Residual function dispersion maps to evaluate multidimensional objective function topography: Near-surface geophysical inverse problems

Oleg Bokhonok (IAG-USP & IPECI/UNISANTOS), Liliana A. Diogo (IAG-USP), Cassiano A. Bortolozo (IAG-USP), Carlos A. Mendonça (IAG-USP) Evert Slob (UT Delft)

This paper was prepared for presentation during the 14th International Congress of the Brazilian Geophysical Society held in Rio de Janeiro, Brazil, August 3-6, 2015.

Abstract

This paper shows an alternative way to obtain topography of the multidimensional objective function. This strategy is based on the dispersion analysis of the results obtained from multiple runs of a stochastic optimization algorithm. The inverted parameters are represented in two-dimensional maps, but it does not mean that the other model parameters were kept fixed. Analyzing the solution dispersion in a map format is easy to differentiate the overall ambiguity of local minima. We called this approach as Residual Function Dispersion Map (RFDM). Near-surface geophysical inverse problems: the estimation of geoeletrical parameters by inversion of apparent resistivity curve and estimation of elastic parameters by inversion of exact reflection coefficients of reflected P –wave were used to illustrate the proposed methodology.

Introduction

The inversion of the near-surface geophysical data is very challenging and deal with nonlinear, multidimensional and multimodal optimization problems. For such problems, it is useful to investigate relative ambiguity of the model parameters, understand if all of them can be determined and select the appropriate optimization algorithm. However, we need the information about number, position, distribution and size of local maxima-and-minima, that is, full knowledge of the objective function topography. The conventional way to get the topography of the objective function is through Residual Function Map (RFM). To construct this map, two parameters are made variable and the others ones are kept fixed. However, the RFM mapping has serious limitations for models with more than two parameters since it is required the knowledge of the exact value of parameters to be fixed, which is possible only with synthetic models, and can mask local maxima-and-minima of the function. To overcome conventional methodology limitations used to construct RFM we adopted an alternative way to obtain topography of the multidimensional objective function.

Method

Our strategy is based on the dispersion analysis of the results obtained from multiple runs of a stochastic optimization algorithm. The inverted parameters are represented in two-dimensional maps, but it does not mean that the other model parameters were kept fixed. Analyzing the solution dispersion in a map format is easy to differentiate the overall ambiguity of local minima. We called this approach as Residual Function Dispersion Map (RFDM).

The estimation of geoeletrical parameters by inversion of apparent resistivity curve and estimation of elastic parameters by inversion of exact reflection coefficients of reflected P –wave are shows advantages of this approach for analyzing nonlinear inversion problems dealing with multidimensional and multimodal objective functions.

<u>VES forward modeling</u> was developed using the linear filtering technique described in the work of Johansen (1975), for the Schlumberger array. The parameter vector used for geoelectrical inverse problem was m = {ρ1=30, ρ2=200, ρ3=50, ρ4=600, h1=2, h2=40, h3=80}, where ρ is resistivity (ohm.m) and h is thickness (m). The examples used an AB/2=200m array.

<u>Pre-stack seismic amplitude forward modeling</u> corresponded to exact calculation of reflection coefficient R_{PP} through Zoeppritz equations (Cerveny, Molotkov & Psensik, 1977) with vector parameters defined by m= {V_{P1}=1500, V_{P2}=3750, V_{S1}=452, V_{S2}=2165, ρ1/ρ2=1.588}, where V_P is P-wave velocity (m/s), V_S is S-wave velocity (m/s) and ρ1/ ρ2 is density ratio. We analyzed a wide window of source-receiver offsets: from 1 m to 192 m, with 1 m interval.

Results

Figure 1 shows examples of RFDMs (Figure 1-a, b) versus RFMs (Figure 1-c, d) obtained by mapping objective function f(m) generated by least squares criterion (L2-norm) for geoelectrical inversion of apparent resistivity curve. This figure show similarities between RFDMs e RFMs topography, but some differences can be noted as well. For example, observing RFDM (Figure 1-a) we note that ambiguity in estimating the resistivity of the second layer is smaller than that on the RFM mapping (Figure 1-c). The cut-off observed at the top of RFDM (Figure 1-b) can´t be seen on RFM (Figure 1-d). Figure 2 shows examples of RFDM (Figure 2-a, b) versus RFM (Figure 2-c, d) obtained by mapping the objective function f(m) generated by least squares criterion (L2-norm) for inversion of exact reflection coefficients of reflected P – wave. In this case, the use of RFDMs is very useful because RFMs are completely unable in recognizing a field with well defined solutions.

