Seismic modeling of traveltimes with rays parameterized by time and arc lenght

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Abstract

The seismic waves traveltime modeling is made by means of ray tracing. The most of works that perform such kind of modeling uses numerical ray equation solutions, where rays are parameterized by a variable that does not have physical meaning, and, then the traveltime is obtained by an indirect way. This work proposes the use of the own time and arc lenght as ray parameters. These variables are physically known and, for this reason, they provide times, numerically calculated, along shorter ray trajectories. It allows to reduce error accumulation for the final traveltimes values.

Introduction

The ray theory is an integrated part of many techniques of the sismology, because it carries the possibility of quantities computation associated with the wave, such as: traveltimes and amplitudes. Based on this, sismological modeling, that uses seismic ray tracing, is largely used as tool in oil industry and in research center to constrution of subsurface image of the earth. It increases the probability to set right in hidrocarbons exploration. The model provides a sense of how each physical parameter of the medium influences the data, in the way that traveltime observation can be explicated by means of the features of the model. Along the path of seismic rays, it is possible to calculate the traveltimes of compressional waves, and, in this way, these times are used in a possible work of seismic inversion, mostly in seismic tomography.

The modeling is done in parameterized seismic velocity fields by mathematical functions, such that when applying such functions in the ray equations, the result is a set of ODE's (ordinary differential equations), that when is correctly solved, generates ray trajectories and traveltimes.

In a vast majority fo situations, the equations of ray are solved by numerical methods, and, thus, the obtained results are approximations, that are less accurate than those developed by analytical methods. Works such as: Figueiró et al., 2005; Teles and Figueiró, 2009; Mendes, 2009; dos Santos, 2008; dos Santos and Figueiró, 2006; and de Souza and Figueiró, 2004; are examples of such studies that make use of the methodology treated here to calculate traveltimes, and use ray equations as shown by Eq. (1).

The resolution of the ray equations, that serves as the basis for such modeling, is done by numerical procedures, and as is well known, accumulation of errors occurs. The question that is done, with respect to such procedure, is that, when using the Eqs. (1), the calculation of traveltimes is done indirectly, it increases the number of approximations and calculations become less accurate, therefore: the use of another type of ray parameter is able to improve the accuracy of calculated traveltimes?

The proposal, established here to perform traveltimes modeling through ray tracing, is the use of rays parameterized by time (T) and arc lenght (S), because they reduce calculations in the modeling algorithm, and they decrease numerical instabilities, increasing, thus, the accuracy of the method.

Ray Equation

Rays play a fundamental role in various branches of physics. For this reason, it is not surprising that many approaches can be used to obtain rays by means of a system of equations. The most common approach, is based on the high-frequency asymptotic solution of the acoustic wave equation, that has as result the eikonal equation ($||\nabla T||_2^2 = 1/V^2$). It can be found a detailed discussion and the derivation of such equation in Santos, 2014. To perform ray tracing, it is used isotropic media models, that have smooth variations, and in these cases, it applies the eikonal equation, which is written in cartesian coordinates as:

$$\rho_i \rho_i = \frac{1}{V(x_i)^2}, \quad \text{with} \quad \rho_i = \frac{\partial T}{\partial x_i} \quad \text{and} \quad i = 1, 2 \ e \ 3, \quad (1)$$

where $T = T(x_i)$ is traveltime, ρ_i are the vector components of slowness $\vec{\rho} = \nabla T$ that defines the normal direction of the wavefront at position $\vec{\chi} = (x_1, x_2, x_3)$ and it is tangent to the ray, and finally $V(x_i)$ is the representation of the acoustic wave velocity $v = \alpha$. The Eq. (1) can be presented by the Hamiltonian as $\mathscr{H}(x_i, \rho_i) = 0$, where $\mathscr{H}$ can take several different forms, such as $\mathscr{H}(x_i, \rho_i) = (\rho_i \rho_i - V^{-2})$, $\mathscr{H}(x_i, \rho_i) = (\rho_i \rho_i^{1/2} - V^{-1})$ or also $\mathscr{H}(x_i, \rho_i) = \frac{1}{2}(V^2 . \rho_i \rho_i - 1)$. The ray equations system is found by substitution of one of the forms in characteristic Hamiltonian system (Bleistein, 1984), and this system is solved in terms of a characteristic. In general characteristics are trajectories in 3D space $x_i = x_i(u)$, where $\mathscr{H}(x_i, \rho_i) = 0$ is satisfied, and u is a parameter of the trajectory (Červeny, 2001).

