On seismic absorption correction

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Abstract

Seismic profiles suffer from gradual decrease in resolution due to the medium absorption. This loss is particularly severe in the deepest layers of the seismic image, preventing the detection of thin layers. Several techniques have been proposed to compensate from the attenuation and the dispersion of the wave, among which several variants of the inverse Q filtering. Such methods usually require processing power, while more direct signal filters achieve similar quality with algorithms faster by several orders of magnitude. The present work aims at comparing and analyzing the computational complexity of three existing methods of for absorption compensation: inverse Q filtering, truncated inverse Q filtering and translated IIR filters. Improvements on the gain and execution speed of translated IIR filters are proposed, compared and experimented on real data provided by the *Agência Nacional do Petróleo, Gás Natural e Biocombustíveis*.

Introduction

Seismic data analysis (Yilmaz, 2001; Rosa, 2010) aims at producing a faithful image of the depositional layers of the earth subsurface. However, the deepest layers are inherently blurred due to the medium absorption of the seismic wave.

Following the seminal model of Kolsky (1953), this absorption alters the wave by attenuation of its energy and dispersion of its velocity. The effect of the absorption increases with the frequency of the wave (Kolsky models the increase as linear), effectively removing the highest frequencies, and thus the ability to detect very thin and deep layers.

Compensating for the absorption can be performed on a limited bandwidth basis (removed frequencies cannot be recovered) by correcting the dispersion and amplifying the signal according to its spectrum. In real data with full compensation, the amplitude of high frequencies would be multiplied by several order of magnitudes. However this would also amplify acquisition noise (which has typically with high frequencies), inverting the signal-to-noise ratio! This restricts practical seismic processing to only partially compensate for the attenuation.

Such compensation can be derived directly from Kolsky's model, a process referred to as *inverse Q filtering* (Gelius, 1987; Wang, 2008). This can be effectively computed by approximating instant frequencies using windowed Fourier transform (Hargreaves and Calvert, 1991), as described at page 2. Further speed improvements may be achieved using truncation of the convolution kernels, *cf.* page 2 (Varela et al., 1993).

Another approach benefits from the similarity of the medium absorption as reflection/refraction division of the wave energy and the recursive formulation of Infinite Impulse Response (IIR) filters, as detailed at page 3 (Duarte, 1993). Such approach is orders-of-magnitude faster than inverse Q filtering, and its gain can be modified to match truncated convolution kernels (see page 4). Moreover, we propose here an improvement on the implementation of the method (page 4) to improve the speed performance of the algorithm, maintaining the same accuracy.

Contributions. In this work, we relate the three models above (inverse Q filter, truncated inverse Q filter and translated IIR filter) and analyzes their computational complexity. We use this relation to devise gain parameters for the IIR filters from approximation of Kolsky's model, as suggested in Rosa (2010). We also propose a modification of that method to reduce its computational complexity. Finally, we test the performance of those methods on a marine seismic line of Sergipe-Alagoas Basin, provided by the *Agência Nacional do Petróleo, Gás Natural e Biocombustíveis* (ANP). The line was reprocessed (by non-professional geophysicists), without and with the different corrections of the losses by absorption applied before stacking. We observe an increase in the resolution of the geological subsurface in all methods tested in comparison with the data without correction and confirm the speed advantage of the original and modified IIR approach. This work is part of the first author's dissertation (Ribeiro, 2015).

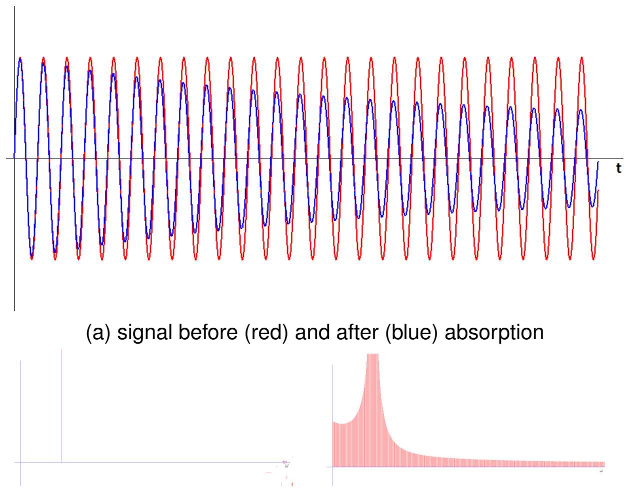
Kolsky's attenuation-dispersion model

The reference model for seismic absorption remains Kolsky's attenuation-dispersion model (1953), which can be formulated as a filter of the instantaneous frequency amplitudes $S(\omega)$ of the seismic signal while traversing a layer between depths τ and $\tau+\Delta\tau$ (Rosa, 2010):

