bandwidth structure of the coefficient matrix formed by the collection of the elements in the mesh.

The overall matrix is never explicitly formed in this original implementation as the progressive Frontal Solution method (Irons, 1970) is used. This method was designed to minimize "core memory" at the expense of speed.

The core EM finite element is designed to have 8 nodes around the edge of an arbitrary quadrilateral. This element is solved for 21 spatial transform values logarithmically spaced from 10^{-5} m^{-1} to 0.1 m^{-1}. Depending on the mesh size, the along strike fields for each additional transmitter position can be computed for less than 2.5% of the computational cost of the initial decomposition of the coefficient matrix.

The frequency-domain field components and sensitivities are initially computed at appropriate nodes in the Fourier domain from shape function interpolation and/or differentiation of the along strike field components. The Fourier domain fields and sensitivities are then splined and appropriately Fourier transformed into the Cartesian domain. For accuracy, computations are based on pseudo-receiver positions that correspond to nodes in the finite-element mesh. These fields and sensitivities are splined and interpolated onto actual receiver positions for the frequency-domain response.

For time-domain modelling, the model response and sensitivity are computed from the fields and sensitivities at 28 frequencies logarithmically spaced from 1 Hz to 100 kHz. These are splined and extrapolated back to zero frequency. The response and sensitivity are computed out to several pulse lengths and then folded back into one, and differentiated if necessary, before being convolved with the transmitter waveform in time-domain.

Deficiencies in the published code

Several authors have published on this same work, noting difficulties (Pirttijärvi, M., 2014 and Belliveau, P. et al, 2014). The modified original codes are not generally in the public domain, nor has there been a true consensus to date on the original deficiencies. The following list, however, is indicative

- Inability to handle sub-vertical lateral high resistivity contrasts. This problem, illustrated in Figure 3, is most apparent in the case shown but gives erroneous solutions for most other models.

- The forward model is inaccurate in the presence of high topographic relief (gradients >20%), because an incomplete set of terms was used to describe the topographic effect.

Only simple rectangular mesh, layered earth case studies are published and do not reflect the original deficiencies. These deficiencies stem from two algorithmic bugs that were not identified in the original testing. New tests reflecting some physical modelling of graphite blocks with stepped geometry were referenced to confirm expected outcomes. The original inversion strategy has been replaced in most other implementations to improve stability.

Modifications to the published code

In our case, further work has been done on these codes as follows:

- optimize time taken to solve large systems using an MPI strategy (deploy multiple processors in parallel). The aim has been to make 1D inversion close to a real time process and to speed up 2.5D inversions so that results are available within a few hours.

- allow the use of a starting or reference geology model to influence the inversion.

- add a completely rewritten Inversion Solver based upon an L2 norm objective function which also includes the misfit to the reference model and a model smoothness function.

- The solver uses a Generalized Singular Value Decomposition (GSVD) method (Paige, C. C. et al, 1981), where generalized singular values of the sensitivity and model norm matrices are used as weights in determining changes in the cell conductivities at each iteration.

- An adaptive Tikhonov regularization scheme is used to solve for changes in the conductivity model at each iteration. An RSVT (Relative Singular Value Truncation) parameter allows dampening of changes in non-sensitive cell conductivities for each data point during the initial stages of the solution.

Solution for model (m) changes at (n+1) iteration is:

$$(G^T G + \beta L^T L)\delta m$$
$$= G^T \delta d - \beta L^T L\big(m^{(n)} - m_0\big)$$

Where:

G = Sensitivity matrix: $G_{ij} = \frac{\partial d_i}{\partial m_j}$

L = Model Roughness or Model Norm Matrix

$$(m - m_0)^T L^T L(m - m_0) = \varphi_m(m - m_0)$$

$$\varphi_m\left(\underline{m}, \underline{m_0}\right) = \alpha_s \iint (m - m_0)^2 \, dxdz$$

$$+\alpha_x \iint \left(\frac{\partial(m - m_0)}{\partial x}\right)^2 dxdz$$

$$+\alpha_z \iint \left(\frac{\partial(m - m_0)}{\partial z}\right)^2 dxdz$$

β = Regularisation parameter

$\delta m = m^{n+1} - m^n$

δd = data misfit at the nth iteration

m_0 = Reference model

m^n = Model at the nth iteration

Regularisation parameter β and the inversion solution are determined by using GSVD on matrices G and L1. L1 is one of the solutions for the model norm matrix L which has an infinite number of solutions. It has an infinite number of solutions because only $L^T L$ is known. It can be shown however that using any other of the infinite number of solutions for model norm matrix L, results in the same GSVD decomposition.

Test results

The original published results for some models are re-produced, followed by improved results from the new inversion strategy described above.

The model below consists of two conductive 100m cubes embedded at 50m depth in an otherwise uniform 1000 Ωm half-space. One of the targets has a resistivity of 1 Ωm and the other has a resistivity of 10 Ωm.
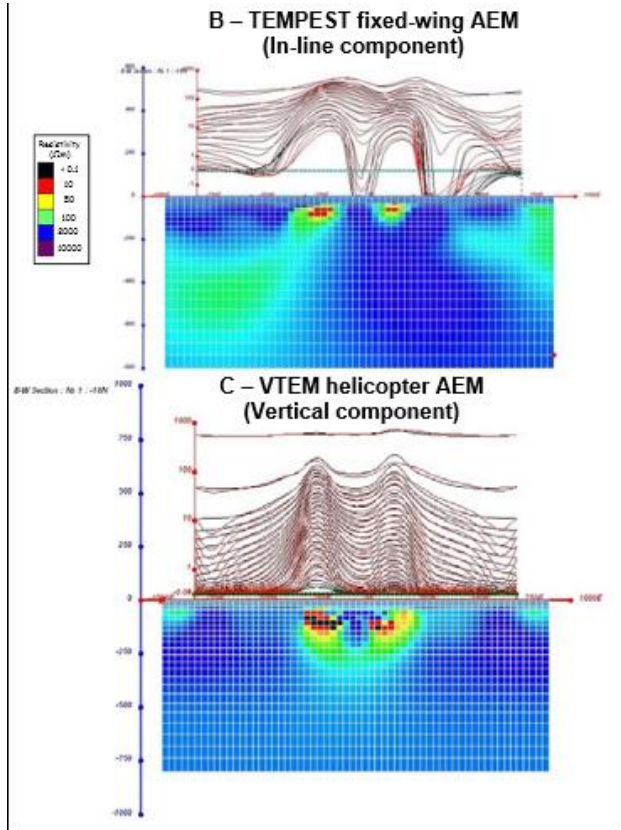

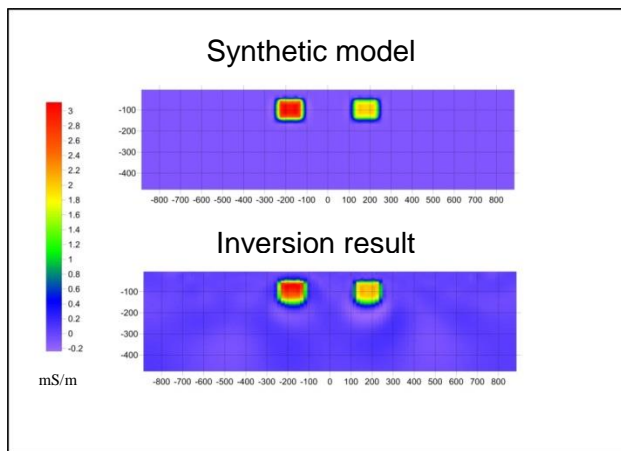Figure 1. Published CSIRO 2.5D ArjunAir inversion results are shown above.

Figure 2. New 2.5D inversion results for the same synthetic model are shown above.