The equation of the Hamiltonian which will be adopted in this work is $\mathscr{H} = 1/n . [(\rho_i \rho_i)^{n/2} - V(x_i)^{-n}]$, where the value of n determines the parameterization of rays. In

most cases, it is used the ray equations, with $n = 2$ in the characteristic system, it makes the ray to have a as parameter $u = \tau$, where τ is an variable which do not have a physical meaning of interest. Therefore it has:

$$\frac{dx_i}{d\tau} = \rho_i, \quad \frac{d\rho_i}{d\tau} = \frac{1}{2}\frac{\partial}{\partial x_i}\left(\frac{1}{V^2}\right), \quad \frac{dT}{d\tau} = \frac{1}{V^2} \quad i = 1, 2 \ e \ 3.$$
(2)

With the objective to use the ray parameterized by time T and with arc length S, it is made $n = 0$ and $n = 1$ in the caracteristic system, which results, respectively, in Eqs. (3) and (4):

$$\begin{cases} \frac{dx_i}{dT} = (\rho_k\rho_k)^{-1}.\rho_i, & \frac{d\rho_i}{dT} = -\frac{\partial(ln(V))}{\partial x_i}, & \frac{dT}{dT} = 1 & i = 1, 2, 3. \end{cases}$$
(3)

and

$$\begin{cases} \frac{dx_i}{dS} = (\rho_k\rho_k)^{-\frac{1}{2}}.\rho_i, & \frac{d\rho_i}{dS} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\right), & \frac{dT}{dS} = \frac{1}{V} & i = 1, 2, 3. \end{cases}$$
(4)

Ray tracing algorithm and traveltime calculation

Trajectory ray equations are obtained when velocity field is substituted inside the ray equation system described above, having as result, a particular ODE system. When these equations are conveniently solved, it is produced a result that provides the position in the ray and the time that it spents to travel the distance from the source until such position. In this way, all points of ray path are obtained.

The resolution method of ray equations can be analytical, in such case is found an algebraic solution to such equations (Santos e Figueiró, 2014). Such solution can be numerical, when it is obtained numerically and it has an approximative character.

In this work, the ray tracing is done in a numerical way, and it is used the Euler method. It is equivalent to the Taylor method truncated after the first term, it can also be understood as the Runge-Kutta method of order second (Butcher, 1987). For the use of this method, it is necessary to have knowledge of boundary conditions of the problem for a given position $\vec{\chi}(u_k)$ and, it must have knowledge of the direction of $\vec{\rho}(u_k)$ at the same time. From this stage, a new position and a ray direction are found with use of the Eq. (5):

$$\begin{cases} \vec{\chi}(u_k + \Delta u) \approx \vec{\chi}(u_k) + \frac{d\vec{\chi}(u_k)}{du}.\Delta u \\ \vec{\rho}(u_k + \Delta u) \approx \vec{\rho}(u_k) + \frac{d\vec{\rho}(u_k)}{du}.\Delta u, \end{cases}$$
(5)

where Δu is the step of the parameter, and it is intimately linked to time $T(x)$ as can be seen in the Eqs. (3) and (4). The ray is a polygonal line, where the nodes are the positions $\vec{\chi}(u_k + n.\Delta u)$, with n being a positive integer, that represents the number of steps from the source up to the arrival point.