$$S(\tau+\Delta\tau,\omega) = \exp\left(-\frac{\omega\Delta\tau}{2Q} - i\frac{\omega\Delta\tau}{\pi Q}\ln\frac{\omega}{\omega_0}\right)\cdot S(\tau,\omega), \quad (1)$$

where ω_0 is the reference frequency and Q is the anelastic attenuation factor, usually referred to as *Quality factor*.

In the above equation, the exponential decay factor $\exp\left(-\frac{\omega\Delta\tau}{2Q}\right)$ models to the energy attenuation and the phase shift $\exp\left(-i\frac{\omega\Delta\tau}{\pi Q}\ln\frac{\omega}{\omega_0}\right)$ models the velocity dispersion. This model applied to a synthetic signal made of a single sine is illustrated in Figure 1.



(a) signal before (red) and after (blue) absorption

(b) original spectrum (c) spectrum after absorption

Figure 1: Direct simulation on a synthetic signal made of a single frequency, and its effect on the spectrum.

Inverse Q filtering

Kolsky's model cab be used to compensate for the absorption by applying the inverse filter, a method which is called inverse Q filtering (Gelius, 1987; Wang, 2008). The inverse filter is obtained by simply inverting the signal of the exponential arguments (as long as the bandwith of the signal is preserved) in Equation (1):

$$S(\tau+\Delta\tau,\omega) = \exp\left(+\frac{\omega\Delta\tau}{2Q} + i\frac{\omega\Delta\tau}{\pi Q}\ln\frac{\omega}{\omega_0}\right)\cdot S(\tau,\omega)\,. \quad (2)$$

In order to implement such filter, one would ideally measure the instantaneous frequencies of the signal, which is impossible with a single profile. However, this can be approximated by windowed Fourier transforms of the signal (Hargreaves and Calvert, 1991), as summarized in the algorithm of Figure 2. The computational complexity of this algorithm is of order $O\left(N^2 \log N\right)$, where N is the number of samples since the algorithm computes a Fourier Transform for each sample.

```
// N - size of the inputs
// X - input seismic profile
// Y - output seismic profile
// Q - anelastic attenuation factor
// w - reference frequency
{
  fftw_complex X_[N], Y_[N] ;
  fft(N, X, X_) ;

  for(int i = 0; i < N ; ++i)
  {
    for(int j = 0; j < N; ++j)
    {
      double k = ((0.001*j)/w ;
      Y_[j] = exp( (i*j)/Q * (0.5 + I*log(k)/PI)
    ) * X_[j]  ;
    }
    fft_inv(N, Y_, Y) ;
  }
}
```

Figure 2: Inverse Q filtering pseudo-code.

Figure 3 shows the simulation of the absorption with Equation (1) and the correction with inverse Q filtering on a sum of sinusoidal signal.

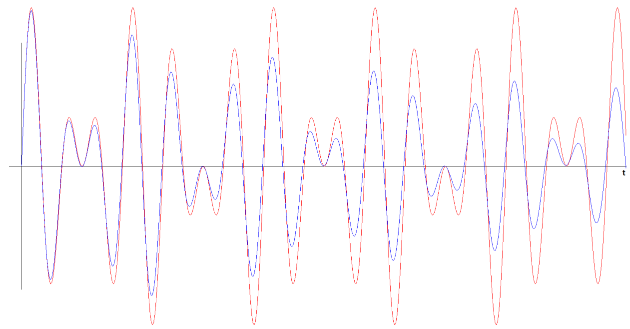


Figure 3: Simulated absorption signal (blue) and the result of inverse Q filtering for the same Q (green), matching almost exactly the original sum of sinusoidal signals (red).

Approximate inverse Q filter by truncation

It is possible to process the original signal $s(t)$ from its frequencies $S(\omega)$ directly translating multiplication in the Fourier domain by convolutions:

$$s_{\tau+\Delta\tau}(t) = \text{InvFourier}\left(\exp\left(\frac{\Delta\tau}{2Q}\cdot G(\omega)\right)\right) * s_\tau(t),$$

com $G(\omega) = \omega + \frac{2i}{\pi}\omega\ln\frac{\omega}{\omega_0}$.

