Robust Picking with the Eikonal Equation

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Abstract

There are many applications in geophysics, where a path must be selected inside a plane, volume, or multidimensional space, such that some measure of the properties inside that space must be minimized (or maximized) along the path, subject to some additional smoothness constraints. We present a method, based on the solution of the Eikonal equation, that naturally produces a smooth path that minimizes a traveltime function equivalent to the original problem. The robustness of the method is illustrated by three examples from different applications.

Introduction

Picking is a common task in several geophysical problems. It can be defined as the selection of a function representing a path across a multidimensional volume, so that the integral of some property along this path is maximized (or minimized). Depending on the particular problem to be solved, the property may represent semblance as a function of time (or depth) and velocity; local correlation as a function of time and static time displacement (or lag); or amplitude as a function of time and offset. Although the method presented here can be applied to all of these problems (as we shall see in the examples), for didactic reasons we will use one particular problem to describe it. The problem we will consider is to define a smooth function that best represents the time variant displacements between two traces, in order to maximize a measure of their similarity.

Most algorithms designed with this purpose (estimating time shifts between traces as a function of time and space) are based on nonlinear approaches (Kruse, 1988; Martinson and Hopper, 1992; Liner and Clapp, 2004). Our approach is based on a linear solution that uses windowed local crosscorrelation (or running crosscorrelation) as a measure of local similarity. The goal is to obtain a function of time and space of the displacements (or lags) that maximize the sum of the corresponding correlations.

In most cases the cross-correlation volumes are affected by noise and cycle skipping, leading to strong events unrelated to the correct displacement. As a result, smoothing filters have to be applied when the displacement function is constructed by selecting the maximum crosscorrelation for each time, and position. One example of an algorithm based on this process is the Residual NMO Correction, when crosscorrelations are computed between neighbor traces in a common image gather (or between each trace and a reference trace). Another example is the computation of traveltime residuals between recorded and synthesized traces in some formulations of Full Waveform Inversion.

An alternative, which produces a more stable and naturally smooth displacement function, is based on the eikonal equation (Cohen and Kimmel, 1997, Deschamps and Cohen, 2001). The strategy adopted here is very similar to the one described by Tao et al. (2012), where they use Eikonal based picking to perform velocity analysis for wide-azimuth data. Let $C(t,l)$ represent the running crosscorrelation between two traces, where t is the time along the traces and l is the displacement (or lag) of the correlation. For each pair of traces, we need to estimate a function $l(t)$ that maximizes the path integral $C(t,l(t))$. The first step is to transform $C(t,l(t))$ in a velocity model $v(t,l(t))$, with positive definite velocities which are proportional to the correlation values. Both t e $l(t)$ are treated as spatial entities.

By solving the Eikonal equation for a horizontal linear source on the top of this model (at $t=0$), we obtain the traveltimes $\tau(t,l)$. The isochrones are interpreted as downward propagating wavefronts, and the position of the wavefront that first touches the bottom of the model ($t=t_{Max}$) corresponds to the part of the wave that majorly traveled through the highest velocities, that is, through the highest correlation values. Starting at this position (l where $\tau(t_{Max}, l)$ is minimum), a ray normal to all isochrones (wavefronts) $\tau(t, l)$ is traced back. This trajectory corresponds to the desired function $l(t)$.

Theory

One wishes to find a function that represents the set of time shifts to be applied to a trace in order to maximize its correlation with a reference trace. Let's define the functional

$$E(s) = \int_s C(t,l)\, ds, \tag{1}$$

where $C(t, l)$ corresponds to the running correlation between two traces, t is the time along the traces, and l the displacement (or lag) of the crosscorrelation that can be a sub-multiple of the sampling interval. The integral in (1) is computed along the path s, defined by the function $l(t)$, from the starting time $t = 0$ to the ending time $t = t_{Max}$. The problem can be formulated as the search for the path s that maximizes the functional $E(s)$, or in an equivalent manner, as the search for the path that minimizes the functional $\tau(s)$ defined by:

$$\tau(s) = \int_s \frac{1}{C(t,l)}\, ds. \tag{2}$$

By a change of scale, we can replace: $t \to z$, $l \to x$, and $C(t, l) \to v(z, x)$. Equation (2) can be rewritten as:

$$\tau(s) = \int_s \frac{1}{v(z,x)}\, ds. \tag{3}$$

As a result, the problem becomes equivalent to find among the different paths the one that minimizes the traveltime along the ray, starting at the surface (time zero) and subjected to Fermat's principle.