Old and new forward results for a model with sub-vertical, lateral high resistivity contrasts, where conductive cover overlies a resistive basement, are illustrated in Figure 3.
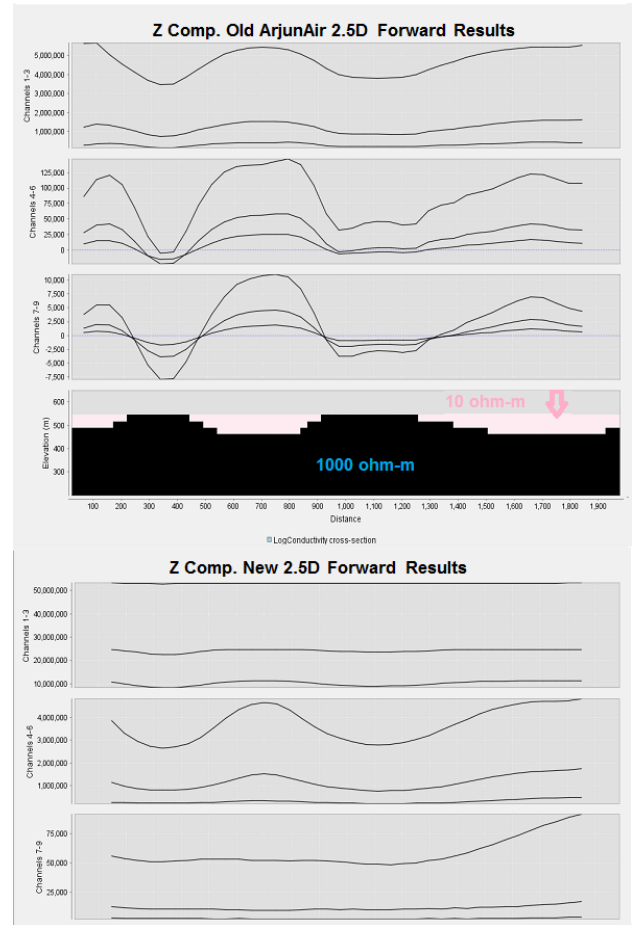

Figure 3. 2D Forward Modelling results of conductive cover over a resistive basement.

The high amplitudes and negative later time responses in the old ArjunAir results demonstrate the original problem.

User Interface

The user must specify the EM survey system and configure a specification of the necessary units, channels, signal type and response curves. The aim is to quickly unify any/all systems to a common SI units system and simplify the way any system is described in terms of layout of transmitter and receiver, waveforms used, on-off times etc.

- The actual delivered geophysical database can be used directly, in its binary form.

- Each EM profile includes the observed responses, the clearance above the ground, the vertical elevation of the ground, and (optionally) a magnetic measurement (eg TMI).

- A depth section within a 3D geological modelling space is automatically created for each EM profile.

- A simple wizard with a preview window and section navigation capability is provided to show the observed profiles in frequency grouped order, together with either forward or inverse calculations,

and the depth section with geology or resistivity/conductivity responses.
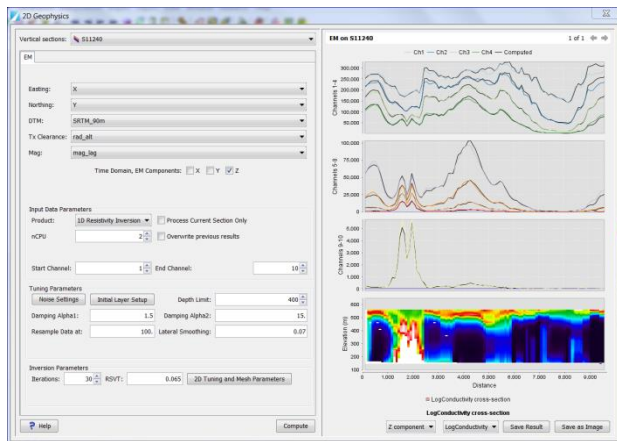