In order to improve the speed of the previous algorithm, Varela et al. (1993) proposed an approximation of inverse Q filters that limits the number of convolutions required, truncating the Taylor serie of the exponential factor:

$$\exp\left(\frac{\Delta\tau}{2Q}\cdot G(\omega)\right) \approx 1 + \frac{\Delta\tau}{2Q}G(\omega) + \frac{1}{2}\left(\frac{\Delta\tau}{2Q}G(\omega)\right)^2 + \frac{1}{3!}\left(\frac{\Delta\tau}{2Q}G(\omega)\right)^3 + \frac{1}{4!}\left(\frac{\Delta\tau}{2Q}G(\omega)\right)^4 + \dots$$

If $g(t)$ is the inverse Fourier transform of $G(\omega)$, we obtain for $\tau = 0$:

$$s_{\Delta\tau}(t) = s(t) + \frac{\Delta\tau}{2Q}(s*g)(t) + \frac{1}{2}\left(\frac{\Delta\tau}{2Q}\right)^2 (s*g*g)(t) + \dots \quad (3)$$

According to Varela et al. (1993), $g(t)$ is given by:

$$g(t) = \begin{cases} \frac{1}{4} & , \text{ if t = 0} \\ \frac{-8}{t^2} & , \text{ if t = 1, 3, 5, \dots} \\ 0 & , \text{ otherwise} \end{cases} \quad (4)$$

Figure 5 shows an implementation of Varela's algorithm when truncating the Taylor serie at step M. This algorithm has order of complexity of $O(N^2)$ for fixed M. Rosa (2010) suggest to use $M \approx 50$ to maintain performance. The result of the method on the sine signal of Figure 1 is shown in Figure 4.

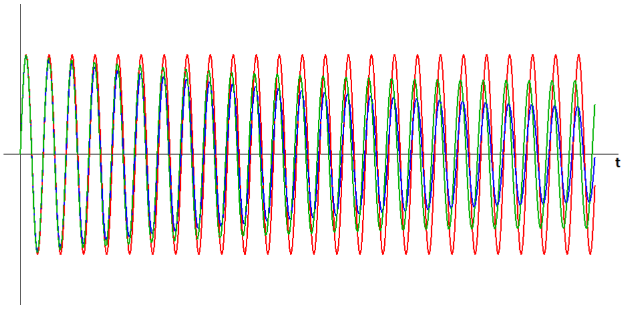


Figure 4: Simulated absorption signal (blue) and its truncated correction (green), with a clear loss of amplitude compared to the original sum of sinusoidal signals (red).

```
// N - size of the inputs
// X - input seismic profile
// Y - output seismic profile
// Q - anelastic attenuation factor
// M - truncation threshold
{
   double z[N] ;
   double inc = (M_PI)/Q ;
   for(int i = 0; i < N; ++i)
   {
      z[i] = x[i] ;
      y[i] = x[i] ;
   }

   for (int k = 1; k<=M; ++k)
   {
      double w[N] ;
      w = convolve(z,g);
      for (int i = 0; i < N; ++i)
      {
         z[i] = (double)i*(inc/k) * w[i] ;
         y[i] += z[i] ;
      }
   }
}
```

Figure 5: Pseudocode of approximate inverse Q filter by truncation.

Translated IIR filtering

The process of absorption can be modelled as a sequence of energy split when the wave refracts on the layer's interfaces. Following Duarte (1993), this can be modeled as a recursive filter that removes a portion β of the signal, transmitting $\alpha = 1-\beta$: the sampled signal $y_{j+1}[i]$ at step $j+1$ of the recursive process is computed from the previous step by:

$$y_{j+1}[i] = \alpha \cdot y_j[i] + \beta \cdot y_j[i-1] \ , \ \forall i \geq j.$$