An important part of the method is the transformation of $C(t, l)$ in $v(z, x)$, which we call the mapping function. There are many possibilities and, for each particular problem to be solved, a different function, or parametrization may be required. Some of the options comprise the range of velocities to be mapped, how to treat the negative values of correlation, raising the correlation to a power in order to control the contrasts in the model.

After the mapping, the solution can be decomposed in two steps:

- In the first step the Eikonal equation is solved for the traveltimes $\tau(z, x)$ associated to a linear horizontal source at $z = 0$. The choice of the linear source in $z = 0$ (actually $t = 0$) corresponds to the assumption that the displacements (lags) do not change at $t = 0$, that is

$$\left| \frac{d\, l(t)}{d\, t} \right|_{t=0} = 0. \tag{4}$$

  Another choice could be a point source at $x = 0$ (or $l = 0$), meaning that the displacement is zero at $t = 0$.

- In the second step we find the position x_s where

$$\frac{\partial \tau(z_{Max}, x)}{\partial x} = 0. \tag{5}$$

  This position corresponds to the end of the path (ray) associated to the desired solution. Starting at this position, the ray $x(z)$ (that is, the function $l(t)$) is traced upward using the traveltimes $\tau(z, x)$ calculated at the first step.

Figure 1 summarizes the application of both steps.

Examples

This strategy for selecting the optimal function $l(t)$ was successfully tested in three distinct applications. The first application comprises the estimation of traveltime residuals for a Full Waveform Inversion algorithm (FWI) with the one-way wave-equation (Guerra and Cunha, 2013). This approach is similar to the one described by Ma and Hale (2013), except that they used dynamic image warping to estimate the traveltime residuals. Given two common shot records (field and synthetic) the goal is to build a record with the same dimensions, such that each sample represents an estimate of the instantaneous (local) time shift between the corresponding traces of the two shots. With this purpose, running cross-correlation panels $C(t, l)$ are generated for each pair of traces. Paths that maximize the integrated correlation are then computed with the Eikonal based method. Figure 2 shows, superimposed to one of those panels, the paths obtained by two different methods. Part **(a)** presents the solution based on the selection of the maximum correlation value for each time, followed by smoothing in time and space. We refer to this as the traditional method. Part **(b)** presents the solution proposed here, which we refer to as the Eikonal method.

The Eikonal based method is less susceptible to cycle skipping, leading to results that are not only more consistent when compared to the results from the traditional method, but also present a naturally smoother behavior, both in time and space. The robustness of the Eikonal method becomes clear by analyzing Figures 3 and 4, where final results for

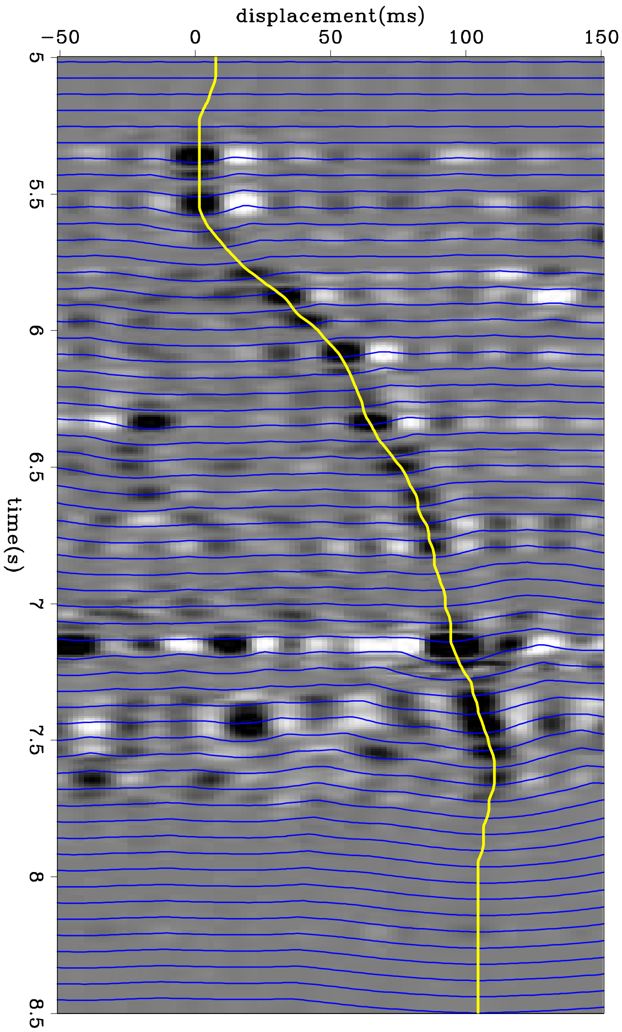

Figure 1: Background in grey scale corresponds to values of correlation as a function of time and time displacement (lag) between two traces (positive values in dark shades and negative values in light shades). When transforming correlation values in velocity, dark shades are mapped to high velocities while light shades are mapped to low velocities. The blue lines correspond to isochrones (wavefronts) associated to a linear source at the top of the model. Wavefront acceleration at high velocity regions becomes apparent. The portion of the traveling wavefront that on average travels through regions with highest velocities (higher correlations) will be the first part of the wavefront to "touch" the bottom of the model. From this position a ray is traced upward (yellow line), which corresponds to the desired function $l(t)$.

