Shear Wave Velocity Estimation in slow siliciclastic formations using empirical models.

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Abstract

In this contribution, we propose an alternative way to calibrate and estimate S-wave velocities using a regression analysis methodology. Prediction of S-wave velocities is critical in locations where sonic logs only have P-wave velocities available or for some reason only in discrete intervals, generally in slow formations. The proposed method applies two robust quadratic models for estimating S-wave velocities, assuming V_p, clay volume and, fractional effective porosity as parameters affecting V_s. Considering S-wave velocities varying according to quadratic models, the estimation process confirms previous results showing V_p as the most important dependence parameter in the regressions. The preliminary results corroborate the idea that the use of the proposed quadratic model by regression analysis can represent a suitable procedure for predicting shear wave velocities in slow siliciclastic rocks.

Introduction

Shear wave velocity has valuable applications in subsurface exploration, such as, AVO analysis, elastic impedance calculations, geomechanic properties, multicomponent seismic interpretations. In general, the logging acquisition program has full suites of logs, among them, sonic logs. In wireline sonic tools, there are two types of sources, monopole and dipole. P and S-wave logs are measured directly by monopole sources, but only in fast or hard formations. It is impossible to obtain shear wave travel times directly from the full waveform acoustic logs in slow or soft formations, because there are no refracted shear wave arrivals (Cheng and Toksöz, 1983). This limitation happens because the formation shear velocity is lower than the compressional mud-wave velocity. A quick way to identify slow (soft) formations is through the average values of the P-wave velocity, generally inferior than 4000 m/s (superior than 75 $\mu s/ft$). In slow formations, S-wave log can be measured using only dipole source. According to Cheng and Toksöv (1983), the characteristics of the full waveform acoustic log microseismograms in slow or soft formations are distinctly different from those fast formations.

The use of empirical models becomes alternative way for predicting shear wave velocities. The estimation of any physical rock property implies in adopting a mathematical model (Augusto et al., 2009). However, the chosen model hardly contains the full set of parameters affecting the rock property under study. Han et al. (1986) used the application of a multivariate linear regression methodology for predicting S-wave velocities taking into account porosity and clay volume fractions and, Castagna et al. (1993) used a parabolic model for limestone with P-wave velocity as variable. For Castagna et al. (1993), Vp/Vs ratio is linear for siliciclastics, but not for carbonates. In this way, we propose a regression analysis methodology similar as in Han et al. (1986) and Castagna et al. (1993).

This paper considers three wells drilled in Brazilian offshore continental margins with s-wave sonic log information available. The objective of this study is testing and analysing the response of two quadratic empirical models to estimate S-wave velocity in the slow siliciclastic formations of these wells. The information of some geophysical well logs to calculate petrophysical properties and then, use them for least-squares estimation of regression coefficients of each proposed models. The fractional effective porosity, clay volume, and compressional wave velocity are the set of assumed independent variables in the proposed quadratic models. After this, we extend the regression models to other two wells and compare with s-wave velocity measured and Castagna model (1993).

Methodology

In order to test our methodology, we take into account three wells (hereafter called Well-A, Well-B and Well-C) drilled in Brazilian continental margins with all well logs necessary to our study, ie., gamma ray (GR), Neutron Porosity ($NPHI$), bulk density ($RHOB$), Compressional sonic (DTC) and Shear sonic (DTS). The sonic logs here used were acquired using dipole source. The shear-wave sonic log is needed to allow calibration with S-wave velocity (V_s) model established at the end of the methodology. From the Well-A logs, regression models will be used to extract all constant values and establish the final equation. After then, the calibration can be extended to Well-B and Well-C.

1) Shaliness (V_{clay} or clay volume) estimation:

The Larionov (1969) equation for shaliness estimation in unconsolidated sediments was used:

$$V_{clay} = 0.083 \cdot \left[2^{3.70 \cdot IGR} - 1.0\right]. \tag{1}$$

In the preceding equation, the gamma ray index is given by:

$$IGR = \frac{GR_i - GR_{ss}}{GR_{sh} - GR_{ss}}, \tag{2}$$

where GR_i is the i^{th} gamma ray log reading, GR_{ss} and GR_{sh} are the minimum and maximum readings in the gamma-ray log taken in the sandstone and in the shale point, respectively.