To calculate the traveltime that the wave spend to make certain route, firstly is calculated the time spent between successive nodes of a polygonal line, and, after, it is made a sum of all times wasted in each polygonal straight line segment. As can be see, the equations are different, depending on the parameterization adopted, therefore, the

manner used to perform the calculation of time depends on the used parameter. It is the main relevance of this study. When it is used the Eqs. (2), the times are calculated as the ratio of the distance between nodes by the average speed, on the segment length that links successive nodes positions that are found numerically. When using the ray parameterized by S, the time spents between these nodes are calculated with the ratio between the modulus of the step, ΔS, and the average speed. Finally, when the ray is parameterized by T, the transit time between successive nodes is equal to the magnitude of the step ΔT and so the calculation is much more economic.
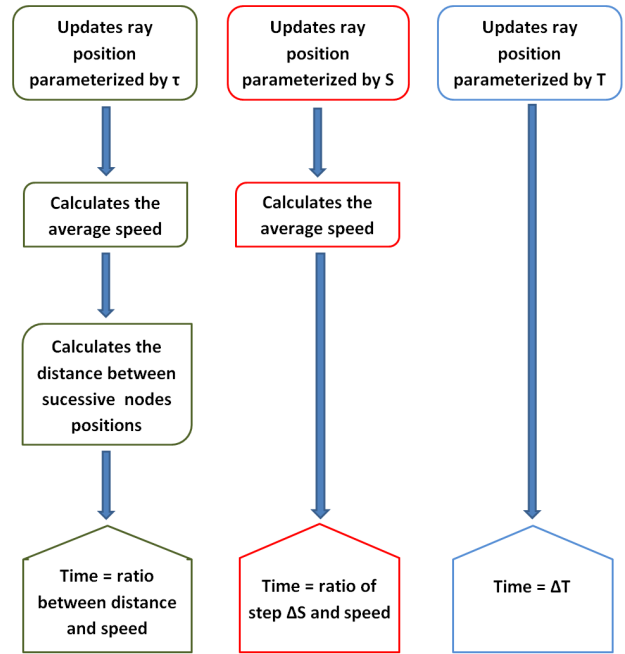

Figure 1: Flow charts for the time calculation between nodes when ray is parameterized by τ, S and T; respectively.

The Fig.1 shows flowcharts for step time calculation for each case of traveltime parameter. As can be seen, the algorithm for transit times calculation, with the ray parameterized by τ, involves some approximations which can be prevented with the ray parameterized by S and, mainly, by T. In this way, it can be seen that these last two, reduce, considerably, the action of numerical approximations, making modeling much more precise for traveltime calculation.

Initial Conditions

It is known that each ray and traveltime are completely specified when it is known boundary conditions. In this work, the method described by Eq. (5), from a point known by initial conditions, makes possible to visit new positions. Therefore, the method developed is connected to the knowledge of the starting point, components of the slowness vector and the traveltimes at this point. It means, at the source:

$$x_i = x_{i0}, \quad \rho_i = \rho_{i0}, \quad T = T_0.$$
(6)

The quantity ρ_{i0} determines the initial direction of the ray at source, which must satisfy Eq.(7):

$$\rho_{i0}\rho_{i0} = \frac{1}{V_0^2}, \qquad \text{where} \qquad V_0 = V(x_{i0}). \qquad (7)$$

In order to make results more didactic the variables x_1, x_2, x_3, p_1, p_2 e p_3, are replaced, respectively, by x, y, z, p_x, p_y, e p_z. In this work, the rays are coplanar, so they are restricted to the plane $y = 0$ with $p_y = 0$. In this way the ray trajectory and their equations are reduced to the 2-D plane. Therefore, the vectors slowness and position can be written as $\vec{\chi} = (x, z)$ and $\vec{\rho} = (p_x, p_z)$.

The components p_{i0} of slowness vector can be found at source by the angle θ that the vector $\vec{\rho}$ makes with the horizontal at the starting point, as can be seen in the Fig. 2. Eq. (8) shows the components of $\vec{\rho}_0$:

$$p_{x_0} = \frac{1}{V_0}\cos(\theta), \qquad p_{z_0} = \frac{1}{V_0}\sin(\theta), \qquad (8)$$
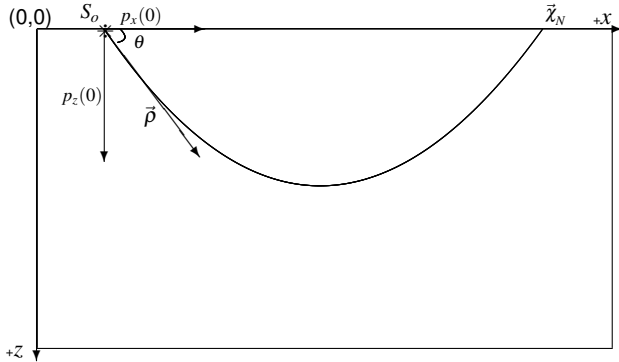
com $0 \leq \theta \leq \pi$.