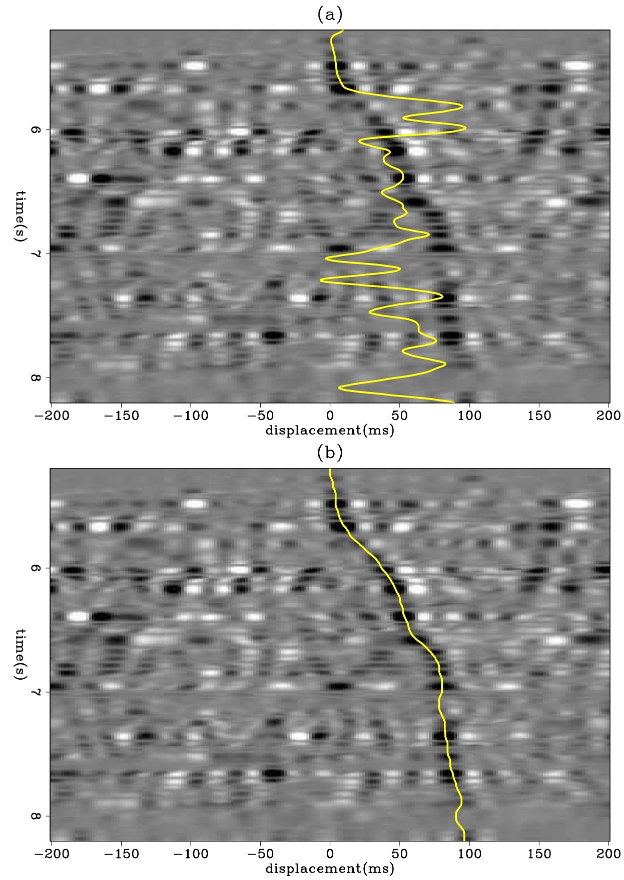

Figure 2: The two figures show panels with correlation values between two traces corresponding to the same offset of the two shot gathers (recorded and synthesized). It is clear that the traditional solution produces unsatisfactory results, while the solution based on the Eikonal leads to stable and consistent results.

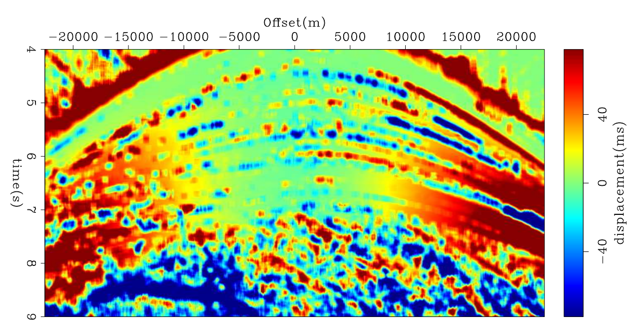

Figure 3: Color scale represents the values for time-shifts (displacement) between traces with same offset from the two data (recorded and synthesized), estimated by the traditional method.

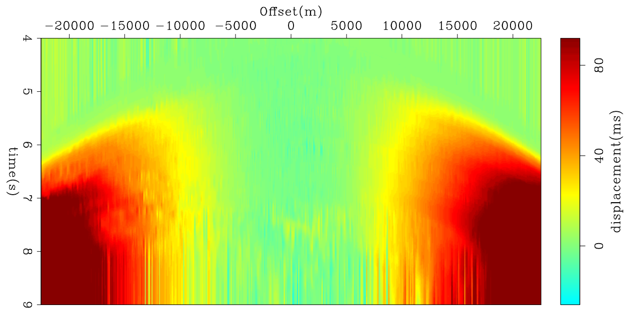

Figure 4: Color scale represents the values for time-shifts (displacements) between traces with same offset from the two data (recorded and synthesized), estimated by the Eikonal method. Not only the outcome is more stable, but also more compatible with expected displacements.

all offsets (*Displacement(t,offset)*) are compared for both methods.