2) Effective porosity estimation (ϕ_e):

Using the bulk density log, the following approximation allows fractional effective porosity estimation (ϕ_e):

$$\phi_e = \underbrace{\frac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f}}_{\phi_t} - V_{clay} \cdot \frac{\rho_{ma} - \rho_{sh}}{\rho_{ma} - \rho_f}, \quad (3)$$

where ϕ_t is the fractional total porosity and ρ_b is the bulk density log reading.

$$\phi_t = \frac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f}, \quad (4)$$

We take mean value from bulk density log to set ρ_{ma}. For the fluid formation density, we adopt $\rho_f = 0.89 g/cm^3$, while ρ_{sh} is taken from the maximum value of the difference between the NPHI log and the total porosity log, i.e.

$$max(\phi_N - \phi_t), \quad (5)$$

Note that, in equation 3 the clay volume corrects the total porosity yielding the effective porosity.

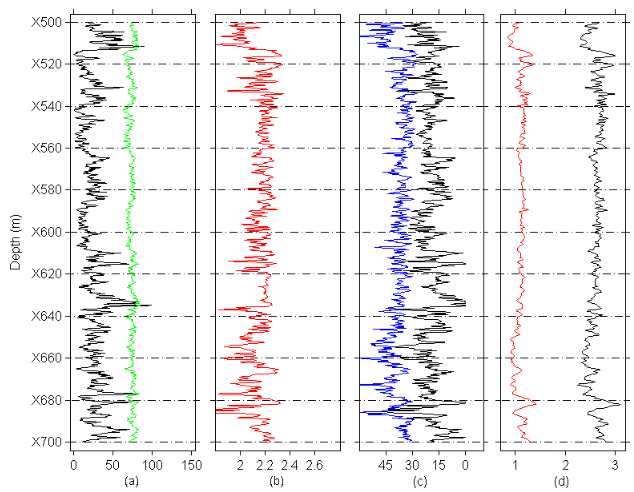

Figure 1: Well-A Logs: (a) gamma ray (GR, in green) in API units and shaliness (V_{clay}) in %, (b) density ($RHOB$) in g/cm^3, (c) neutron porosity ($NPHI$, in blue) in pu units and effective porosity (ϕ_e) in %, and (d) Velocities converted from Compressional slowness (DTC, in black) and Shear slowness (DTS) in km/s.

3) Regression Analysis and Calibration:

Assuming V_p, V_{clay} and ϕ_e as the main parameters affecting V_s, we propose the empirical equations:

$$V_s = A x^2 + B y^2 + C z^2 + D x y + E x z + F y z + \\ + G x + H y + I z + J \quad (6)$$

and

$$V_s = A x^2 + B y^2 + C z^2 + D x y + E x z + F y z + \\ + G x y z + H x + I y + J z + L \quad (7)$$

where the variables x, y and z were among the three dependence parameters V_p, V_{clay} and ϕ_e, respectively. The least-squares methodology is responsible for determining the lithological parameters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J* and *L*. The quadratic models proposed are very similar, differing in an additional term that takes into account the intercorrelation of the all variables involved. The parabolic model from Castagna et. al (1993) is often used in S-wave velocity prediction, according the following equation:

$$V_s = -1.031\, V_p^2 + 1.017\, V_p - 0.055 \quad (8)$$

For calibration, we compared S-wave velocity logs at all wells and the V_s regression models (equations 6, 7 and 8) in order to inspect the misfits in the least-squares regression models. The comparison of measured S-wave velocities to the results of the parabolic Castagna model (1993) is another way to inspect the reliability of these proposed quadratic regression models. Determination of correlation coefficients between observed and predicted velocities and plotting curves allows to analyzing the confidence on the resulting V_s regression models.

Results

Following the methodology described above, we applied regression analysis to Well-A, with the log curves shown in Figure 1. Using the dependence parameters V_p, V_{clay} and ϕ_e the proposed regression methodology yields two models for S-wave velocity variation in the analyzed interval. The least-squares coefficients are exhibited in Tables 1 and 2, and in the equations 9 and 10. As a measure of confidence on the regression models, the correlation coefficient is also incorporated into the tables. The wells were drilled in Oligocene siliciclastic sequences in Brazilian continental margins.

