Reverse time migration with amplitude weighted by seismic illumination

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Abstract (Font: Arial Bold, 9)

This work presents comparative results of a 2D frequency domain reverse time migration algorithm based on the least-square formulation. Different approximations were taken into account in order to obtain the pseudo-inverse of the Hessian of the least-square function so the correct reflector amplitudes could be found.

Introduction

Reverse time migration is already established as an appropriate technique for imaging geological complex regions and for velocity model building. Although its advantages do not always appear clearly in practice due to problems such as limited acquisition geometry and low illumination.

In order to recover the amplitude as close as possible to the true one, we formulate the migration problem as an inverse problem based on a least-square functional (Tarantola 1984, Lailly 1983). Using this approach those authors found that the gradient of this functional with respect to the reflectivity corresponds to the conventional migration algorithm and the correct amplitude can be achieved by applying the pseudo-inverse of the Hessian of the least-square functional to its gradient. This procedure would correct illumination effects present in the migrated image as the Hessian carries information concerning the geometry acquisition.

Despite this possibility, in practice, it is not possible to actually compute the exact hessian because of its size. Considering this fact, we assume that the hessian is composed only by its diagonal terms, making the calculation of its diagonal a quite simple process. Other than, that we assume three different considerations to compute the diagonal of the hessian. The simplest one assume a infinite and continuous receiver coverage, the second one imposes a homogeneous velocity model and the third and more sophisticated approach considers a random-phase encoding that considerably reduces the computational cost but adds a crosstalk noise that can be minimize if the proper number of realizations is apply. The results obtain show a numerical comparison of all these approximation for a synthetic velocity model.

Method

Considering the acoustic wave equation for constant density in the frequency domain

$$(\omega^2\sigma^2 + \nabla^2)u(x,\omega;\sigma) = -F(\omega)\delta(x - x_s)\,,$$

where σ stands for the slowness, $u(x,\omega;\, \sigma)$ the pressure field created by a point source at x_s, $F(\omega)$ the source amplitude and ω the angular frequency. For a background velocity model associated with the slowness σ_0, the pressure field is obtained similarly

$$(\omega^2{\sigma_0}^2 + \nabla^2)u_0(x,\omega;\sigma_0) = -F(\omega)\delta(x - x_s)\,.$$

We now define the reflectivity and the scatered field as

$$r = \sigma - \sigma_0$$

$$u_{sct}(x,\omega,r;\sigma_0) = u(x,\omega,r;\sigma) - u_0(x,\omega,r=0;\sigma_0)\,,$$

repectively. Tarantola's approach defines migration as a least-square inverse problem where we attempt to find a reflectivity model the minimizes the following error functional

$$E(r) = \frac{1}{2}\sum_{\omega}\sum_{s=1}^{N_s}\sum_{r=1}^{N_r(s)}\left\| u_{sct}^{(s)}(x,\omega,r;\sigma_0) - d^{(s)}(x_r,\omega)\right\|.$$

identify the Following Tarantola's method it is possible to migrated image m as

$$m = -H^{-1}\nabla E(r)\,.$$

With H representing the Hessian of the error functional. Which means that the minimum of a quadratic function is obtained by the gradient pre-multiplied by the inverse of the Hessian and the best amplitude-preserving migration image is obtained by choosing H as the exact Hessian of

the error function. In practical terms, however, it is not possible to compute the hessian because of its size. In this work we propose to approximate the pseudo-inverse of the Hessian with a diagonal matrix. This approximation would be fully valid in the high-frequency limit but may be inadequate with a finite set of frequencies.
Calculating explicity the expressions for gradient and hessian we get

$$\frac{\partial E}{\partial r_i} = -\Re\left\{\sum_{\omega}\sum_{s=1}^{N_s}\sum_{r=1}^{N_r(s)}\frac{\partial u_{sct}^{(s)}}{\partial r_i}(x,\omega,r=0,\sigma_0)d^{(s)}(x_r,\omega)^*\right\}$$