The second application refers to the estimation of the time-shifts required for residual moveout correction (RMO). The results are illustrated by Figure 5. Although the differences between the data corrected by the two methods are more subtle when compared with the non-corrected data, the Eikonal method has the advantage of requiring less smoothing than the traditional method. The number of traces (CIGs) used for the smoothing operator was independently chosen to produce the best result for each method. The traditional method required an operator two and a half times longer for this specific case. Figure 6 compares the time-shifts without smoothing estimated by the two methods.

The third application is the automatic picking of specific events in common shot records, as part of a wave-equation based internal multiple attenuation process. To use the Eikonal method in this particular case, it is necessary first to apply a NMO correction to the original gather, in order to better align the desired event. Next, the gather is transposed, with the time axis becoming x and the offset axis becoming z. Then the amplitudes are converted to velocity values, defining the model where the Eikonal is solved and ray-tracing is performed. One example of this application is presented in Figure 7.

Conclusions

Any picking process aiming at the construction of a continuous and smooth path that maximizes (or minimizes) the integration of a multidimensional function along the path, is well suited for the use of the Eikonal based method. In this method, picking is re-

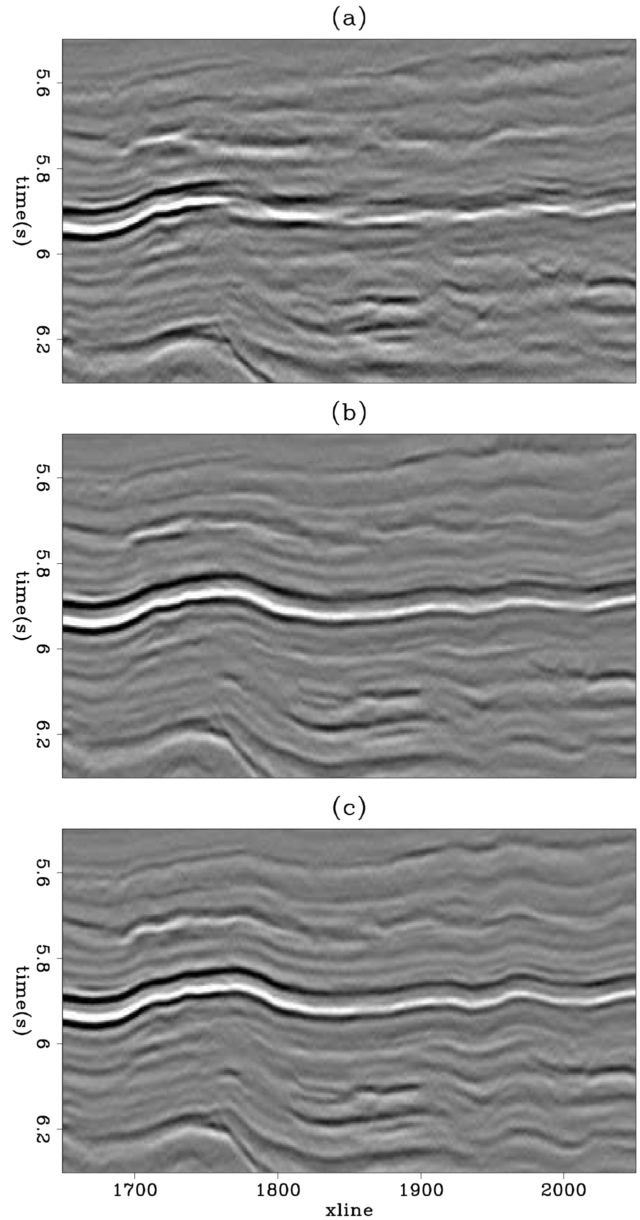

Figure 5: The three sections correspond to a window of migrated and stacked data. **(a)** without RMO correction. **(b)** With RMO using the traditional method for estimating the time-shifts, with 50 traces smoothing. **(c)** With RMO using the Eikonal method for estimating the time-shifts, with 20 traces smoothing.