This model turns out to be a translated Infinite Impulse Response (IIR) filter applied to the initial signal $y_0 = x$. A non-recursive formula for this binomial filter can be obtained by straightforward substitution:

$$y_j[i] = \sum_{k=0}^{j} \binom{j}{k} \alpha^{j-k} \beta^k x[i-k]. \qquad (5)$$

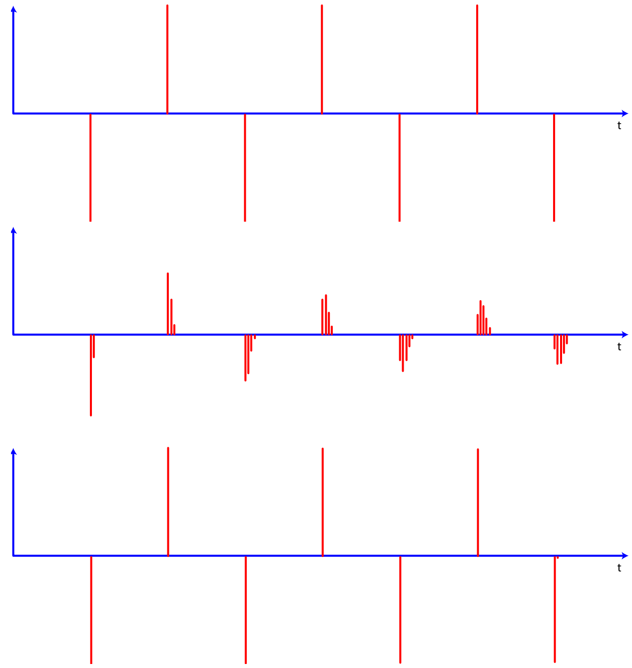


Figure 6: Simulation of absorption using IIR filter (top-to-middle) and correction also using IIR filters (middle-to-bottom) on a synthetic signal.

Duarte (1993) showed that this filter can model the absorption using $\beta = \frac{1}{Q}$ and also compensate for it when using $\beta = -\frac{1}{Q}$ (see Figure 6).

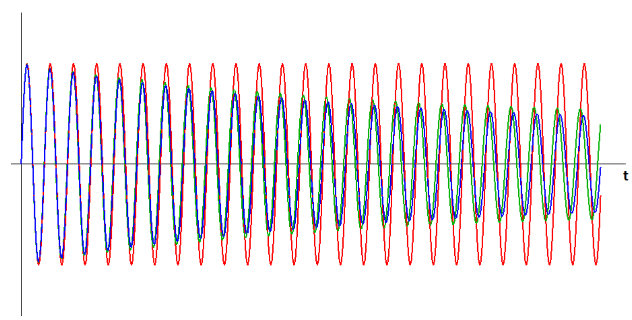


Figure 7: Simulation of absorption using Kolsky's model (blue) and its correction with IIR filters (green), with a small gain on amplitude compared to the original sinusoidal signal (red).

However, when the number of iterations increase, so does the gain of the compensation. In order to avoid amplifying the noise beyond the level of the signal, the algorithm must stop after a maximal number of iterations M. In that case, the maximal gain can be estimated from the previous equation, assuming $\beta << 1$:

$$y_M[i] = \sum_{k=0}^{M} \binom{M}{k} \alpha^{M-k} \beta^k x[i-k] \approx \alpha^M \cdot x[i].$$

Given a target maximal gain Γ expressed in dB, M can be estimated as:

$$M = \frac{\gamma}{20 \cdot \log\left(1+|\beta|\right)}.$$

The result on the sine signal of Figures 1 and 4 is illustrated on Figure 7. Figure 8 shows the implementation of this method, which has complexity $O(N \cdot M) \approx O(N^2)$[1].

```
// N - size of the inputs
// X - input seismic profile
// Y - output seismic profile
// Q - anelastic attenuation factor
// G - target maximal gain
{
  double beta  = -1.0/Q ;
  double alpha = 1.0 - beta ;
  int M = min(N, (G/20)/log10(1+fabs(beta)));

  Y = X ;
  for(int j = 0; j < M ; ++j)
  {
    double xb = Y[j][0];

    for(int i = j ; i < N; ++i)
    {
      double xa = Y[i][0] ;
      Y[i][0] = xa*alpha + xb*beta ;
      xb = xa ;
    }
  }
}
```

Figure 8: Pseudocode for the translated IIR filter.