Figure 2: Trajectory of a ray starting from the source S_0 with certain departure angle θ. It arrives on surface at $\vec{\chi}_N$.

At every point can be seen that the slowness vector $\vec{\rho}$ is tangent to the ray trajectory as shown in Fig. 2, and its components are slowness vetor projections on the axis x and z of the cartesian plane, so it is valid the relationship between $\vec{\rho} = (p_{x_0}, p_{z_0})$ and the slowness at the source, such as:

$$\| \vec{\rho}_0 \| = \sqrt{p_{x_0}^2 + p_{z_0}^2} = V(S_0, 0)^{-1}. \qquad (9)$$

So, it can be noted that at the source, the completed system of initial conditions is:

$$\begin{cases} x(0) = S_0, \\ z(0) = 0, \\ p_{x_0} = V(S_0, 0)^{-1} \cdot \cos(\theta), \\ p_{z_0} = V(S_0, 0)^{-1} \cdot \sin(\theta), \end{cases} \qquad (10)$$

since $u_0 = 0$. This system is used as a basis to generate rays and calculate transit times by means of the Eq. (5).

Results

To perform the experiments are selected four two-dimensional isotropic compressional velocity field models,

which are functions of the variables x and z parameterized by coefficients. In these fields, the speeds are given in kilometers per second (km/s). The choice of models aims to have a maximum resemblance to actual geological situations in order to make applicable such study.

Another important factor, in the choice of models, it is relative to the mathematical equations for velocity fields in order to reduce the complexity of ODE's involved.

In the first two models, the rays are parameterized by time T, and they are represented mathematically by equations of the type shown by Eq. (11):

$$V_n(x, z) = F_n.e^{(A_n + B_n.x + C_n.z + D_n.x^2 + E_n.z^2)} \qquad n = 1 \ e \ 2 \quad (11)$$

where n represents each model. The first, hereby appointed by M_1, tries to represent a layer of salt that has suffered a slight compression in the central region, deforming it as shown in Fig. 3, and this model is parameterized by Eq. (11), where the coefficients have the values $F_1 = 0.0100, A_1 = 4.8400, B_1 = -0.2020, C_1 = 3.5000, D_1 = 0.0595$ and $E_1 = -3.5000$.

For the second case, the model M_2, seen in Fig. 4, tried to portray a geological situation known as antiform anticline, where, also using Eq. (11) the coefficients are $F_2 = 1.0000, A_2 = 0.4500, B_2 = 0.3300, C_2 = 0.4085, D_2 = -0.0880$ and $E_2 = 0.4050$.

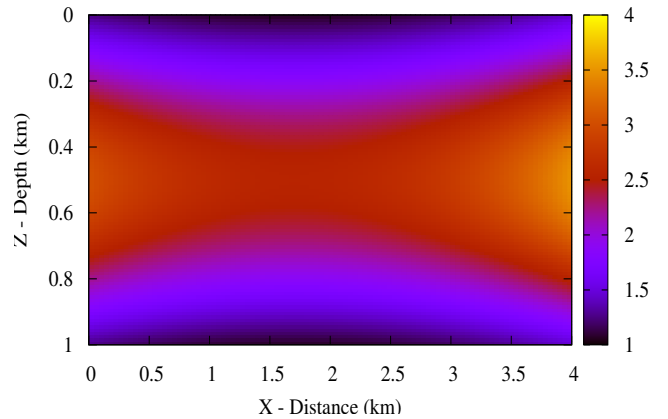The units of these coefficients are compatible with those chosen to x, z and t.

Figure 3: Model M_1, given by Eq. (11) with $n = 1$.