$$V_s = -0.107\, V_p^2 + 0.019\, V_{clay}^2 + 0.053\, \phi_e^2 + 0.048\, V_p V_{clay} + \\ -0.406\, V_p \phi_e - 0.020\, V_{clay} \phi_e + 1.183\, V_p - 0.263\, V_{clay} \\ + 0.648\, \phi_e - 1.147 \quad (9)$$

$$V_s = -0.034\, V_p^2 + 0.295\, V_{clay}^2 + 1.426\, \phi_e^2 + 0.376\, V_p V_{clay} + \\ + 0.282\, V_p \phi_e + 2.071\, V_{clay} \phi_e - 0.466\, V_p V_{clay} \phi_e + \\ + 0.578\, V_p - 1.492\, V_{clay} - 1.999\, \phi_e + 0.077 \quad (10)$$

In well-A (Figure 1), we observe p-wave velocity values up to 3000 m/s configuring slow formation, and constant lithology in the whole interval as demonstrated by gamma ray log curve. Figure 2 shows the regression models results, comparing to measured shear velocity curves (in black). We can observe that P and S-wave sonic logs response to this lithology may explain correlation coefficient value: r = 0.93 (r^2 = 0.86) for equation 6 and r = 0.94 (r^2 = 0.87) for equation 7. The additional regression model of Castagna et al. (1993) is also applied at well-A (r = 0.90 and r^2 = 0.82). The least-squares coefficients of the equations 6 and 7 are shown in the equations 9 and 10, respectively. The use of the additional term in the quadratic model described in equation 7 improves the confidence on predicting S-wave velocities. Nevertheless, the regression models both achieved nearly the same correlation coefficient and a good fit for the predicted curve (quantitative analysis). Despite showing also a good correlation coefficient for the Castagna′s (1993) models, the predicted curve shows large misfits observed in the whole interval under study.

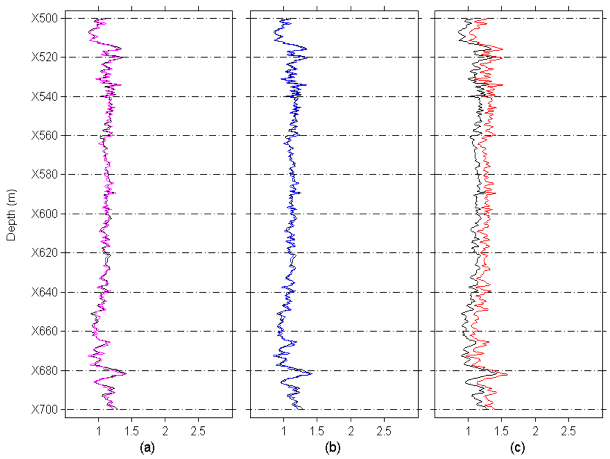

Figure 2: S-wave velocity regression models at Well-A: Measured s-wave velocities (km/s) are in black. S-wave velocities estimated using: (a) equation 6 (magenta); (b) equation 7 (blue) and; (c) equation 8 (red) as in Castagna et al. (1993).

As a final step to this calibration procedure, the regression models for Well-A are extended to Well-B and Well-C. They are around 5 km far away from Well-A. As observed in Figure 3, quadratic regression models (equations 9 and 10) shows good prediction of S-wave velocities in Well-B slow formations for equation 9 (r = 0.94 and r^2 = 0.89)and for equation 10 (r = 0.94 and r^2 = 0.88). Otherwise, Castagna et al. (1993) regression models shows a considerable misfits in the whole interval observed in Figure 3c. Even so, their correlation coefficient reached same values that the equations 9 and 10.

In the following, in Figure 4 shows that regression models applied to Well-C yields results similar to Well-B, with the two quadratic models almost identical and a considerable misfit for Castagna et al. (1993) regression models estimation. Although their correlation coefficient is high, this is caused by the high correlation that the compressional wave velocity has to shear-wave velocity. From these results, it can conclude that Castagna et al. (1993) models generally yields faster shear-wave velocity not compatible to slow formations.

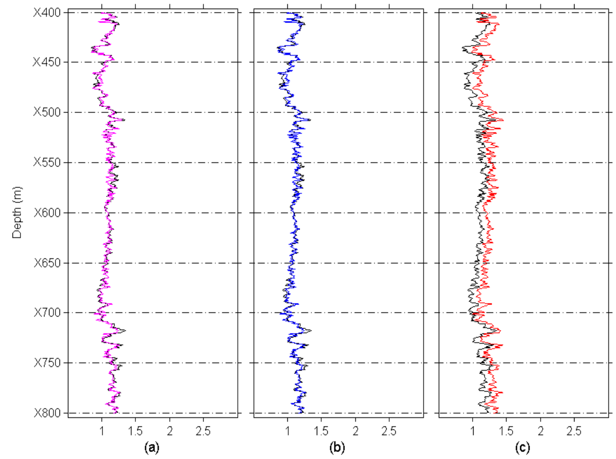

Figure 3: S-wave velocity regression models extended to Well-B: Measured s-wave velocities (km/s) are in black. S-wave velocities estimated using: (a) equation 6, in magenta; (b) equation 7, in blue; and (c) Castagna′s equation, in red.