Gain improvement on IIR filtering

The choice of $\beta = -\frac{1}{Q}$ and $\alpha = 1 - \beta$ is motivated from the physical phenomenon, but it may also be related to the inverse Q filtering as suggested in Rosa (2010). We will end up with a different values for α and β, which will improve the too small compensation observed in Figure 7 when simulating the absorption with Kolsky's model. The idea is to use the truncation (Equation (3)) proposed by Varela et al. (1993) to only two terms as the model for one step of the recursive filter, obtaining:

$$S(\tau + \Delta\tau, \omega) \approx \left(1 + \frac{\Delta\tau}{2Q} G(\omega)\right) \cdot S(\tau, \omega).$$

If we further truncate the convolution kernel g (Equation (4)) to two terms, we obtain:

$$s_{\tau+\Delta\tau}(t) = s_\tau(t) + \frac{\Delta\tau}{2Q} \cdot \big(g(0)\,s(t) + g(1)\,s(t-1)\big).$$

In particular, on the discrete signal $x[i] = s_0(i \cdot \Delta\tau)$:

$$y_1[i] = \left(1 + \frac{\Delta\tau}{8Q}\right) x[i] + \left(-\frac{8\Delta\tau}{Q}\right) x[i-1].$$

If we consider $\alpha = 1 + \frac{\Delta\tau}{8Q}$ and $\beta = -\frac{8\Delta\tau}{Q}$ we can relate parameters α and β of the IIR filter to Kolsky's model, and simulating absorption with the Kolsky's attenuation-dispersion model and correcting using this translated IIR filter with the modified values of α and β, we obtain a better result (Figure 9).

[1]since a larger number of samples do not necessarily implies a longer measure, but rather a higher frequency of measurement, M is likely to increase proportionally to N and we chose to keep it in the complexity notation.

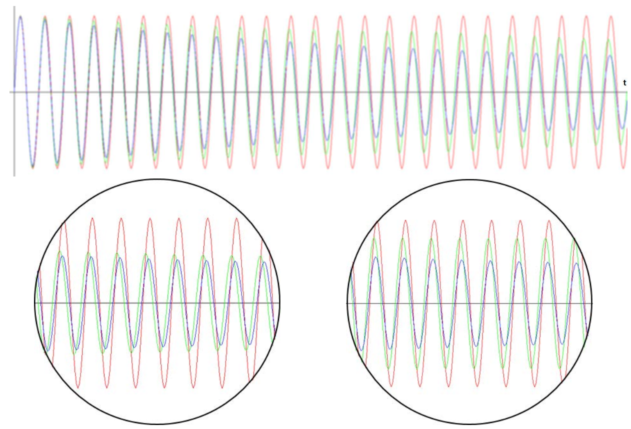


Figure 9: Comparison of the experiment of Figure 7 with modified α and β (bottom right), with a close-up comparing with the original values (bottom right).

Speed improvement on IIR filtering

Observing the non-recursive formulation of the filter (Equation (5)) when the gain is capped (i.e. $k \geq M$), the filter is a convolution with a fixed kernel that can be pre-computed before processing the whole seismic measurement. This convolution would be calculated faster if using a Fast Fourier Transform, reaching a complexity as low as $O(N \log N)$. The convolution kernel is easily derived from Equation (5):

$$\begin{aligned} Y[\omega] &= \sum_{k=0}^{M} \binom{M}{k} \alpha^{M-k} \beta^k \mathrm{e}^{-i\frac{2\pi}{N} j\omega} \cdot X[\omega] \\ &= \left(\alpha + \beta \mathrm{e}^{-i\frac{2\pi}{N}\omega}\right)^M \cdot X[\omega]. \end{aligned} \tag{6}$$

Note that this modified algorithm produces the exact same result, but it is faster in many cases (see Figure 10).

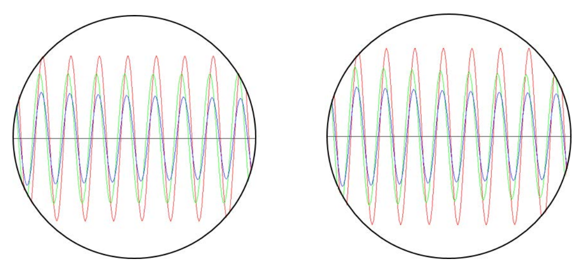


Figure 10: The speed improvement produces the exact same result.