For other two models, it used as parameters to the rays arc length S, and, therefore, the velocity fields are written as shown by Eq. (12):

$$V_n(x, z) = \frac{1}{A_n + B_n.x + C_n.z + D_n.x^2 + E_n.z^2} \qquad n = 3 \ e \ 4. \quad (12)$$

In model M_3, coefficients have the following values: $A_3 = 0.9900, B_3 = -0.4198, C_3 = -0.8986, D_3 = 0.0872$ and $E_3 = 0.6319$. For such values: model tries to represent something like a granitic intrusion as can be seen in Fig. 5. Differently, the model M_4 has resemblance to the continental shelf break model, and it is used: $A_4 = 0.2200, B_4 = 0.0000, C_4 = 0.0000, D_4 = 0.0000$ and $E_4 = -0.1350$ in the Eq. (12). M_4 is shown in Fig. 6.
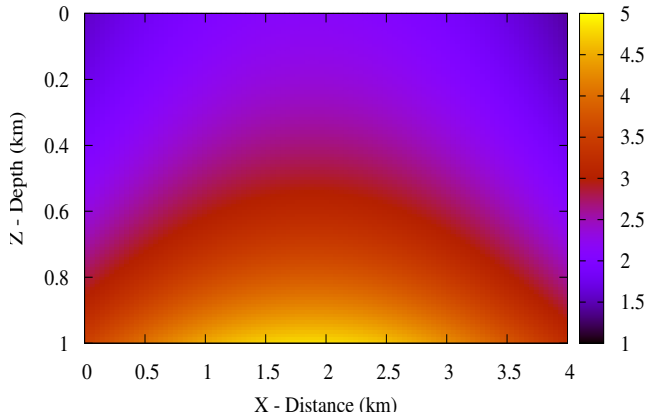
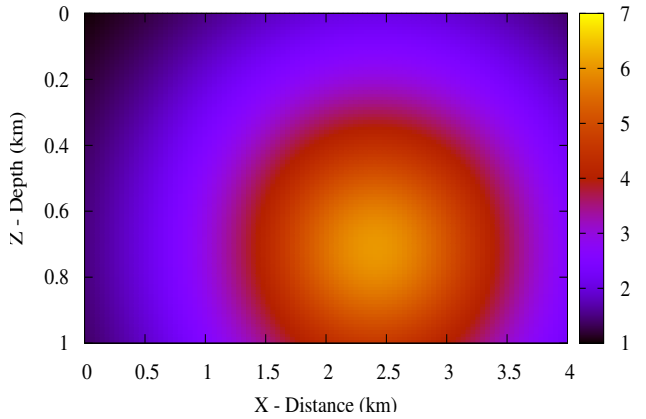Figure 4: Model M_2, given by Eq. (11) with $n = 2$.

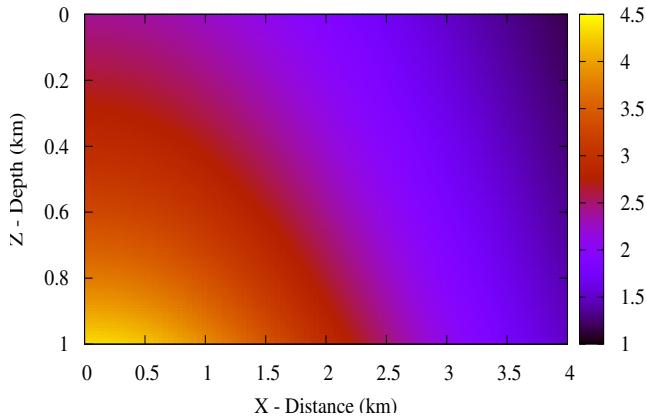Figure 5: Model M_3, given by Eq. (12) with $n = 3$.

Figure 6: Model M_4, given by Eq. (12) with $n = 4$.

In this work ray tracing was done based in shooting methods, where the ray comes from source according to one given direction. Using ray tracing it is possible calculate ray attributes that reach to the surface after travel inside the model without overcome their limits. At the moment of arrival, it is calculated position and the traveltimes of these rays. It occurs, then, plotting the path described by the ray and it is generated graphics of time as a function of the positions, which are known as time profiles. In each of the models, it is choosen a position on the surface to place the source $(\vec{\chi}(S_0, 0))$, and the calculations are made throughout the extension of a split-spread asymmetric pattern, resulting in common shooting families profiles.