Figure 4. Inversion Setup and Profile Viewer

Field example – Bryah Basin AEM Survey

Survey system

The Bryah Basin AEM survey was flown with the SPECTREM$_{2000}$ fixed wing AEM system, Leggatt et al. 2000, with ~5.2km line spacing orientated N-S. The SPECTREM$_{2000}$ is a fixed wing, time domain AEM system employing a bipolar, 100% duty cycle, and a square-wave current pulse which operates at variable base frequencies of 25 Hz and higher. It has a peak moment of 400,000Am^2. These specific characteristics imply that the transmitted current pulse is coupled with ground response so further processing is needed in order to separate the secondary field response. Both X- and Z-component data are recorded and at each station the EM data is deconvolved to remove system response, stacked, transformed to a step response and then binned into 10 time windows (window times: 0.026 – 16.65ms). In this processing scheme, the last window of the decay is subtracted from all the earlier windows in an attempt to remove the transmitted primary present in the recorded response.

Geology

The Bryah Basin is part of the 2.0 to 1.8 Ga Capricorn Orogen separating the Yilgarn and Pilbara Cratons in northern Western Australia. It contains a succession of mafic and ultramafics overlain by clastic and chemical sedimentary rocks. The Basin is host to significant mineralisation, including the DeGrussa and Horseshoe Cu-Au VMS deposits.

The VMS mineralising environment at DeGrussa has been confirmed over a +30km long, 2km wide corridor, which has seen minimal exploration below 100m depth. The existing DeGrussa deposits have a strike length of just 1.2km within this broader 30km corridor.

Structural interpretation from mapping within the underground mine and open pit has proved to be invaluable in improving Sandfire's understanding of the lithological sequence, structural setting and, consequently, the positioning of potential accumulations of VMS mineralisation, giving the geological team a

unique level of insight into the most likely areas where ore zones could occur.

Among the challenges in the study of, and exploration for, these mineral systems is the paucity of outcrop and the extent and variability of a complex regolith cover.

Project scenario

The main purpose of the Bryah Basin AEM survey was to stimulate mineral exploration by mapping the very conductive carbonaceous / graphitic / BIF / iron rich sediments which are present under the regolith or at depth in this area.
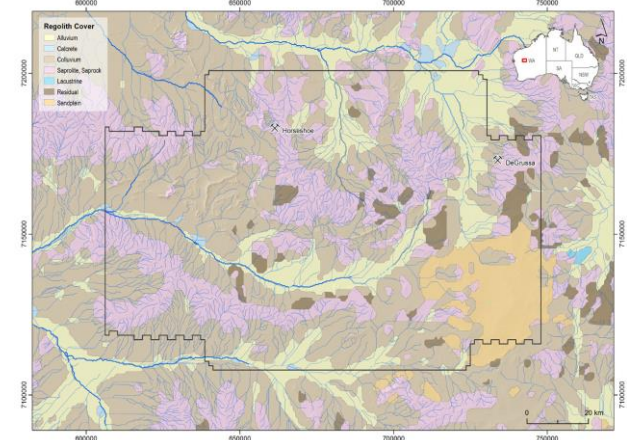

Figure 5. A map of regolith materials across the Bryah Basin. The SPECTREM survey area is outlined by the black polygon. The area is extensively covered by transported cover.

Results

Both 1D (Z component) and 2.5D (X and Z component) inversions were conducted over a survey subset of 3 lines in the DeGrussa neighbourhood.

On sections 11240, 11250 and 11260 a buried conductor at ~100m depth is very similar to anomalies associated with the known VMS copper deposits in the area.

The original SPECTREM CDI Resistivity sections are shown in Figure 8 and are generally similar to the 1D inversions but exhibit poorer depth resolution.

The dramatic improvement from CDI's to 1D to 2.5D inversion in deriving complex geological structure at depth for conductors with complex geometries is illustrated in Figure 7, 8 and 9.

We show a small 3 line subset of this survey in the vicinity of the DeGrussa mine over the 100k published geology. A mapped synclinal feature is clearly visible associated with the high apparent conductivity anomalies on SPECTREM lines 11250 and 11260. It disappears under cover on line 11240 but high conductivities are still associated.