In the initial part $k < M$ of the processing of a single profile, the kernel changes at each sample and Duarte's original algorithm (Figure 8) avoids storing several pre-computed kernels. Therefore our implementation (Figure 11) separates this initial part.

```
// N - size of the inputs
// X - input seismic profile
// Y - output seismic profile
// Q - anelastic attenuation factor
// G - target maximal gain
// A - precomputed binomial kernel
{
  double beta  = -1.0/Q ;
  double alpha = 1.0 - beta ;
  int M = min(N, (G/20)/log10(1+fabs(beta)));

  Y = X ;
  for(int j = 0; j <= M ; ++j)
  {
    double xb = Y[j][0];

    for(int i = j ; i <= M; ++i)
    {
      double xa = Y[i][0] ;
      Y[i][0] = xa*alpha + xb*beta ;
      xb = xa ;
    }
  }

  fftw_complex X_[N-M],Y_[N-M] ;

  fft(N, X>>M, X_) ;

  for (int k = 1; k < N ; ++k)
    Y_[k] = A[k][M]*X_[k] ;

  fft_inv(N, Y_, Y>>M) ;
}
```

Figure 11: Pseudocode for our modified IIR filter

Experiment on real data

In order to compare the results of the correction methods in a more realistic context, we use a seismic data provided by the *Agência Nacional do Petróleo, Gás Natural e Biocombustíveis* (ANP). This line is the segment VB00-52 of Survey VB00 2D BM SEAL in Sergipe-Alagoas Basin, Brazil. Each of the 653 490 profiles was sampled with a 4ms interval during 12s (i.e. 3 001 samples per profile). The line was reprocessed (by non-professional geophysicists) using Seismic Unix (Forel et al., 2005; Cohen and Stockwell, 2007), without and with the different corrections of the losses by absorption applied before stacking, applying the correction methods with $Q = 200$ for all experiments and target maximal gain $G = 60$ (which limits M to 2 173). For the sake of comparison, we applied the absorption compensation after NMO correction. We observe on Figure 12 that all methods produce similar results in quality (at least in this non-professional processing).

Table 1 shows the execution times of each method for a small portion of the line (2.5MB out of 8GB) and the whole data on a laptop computer with 6GB of RAM and Intel Core i5 processor at 2.3GHz. The results shows a clear speedup of the IIR filter already on that small portion (more than 100× faster!), and the improvement of our modification becomes more significant on the whole data.

Absorption compensation	first profiles	whole data
Inverse Q filtering	350.86s	∞
Truncated filtering	107.61s	∞
IIR filter	0.96s	5 960.32s
Modified IIR filter	0.89s	5 765.62s

Table 1: Execution time of the different compensation methods on a fraction of a few lines (around 2.5 MB out of 8 GB) and the wole data.