For the model M_1 the source was placed at $S_0 = 1.5\,km$, generating the trace exposed in Fig 7 by means of the Eq. (3). The calculation of the traveltimes on these rays is the basis for the generation of profiles shown in Fig. 8.
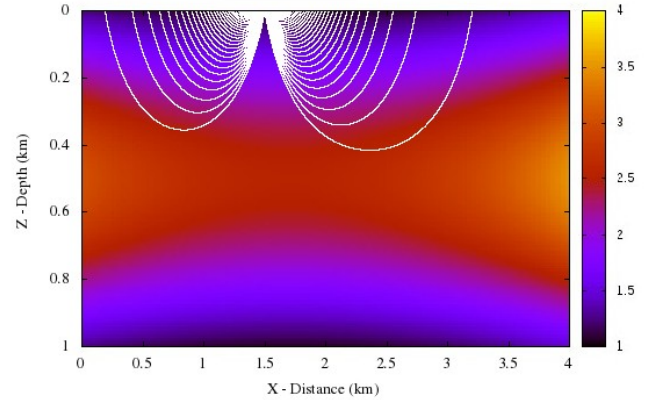

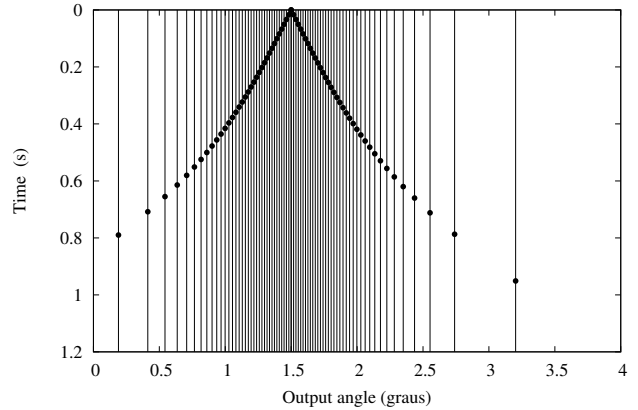Figure 7: Ray field parameterized by time T, obtained for the model M_1.

Figure 8: Traveltimes profile obtained with ray tracing parameterized by time T for the model M_1

In the case of M_2, the source is placed at $S_0 = 1.5\,km$ and the ray tracing is done, too, based on Eq. (3) which resulted in the ray field shown in Fig. 9 and its traveltimes profile in Fig. 10.
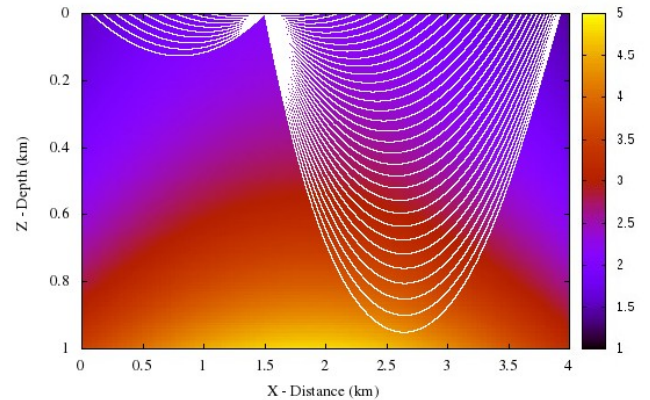

Figure 9: Ray field parameterized by time T, obtained for the model M_2.

For the model M_3, the ray tracing is made with the ray parameterized by S, and it is based on Eq. (4). In
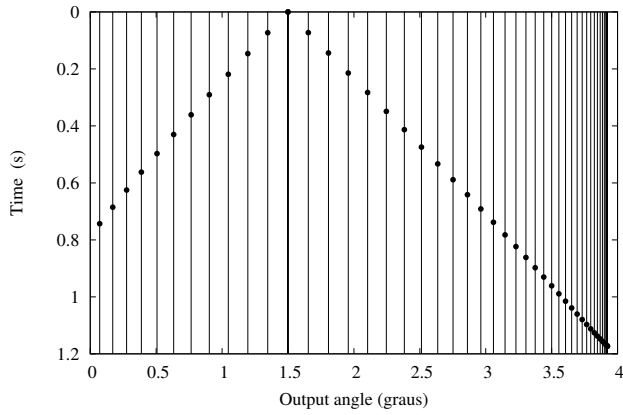
Figure 10: Traveltimes profile obtained with ray tracing parameterized by time T for the model M_2.