Conclusions

Our results suggest that RFDM can be an excellent alternative to RFM in terms of analyzing nonlinear inversion problems with multidimensional and multimodal objective functions and accessing ambiguity for found solutions.

Acknowledgments

This work was kindly supported by CNPq.

References

Cerveny, V., Molotkov, LA. & Psensik, I., 1977. Ray method in seismology, Charles University Press, Prague.

Johansen H.K. 1975. An interactive computer1 graphic-display-terminal system for interpretation of resistivity soundings. Geophys. Prosp. 23,449-458.

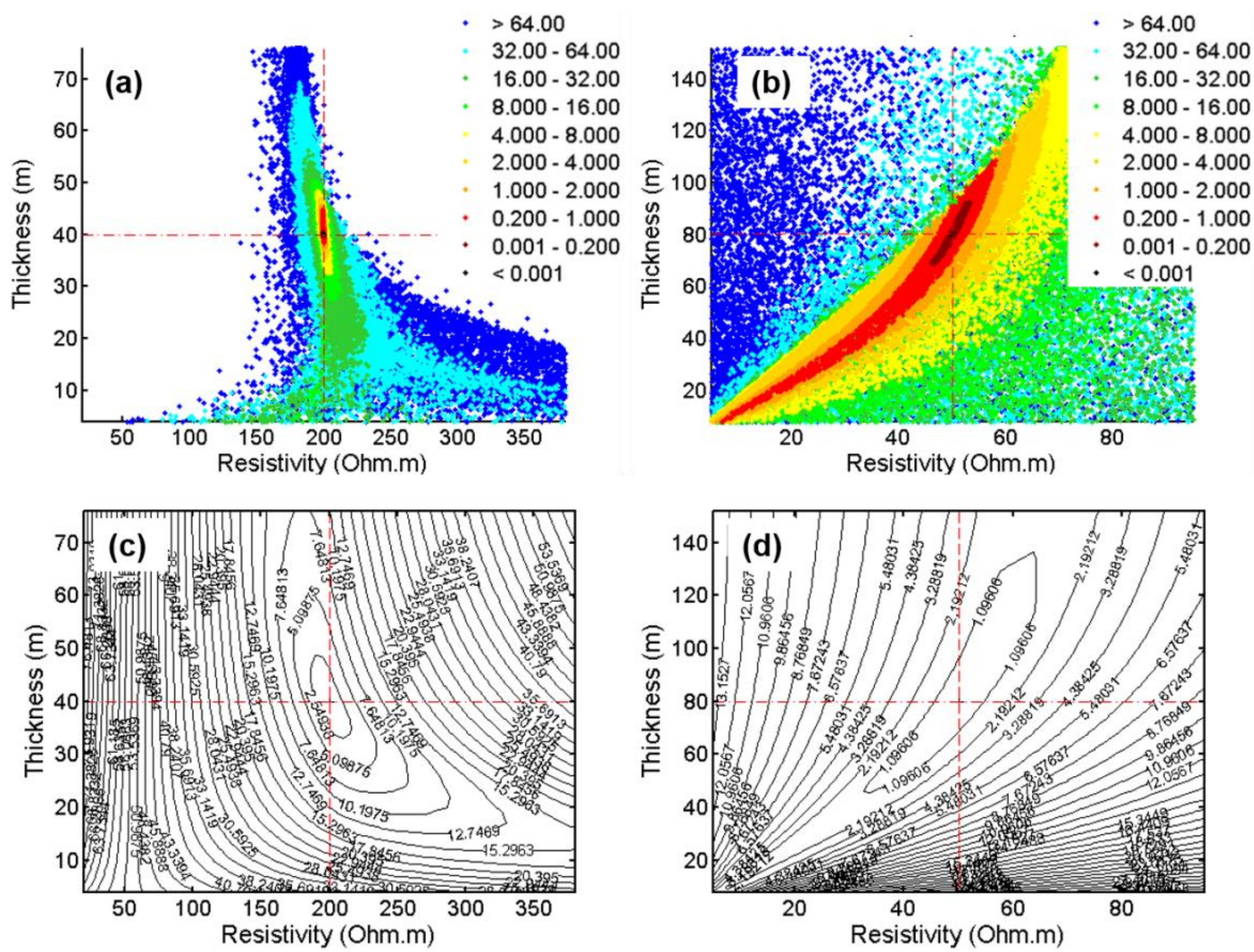

Figure 1: RFMs and RFDMs obtained by mapping objective function f(m) generated by least squares criterion (L2-norm) for geoeletrical inversion of apparent resistivity curve: (a) RFDM of resistivity vs thickness for second layer; (b) RFDM of resistivity vs thickness for third layer; (c) RFM of resistivity vs thickness for second layer; (d) RFM of resistivity vs thickness for third layer.