2.5D inversion misfit graphs of RMS Error % and Model Norm appear in Figure 10. Misfit errors of 11, 8 and 18% are very good for this degree of geological model complexity.
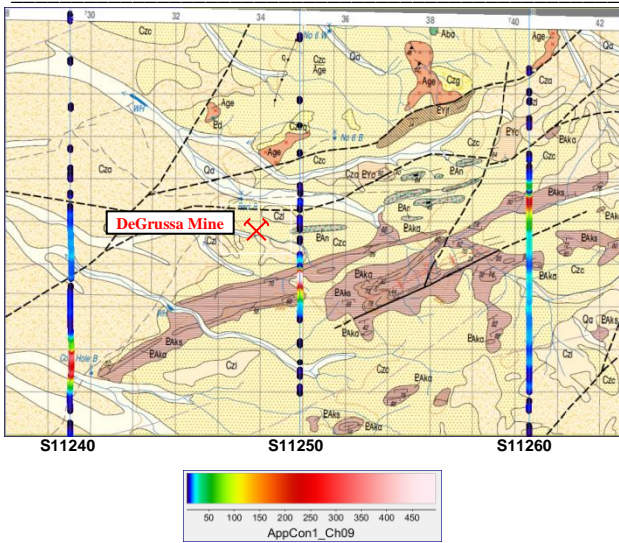
Figure 6. Apparent Conductivity SPECTREM Ch9 on 1:100K Surface Geology Map
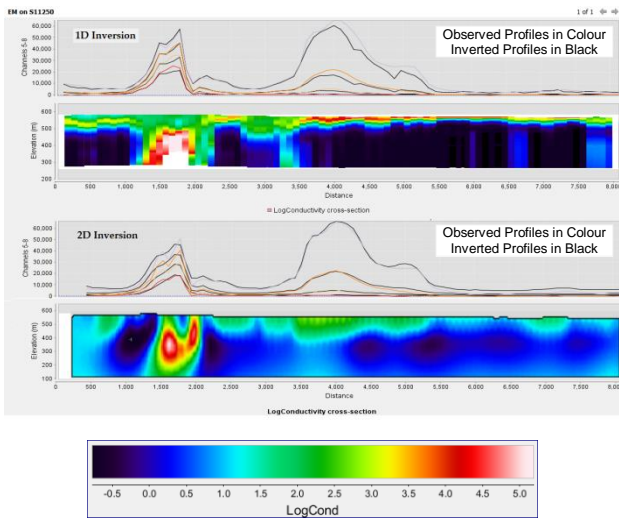

Figure 7. 1D & 2D Inversion Z Component, Ch6 to Ch8 Profiles and Conductivity Sections, S11250; 2D Misfit 8%
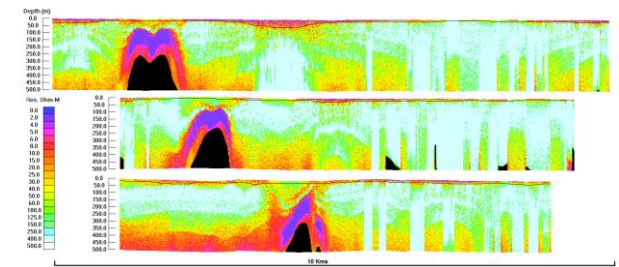

Figure 8. SPECTREM CDI, Resistivity Sections, S11240, S11250 and S11260
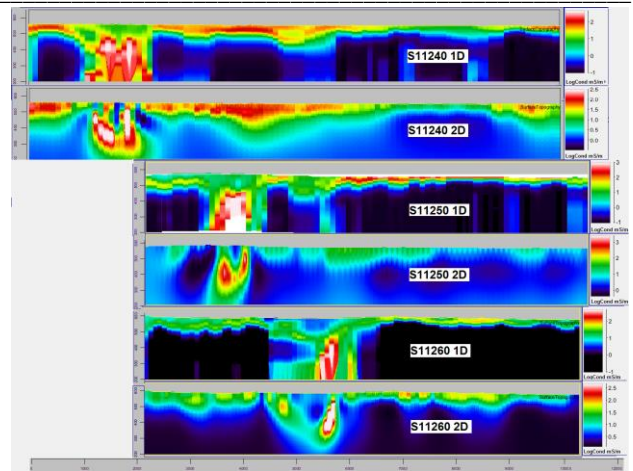