this model, the source is positioned at $S_0 = 2.0 km$, which generated the ray field exposed in Fig. 11 and the traveltime profile is shown in Fig. 12.
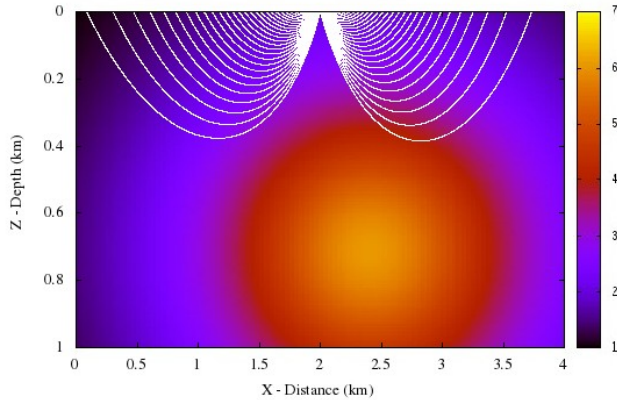

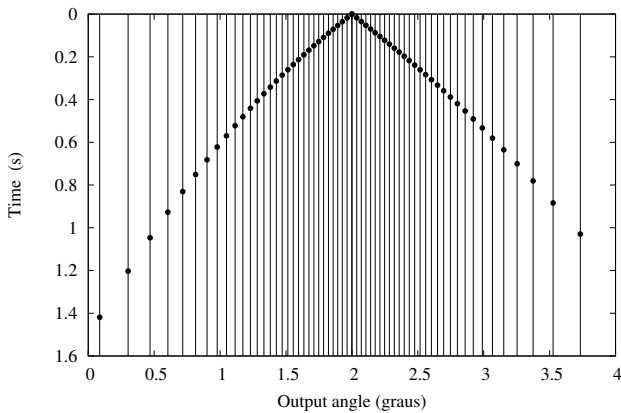Figure 11: Ray field parameterized by arc length S, obtained for the model M_3.

Figure 12: Traveltimes profile obtained with ray tracing parameterized by arc length S for the model M_3.

Finally, for the model M_4 the source is placed in the position $S_0 = 1.0 km$ and once again the ray tracing is based on Eq. (4), resulting in Fig. 13 and these rays were responsible for the generation of the profile shown in Fig. 14.
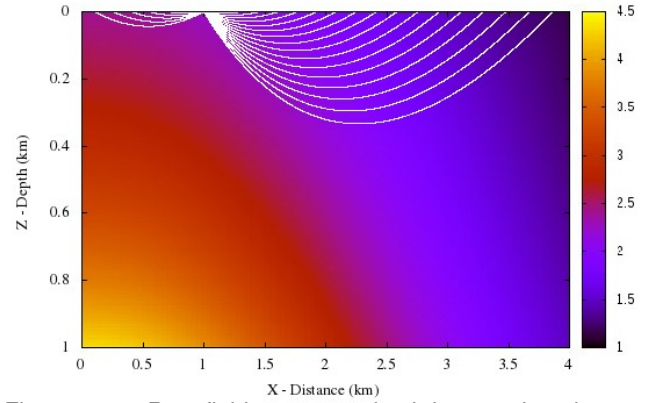

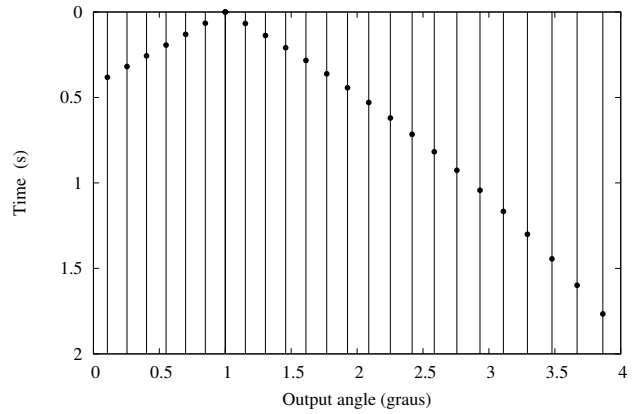Figure 13: Ray field parameterized by arc length S, obtained for the model M_4.