and

$$\frac{\partial^2 E}{\partial r_i \partial r_l} = -\Re\left\{\sum_{\omega}\sum_{s=1}^{N_s}\sum_{r=1}^{N_r(s)}\frac{\partial u_{sct}^{(s)}}{\partial r_i}(x,\omega,r=0,\sigma_0)\left(\frac{\partial u_{sct}^{(s)}}{\partial r_l}(x,\omega,r=0,\sigma_0)\right)^*\right\}$$

respectively. If we consider the linearized case where only small reflectivities are taken into account. Whereas the scattered field can be written using the definition of Green's function as

$$\frac{\partial u_{sct}}{\partial r_i}(x_s,x_r,\omega) = \omega^2 F(\omega)G(x_s,x_i,\omega)G(x_i,x_r,\omega)$$

Substituting the expression above in the equations for the gradient and the diagonal of the hessian it is straightforward to write

$$\frac{\partial E}{\partial r_i} = -\Re\left\{\sum_{\omega}\omega^2\sum_{s=1}^{N_s}F(\omega)G(x_s,x_i,\omega)\sum_{r=1}^{N_r(s)}G(x_i,x_r,\omega)d^{(s)}(x_r,\omega)^*\right\}$$

and

$$\frac{\partial^2 E}{\partial r_i \partial r_l} = -\Re\left\{\sum_{\omega}\omega^4\sum_{s=1}^{N_s}|F(\omega)|^2|G(x_s,x_i,\omega)|^2\sum_{r=1}^{N_r(s)}|G(x_r,x_i,\omega)|^2\right\}$$

With an infinite and continuous receiver coverage for every shot, the term involving the receiver contribution is almost constant at least for a homogeneous model and it could be neglected. However such naïve approach does not give satisfactory results for most cases and the excessive computational related to the calculation of this term cost cannot always be avoid. In the following we present more robust approaches to compute this term.

Migration Weights

The first assumption, as previously mentioned, consists in assuming that the term

$$\sum_{r=1}^{N_r(s)}|G(x_r,x_i,\omega)|^2$$

Does not dependo n the spatial coordinates so it can be disregarded being a constant fator. The resulting migration weight $H^{(1)}$ is

$$H^{(1)} = \sum_{\omega}\omega^4\sum_{s=1}^{N_s}|F(\omega)|^2|G(x_s,x_i,\omega)|^2$$

The second supposition, on the other hand, takes into accound an influence of the receptors. Computing the receptor's Green function for a constant velocity model we get

$$\sum_{r=1}^{N_r(s)}|G(x_r,x_i,\omega)|^2 \propto a\sinh\left(\frac{x_r^{\max}(x_s)-x}{z}\right) - a\sinh\left(\frac{x_r^{\min}(x_s)-x}{z}\right)$$

In two space dimensions, let $x^{min}_r(x_s)$ and $x^{max}_r(x_s)$ be the minimum and maximum receiver positions for a shot located at x_s (Plessix and Mulder 2004).

The last approach (assigned to Tang 2011) calculates the term corresponding to the receiver using a random-phase enconding that allow us to obtain the diagonal of the hessian as its true expression plus a contribution of a crosstalk that would be less effective if more realizations are calculated

$$\widetilde{H}^{(3)}(x,x) = H(x,x) + Crosstalk$$

Results

The SEG/EAGE salt dome model as used to test the effectiveness of the three approximations presented above. The true reflectivity model that we wanted to recover can be seen in figure 1.

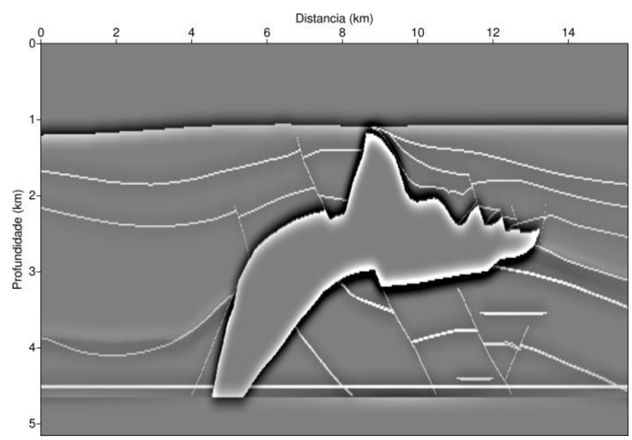


Figure 1: True reflectivity model of SEG/EAGE salt dome model.