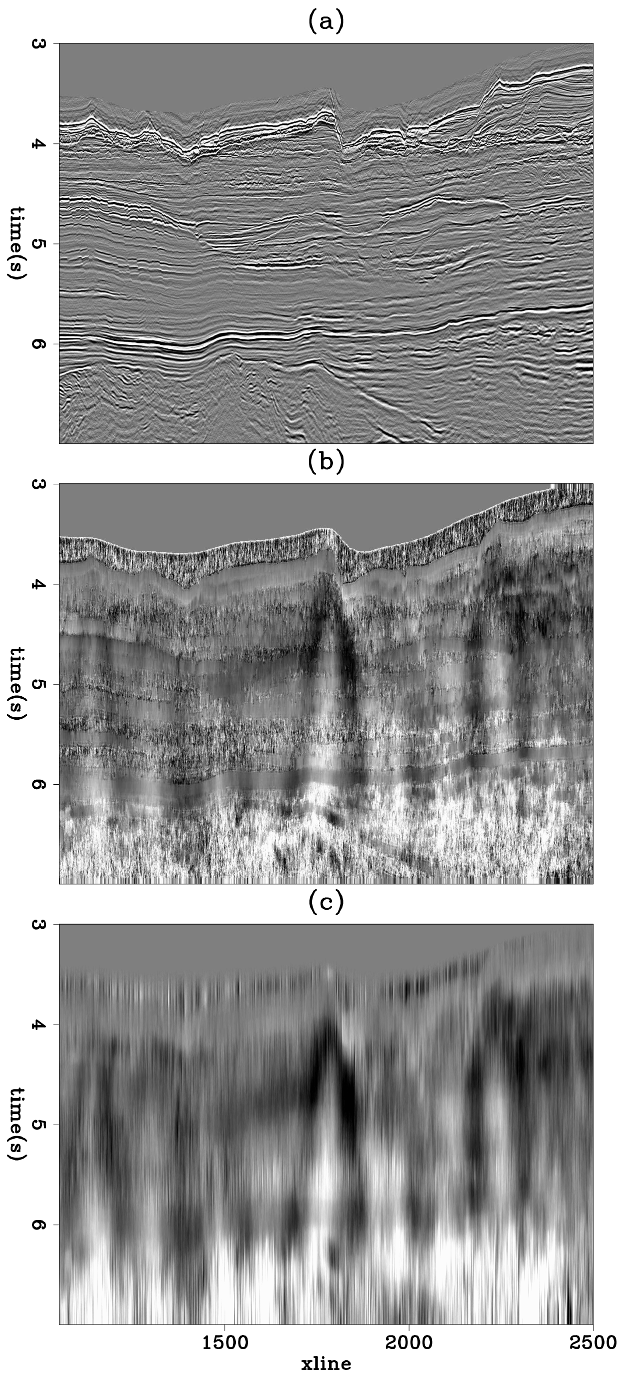

Figure 6: In **(a)** a general view of Figure 5-c. The other two panels show the RMO estimated time-shifts without smoothing, for offset 5.6 Km, using: **(b)** the traditional method, and **(c)** the Eikonal method. As expected, the Eikonal method produces a naturally smoother solution for the RMO time-shifts as compared to the tradicional method. The relevant low frequency features of the estimated shifts in **b** and **c** are well correlated with the water bottom canyons observed in **a**.

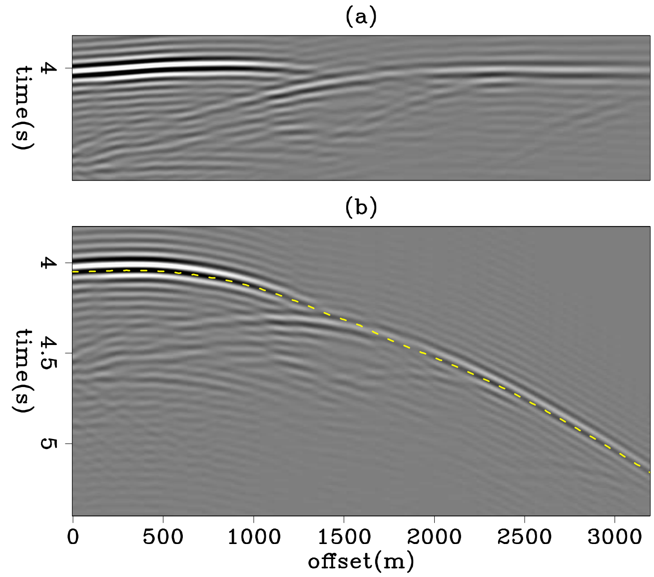

Figure 7: **(a)** NMO corrected common shot gather. Similar to the correlation panels, this gather is transformed to a velocity model (dark shades for higher velocities, and light shades for lower velocities). **(b)** The yellow line shows the result of automatic picking based on the Eikonal method, after inverse NMO correction, superimposed to the original shot gather.

defined as the solution of a well-posed physical problem, with specific initial conditions, a model space, and an equation to be solved. The key to adapt the method for different kinds of applications lies in the translation from the particular problem to the equivalent velocity model, the so called mapping function, and how the initial condition is specified. Examples from three different applications illustrate the robustness of the method. Although not presented here, the method was successfully applied to residual moveout picking in a tomographic inversion application.

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