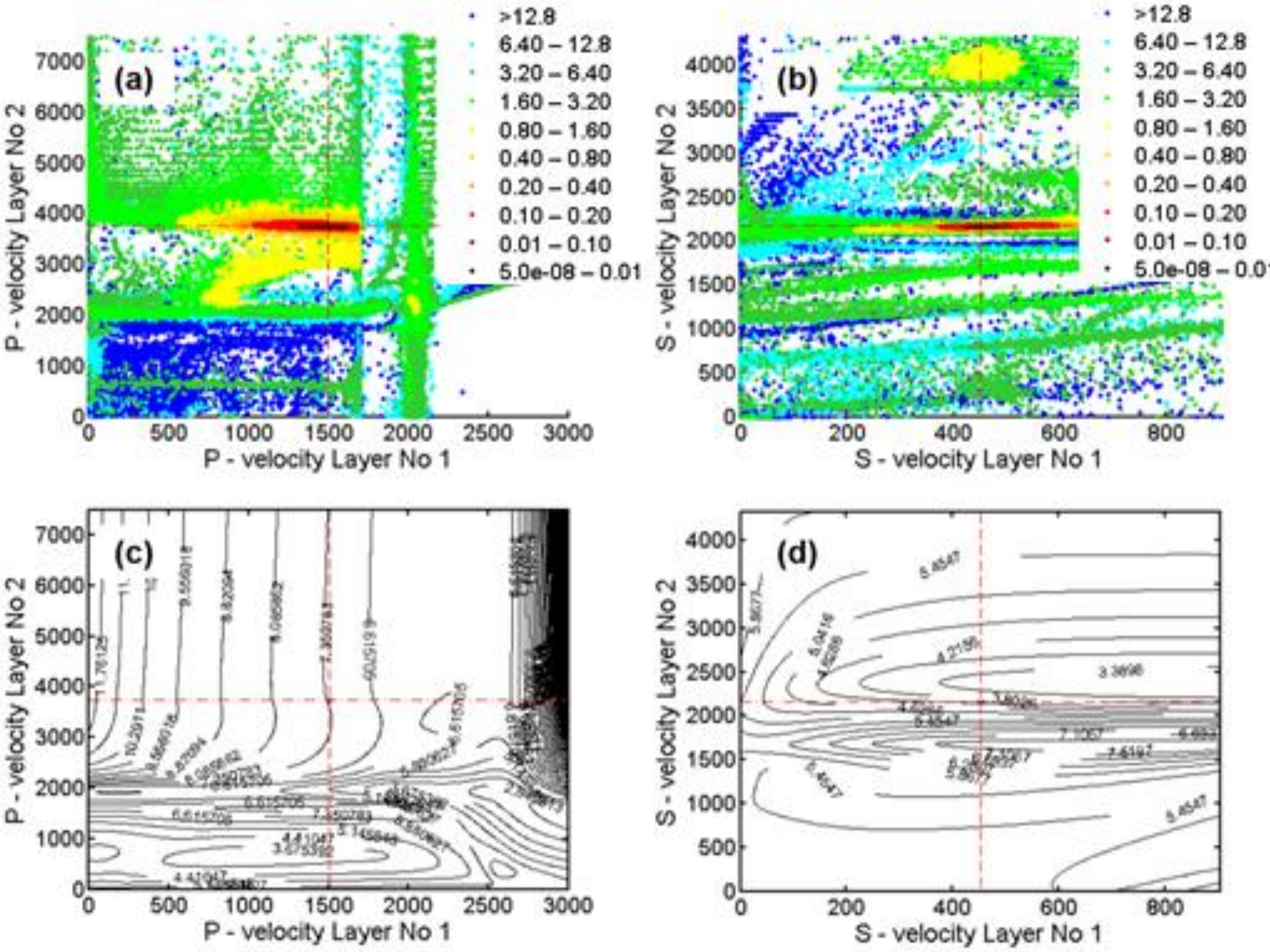

Figure 2: RFMs and RFDMs obtained by mapping objective function f(m) generated by least squares criterion (L2-norm) for inversion of exact reflection coefficients of reflected P –wave: (a) RFDM for first layer P-wave velocity vs second layer P-wave velocity; (b) RFDM for S-wave velocity of first layer vs S-wave velocity of second layer; (c) RFM for first layer P-wave velocity vs second layer P-wave velocity; (d) RFM for first layer S-wave velocity vs second layer S-wave velocity.