Figure 9. Comparing 1D and 2.5D Inversions, Conductivity Sections S11240, S11250 and S11260
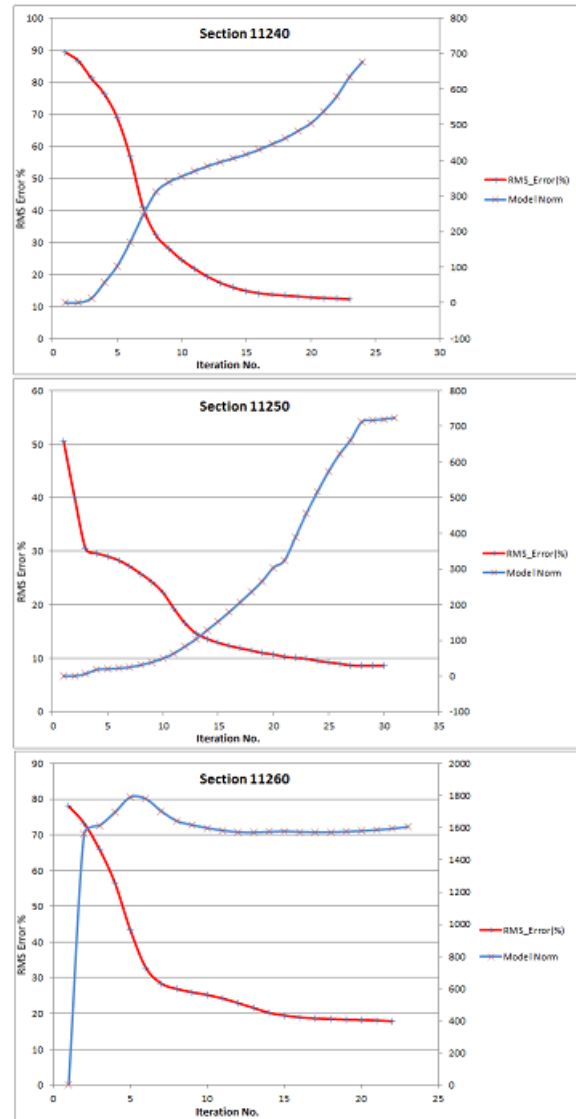

Figure 10. 2.5D Inversion Convergence Graphs for lines S11240, S11250 and S11260 (last with reference model). Note the early plateauing of Model Norm and faster convergence for S11260 even though it is 1.6kms longer.

Conclusions

- The ArjunAir, 2.5D inversion program was substantially rewritten, problems were fixed in the forward code and a new adaptive inversion solver was implemented.

- The new program was parallelised using Intel MPI. The aim was to make 1D inversion close to a real time process and to speed up 2.5D inversions so that results are available within a few hours.

- The new implementation was tested against previous test models and found to produce superior results with fewer artifacts, Figs. 2, 3.

- The problems fixed in the forward segment have been tested against some simple bench models and synthetic examples and their accuracy confirmed, Fig. 3.

- We have compared CDI's, 1D and 2.5D inversion results for 3 SPECTREM lines from the Bryah Basin survey in the DeGrussa deposit area and demonstrated much improved definition of 3D geological structure in the 2.5D conductivity sections, Figs. 7, 8 & 9 .

- The inverted structure is compatible with the 100k geological mapping of the outcrop on two of the survey lines. The third line shows similar structure under cover to the west.

- Applying geological/conductivity constraints to the inversion by means of a reference model derived from the 1D inversion does not significantly change the outcome in this case but improves the 2.5D inversion convergence speed by up to 50%.

Acknowledgments

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