Figure 14: Traveltimes profile obtained with ray tracing parameterized by arc length S for the model M_4.

Discussion and Conclusions

As it is stressed, the seismic modeling using ray tracing is the basis for seismic procedures, such as: migration and inversion, for example. It is known that the latter is highly dependent on the direct problem solution, that is: on the modeling quality. Many of the mentioned procedures use results generated by modeling to solve, also in numerical way, other seismic problems. Therefore, it is necessary to look for resources that tend to make more accurate final results, and this need can be achieved by means of reduction of numerical steps and, so, the number of approximations. In this study, this is achieved through the use of alternative parameterizations for the ray. As can be seen, for traveltimes calculation, the number of numerical steps necessary, to reach the final results, grows depending on the type of ray parameter. The sequence of increasing difficulty is as follows: T, S and τ. Thus, it can be concluded that, to model traveltimes with maximum accuracy and economy in processing, it must be chosen rays parameterized by time T.

Another factor that made interesting the use of alternative parameterizations of rays is the fact that, depending on the complexity of the model and of the function of the velocity field, the ray equations can have a very simpler solution when choosing the appropriate ray parameter. This can be seen in the models M_1 and M_2, which have equation of exponential type. In this case, the most appropriate parameterization for the ray is T. Then, the equations of

the ray are given by Eq. (3), and when using the mentioned velocity field such equation is solved in a very simple way.

References

Bleistein, N.; 1984. Mathematical Methods for Wave Phenomena. Academic press.

Butcher, J.C.; 1987. The Numerical Analysis of Ordinary Differential Equations: Runge-Kutta and General Linear Methods. Wiley-Interscience.

Červeny, V.; 2001. Seismic Ray Theory. Cambridge University Press.

Figueiró, W.M.; Oliveira, S.P. and Novaes, F.C; 2005. Traveltime data profiles obtained using seismic ray tracing methods for the continental slope model parameterized by polynomials. In: 9th International Congress of the Brazilian Geophysial Society, Expanded Abstracts, Salvador, BA, Brazil.

Mendes, F.C.; 2009. Modelagem Sísmica de Curvas de Tempos de Trânsito usando Traçamento de Raios em Campos de Velocidades Parametrizados por Polinômios. Trabalho de Graduação em Geofísica, UFBA.

dos Santos, M.A.B.; 2008. Modelagem Sismológica de Curvas de Tempos de Trânsito com Parametrização por Série Trigonometrica do Campo Global de Velocidades. Trabalho de Graduação em Geofísica, UFBA.

dos Santos, R.H.M. and Figueiró, W.M.; 2006. Modelagem acústica bidimensional usando diferentes parametrizações de campos de velocidades. Brazilian Journal of Geophysics. **24** (1): 103-105.

de Souza A.E.C.M. and Figueiró, W.M.; 2004. Campos de tempos de trânsito obtidos por traçamento de raios sísmicos em campos de velocidades com parametrização polinomal. In: I Brazilian Symposium on Geophysics, Expanded Abstracts, São Paulo, SP, Brasil.

dos Santos, M.A.B.; 2008. Modelagem Sismológica de Curvas de Tempos de Trânsito com Parametrização por Série Trigonometrica do Campo Global de Velocidades. Trabalho de Graduação em Geofísica, UFBA.

Santos, R.S.; 2014. Estudo Avaliativo da Abordagem Numérica do Traçamento de Raios na Modelagem Sísmica de Tempos de Trânsito. Trabalho de Graduação em Geofísica, UFBA.

Santos, R.S. and Figueiró W.M.; 2014. Estudo avaliativo da abordagem numérica do traçamento de raios na modelagem sísmica de tempos de trânsito. In: VI Simpósio Brasileiro de Geofísica, Expanded Abstract, Porto Alegre, RG, Brasil.

Teles, M.R.R. and Figueiró, W.M.; 2009. Modelagem sísmica de tempos de trânsito de reflexão em interface parabólica. In: 11th International Congress of the Brazilian Geophysial Society, Expanded Abstract, Salvador, BA, Brazil.