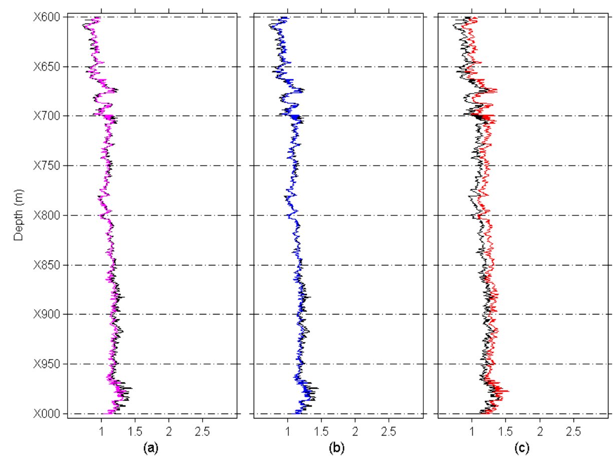

Figure 4: S-wave velocity regression models extended to Well-C: Measured s-wave velocities (km/s) are in black. S-wave velocities estimated using: (a) equation 6, in magenta; (b) equation 7, in blue; and (c) Castagna′s equation, in red.

Discussion and Conclusions

The use of regression analysis represents a suitable procedure for establishing velocity dependence in Siliciclastic slow formations. The results shows that use of Castagna et al. (1993) regression model in slow siliciclastic formation conditions is not appropriate, because it does only take into account P-wave velocity dependence. When two more variables are used the response in the predicted rock property improves. The empirical quadratic models here proposed take into account the effects of the interrelationship between the variables involved, giving more reliability to the estimation. It is indispensable to use P-wave velocity as the main variable when building a regression model to estimate shear-wave velocity. However, restricting an empirical model to use a single variable may cause some misfits.

The preliminary results presented in this contribution are a part of an ongoing research project. The main task of this project is to find the best relationship using petrophysical parameters to estimate of S-wave velocity variation in slow formations, without relying on the proximity of well with such S-wave log available. The following step is to test other petrophysical variables such as effective porosity from NMR log, Stoneley-wave velocity, and mud properties. Another challenge is to try to incorporate some important aspects like compaction and aspect ratio into the empirical model.

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Table 1: The Least-squares coefficients using equation $V_s = A V_p^2 + B V_{clay}^2 + C \phi_e^2 + D V_p V_{clay} + E V_p \phi_e + F V_{clay} \phi_e + G V_p + H V_{clay} + I \phi_e + J$ (eq. 6).

Coeff.	Well-A	-	-
A	-0.10704	-	-
B	0.0194	-	-
C	0.052607	-	-
D	0.047831	-	-
E	-0.40624	-	-
F	-0.020458	-	-
G	1.1829	-	-
H	-0.26307	-	-
I	0.6479	-	-
J	-1.1466	-	-
Corr. Coef.	Well-A	Well-B	Well-C
r	0.93	0.94	0.99
r^2	0.86	0.89	0.97

Table 2: The Least-squares coefficients using equation $V_s = A V_p^2 + B V_{clay}^2 + C \phi_e^2 + D V_p V_{clay} + E V_p \phi_e + F V_{clay} \phi_e + G V_p V_{clay} \phi_e + H V_p + I V_{clay} + J \phi_e + L$ (eq. 7).

Coeff.	Well-A	-	-
A	-0.034285	-	-
B	0.29538	-	-
C	1.4256	-	-
D	0.37605	-	-
E	0.28194	-	-
F	2.0709	-	-
G	-0.46634	-	-
H	1.57814	-	-
I	-1.4923	-	-
J	-1.9999	-	-
L	0.077032	-	-
Corr. Coef.	Well-A	Well-B	Well-C
r	0.94	0.94	0.99
r^2	0.87	0.88	0.97

Table 3: Least-squares coefficients using Castagna equation (1993) $V_s = -1.031 V_p^2 + 1.017 V_p - 0.055$.

Coeff. Corr.	Well-A	Well-B	Well-C
r	0.90	0.95	0.98
r^2	0.82	0.89	0.96