A marine type acquisition geometry has been chosen and a time-domain finite-difference code was used to generate the data used in the migration algorithm. The migration details are show in table 1.

Number of shots	237
Number of receivers/shots	65
Minimum offset	80 m
Maximum offset	2.7 km
Frequency interval	0.125 Hz
Minimum frequency	5 Hz
Maximum frequency	30 Hz

Tabel 1: Migration details used in the SEG/EAGE model.

The migration result without any pre-conditioner is show in figure 2. This is the gradient of the error function presented above. The migration results with the correspondent weight can be seen in figure 3.

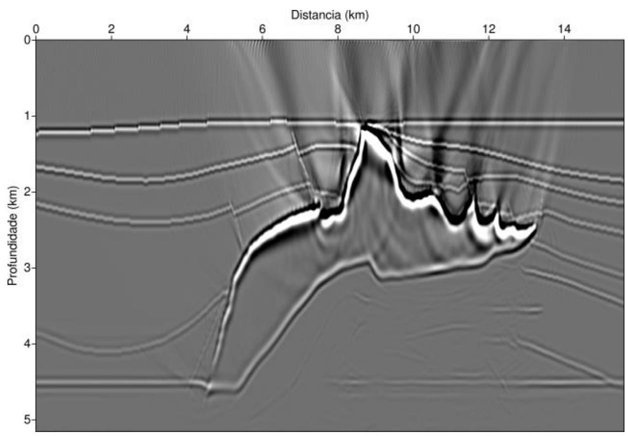
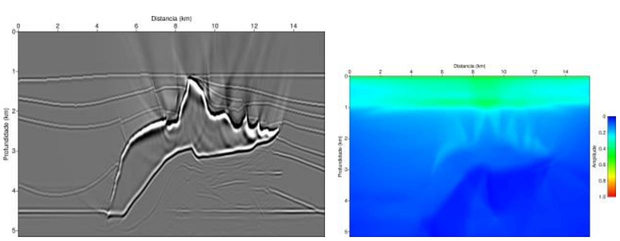
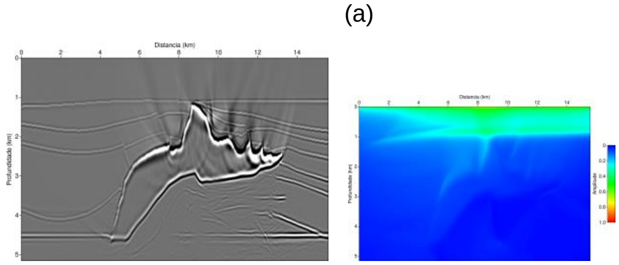


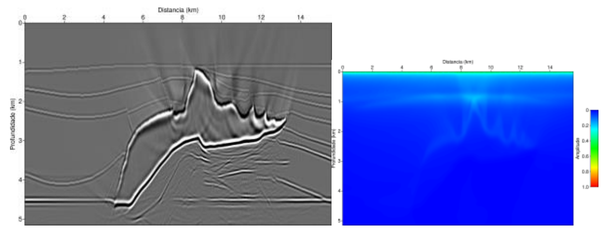
Figure 2: Migration result for the salt dome model without any weight application.



(a)



(b)



(c)

Figure 3: (a) Amplitude-preserving migration result for approximation $H^{(1)}$ and migration weights $H^{(1)}$. (b) Amplitude-preserving migration result for approximation $H^{(2)}$ and migration weights $H^{(2)}$. (c) Amplitude-preserving migration result for approximation $H^{(3)}$ and migration weights $H^{(3)}$.

Conclusions

We have presented different strategies to compute migration weights that can produce acceptable reflector amplitudes when using a frequency domain finite-difference migration algorithm. The comparison of the numerical results using the migrations weights presented here show that for deeper reflector, particularly in geological complex regions, can only give reasonable results when a more robust approximation for the contributions of the receiver are made, as the $H^{(3)}$ approach shows in figure 3.

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