Acknowledgments

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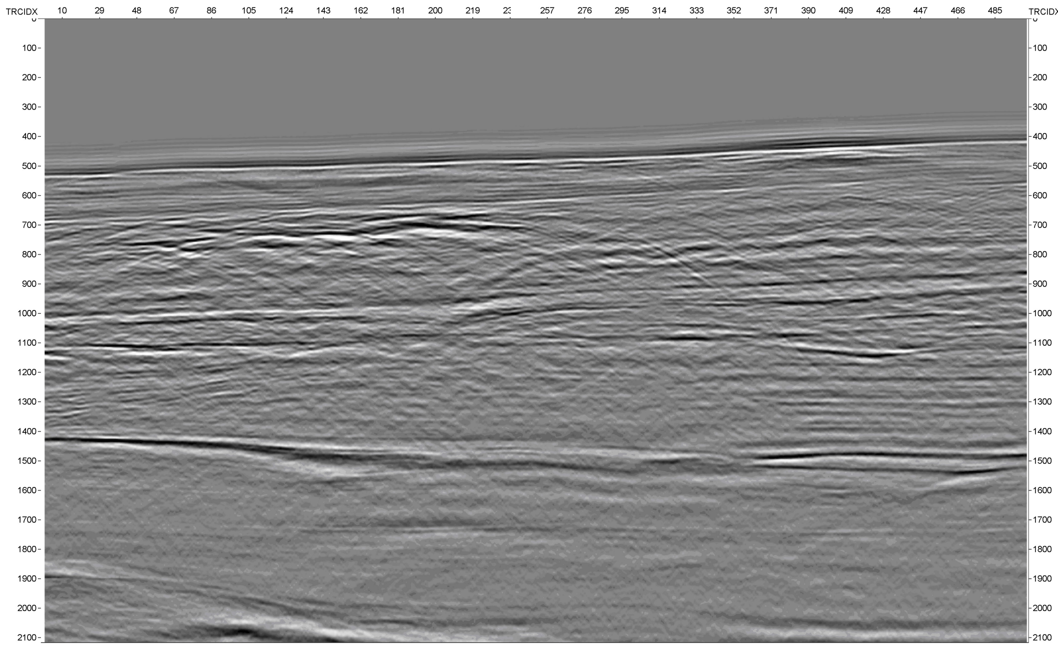
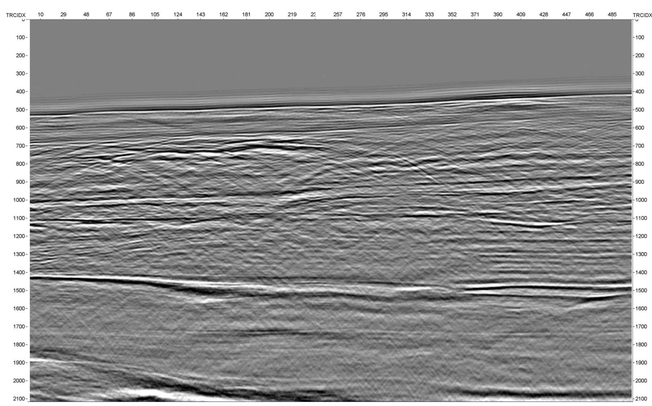
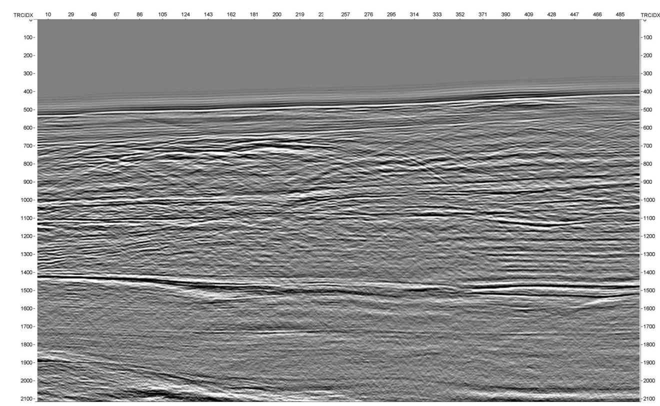


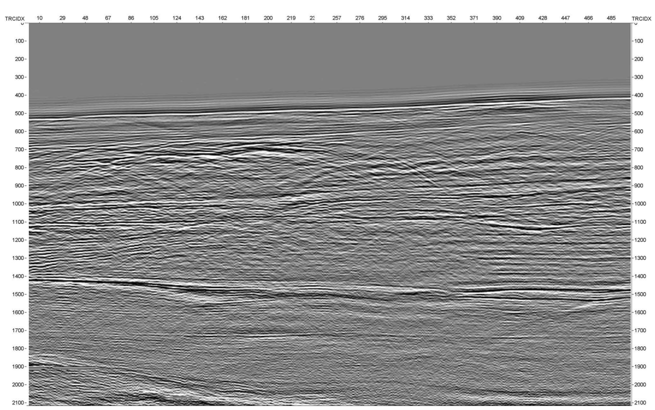
(a) No absorption compensation



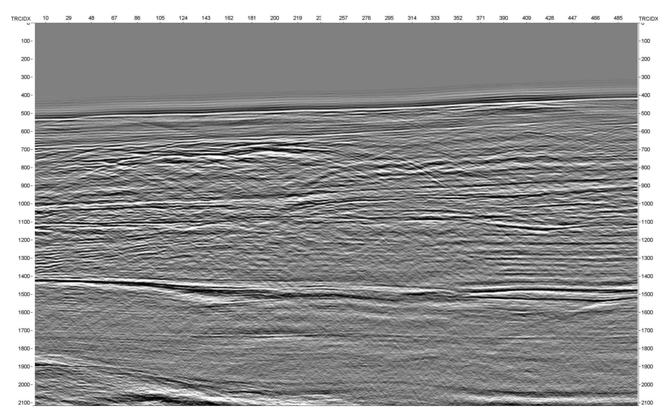
(b) inverse Q filtering



(c) Truncation



(d) Translated IIR filter



(e) Modified IIR filter

Figure 12: Experiments on a real maritime line from Sergipe-Alagoas comparing the different absorption compensation methods.