Time-depth conversion using image ray and normal ray in the tau-p domain: application in synthetic data

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Abstract

The time-depth seismic conversion is an important tool in seismic processing in order to obtain friendly results to the interpreter. The achievement of these sections, in depth, can be accomplished by pre- and post-stacking migration and by ray tracing methods. The tracing of normal and image rays were the methodologies used in this work for such conversion. The image ray converts a post-stacked time-migrated section to depth domain using only an initial direction of propagation. The normal ray converts time-stacked sections into depth sections requiring a set of a priori initial directions, this initial set of directions is obtained by slant stack technique. The sections obtained by image ray tracing showed good accuracy in recovering the depth of reflectors. The sections obtained by normal ray tracing recovered, with great accuracy, the depth of reflectors as well as the good lateral continuity of the same.

Introduction

Several steps are used for the treatment of seismic data with the aim of obtaining an image of the subsurface. One of these steps is the migration. The migration may be applied on pre- or post-stacked sections, and can be performed in time or depth domain. The time migration allocates the seismic events the corrected position in the time domain while the depth migration converts seismic data to the depth domain.

Concomitantly to the migration, there is the tracing method of image rays and normal rays. The image ray is constructed with the initial condition perpendicular to the surface and traced back until the time of the time-migrated section is consumed. The ray is called normal when it focuses perpendicular to the reflector (ČERVENÝ, 1987; ČERVENÝ, 2001; POPOV, 2002). The normal rays are constructed by varying the angle at which the initial rays run from the surface. This angle varies within the limits of the ray parameters. The image ray converts post-stacked time-migrated sections in depth sections. The normal ray, in this work, converts seismic sections in the $\tau\text{−}p$ domain to the depth domain.

The conversion of the sections from $t\text{−}x$ domain to $\tau\text{−}p$ domain coverts the seismic data from the "traveltime domain" to the "intersection time and ray parameter domain". This procedure allows that the initial conditions of the normal ray be satisfied: initial angle with the vertical, initial slowness, initial coordinate from where the ray runs and total time for the ray tracing (HARLAN, 1984; STOFFA, 1981; DIEBOLD, 1981). Also, the conversion to the $\tau\text{−}p$ domain is used in the conventional processing for suppressing multiple waves and and for filtering refractions and guided waves (STOFFA,1981).

The conversion of normal rays requires several rays are set for each point to be converted from the time section. Thus, the computational effort is much higher when compared to the conversion ray image, because it requires only one ray traced.

In this paper, we applied the time-depth conversions using normal and image rays, comparing the results obtained for each methodology, besides the computational effort made by both methods conversions.

Method

The steps used in this work followed the flowchart of Figure 1.

This flowchart was applied in a synthetic data model. Thus, the steps used are different than those ones performed on real data, because of those steps as deconvolution, filtering, gain, stretch correction, mute, among others, were not applied.

The first step is the data edition. We eliminate the coherent and random noise and we organize the traces in the most appropriated form for processing.

The second step is the data ordenation in Commom Middle Points (CMP) gathers. Thus, each gather contains information of the same coordinate of the reflector in depth. (YILMAZ, 2001).

The third step, velocity analysis, estimates the velocity function for CMP stacking. This velocity is used in the further step, the Normal Moveout (NMO) correction.

The fourth step, NMO correction, corrects the traveltime hyperbolic curve to the zero-offset (zo) traveltime.

The fifth step, the stacking, is performed summing all the traces from the NMO-corrected data. The result increases the signal-to-noise ratio and simulates a zero-offset section (YILMAZ, 2001).

From the fifth step on, the methodology used in this paper follows two sub-steps:

1 - The conventional one where we used a Kirchhoff time-migration section and then we converted it to the depth domain using the image ray tracing;

2 - Alternative methodology which was performed the slant stack technique and the depth migration using normal ray tracing.

The seismic migration step is the procedure that places the geological features (non-horizontal layers, faults, synclines etc.) at their real coordinates and collapses the diffractions.

The step of Slant Stack is performed with the transformation to $\tau - p$ domain. The seismic data is decomposed into series of straight lines which will map a section in the $\tau - p$ domain. This process is called Slant Stack so that the data in $\tau - p$ domain is obtained by sum along straight lines in the $x - t$ domain (BESSENOVA et. al., 1974; CHAPMAN, 1978; DIEBOLD & STOFFA, 1981).

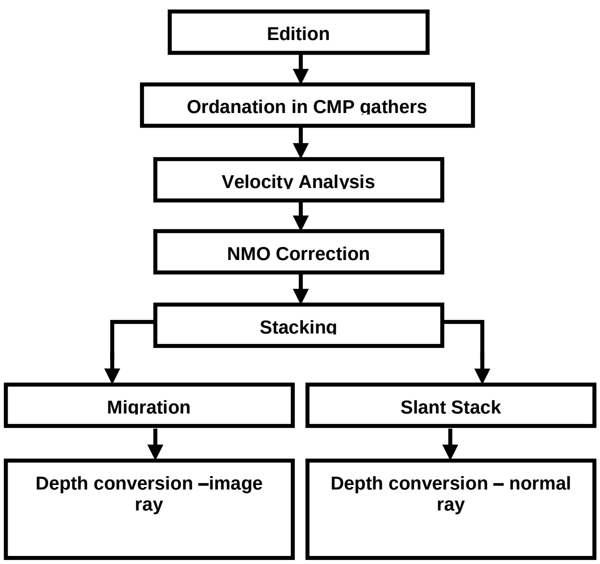

Figura 1 - Flowchart of the steps used in this paper.

Conversions using image and normal rays were performed using the ray theory. This theory is widely used in various steps of seismic processing. It is divided into kinematic tracing (determined by the traveltime along the ray) and dynamic ray tracing (calculation of the distribution of energy propagated in the geological environment) (HUBRAL, 1980; ČERVENÝ, 2001). In this study we used the kinematic ray tracing.

We must remember that for ray tracing, the velocity model must be known. As the processing was performed on synthetic data, the velocity model in depth is known.

The kinematic ray tracing provides the traveltime given by the eikonal equation:

$$(\nabla\tau(\mathbf{x}))^2 = \frac{1}{v^2(\mathbf{x})} \qquad (1)$$

wherein $\tau = \tau(\mathbf{x})$ is the traveltime and $v = v(\mathbf{x})$ is the velocity of the medium. This equation calculates the traveltime for a given set of rays, allowing the construction of wave fronts through the calculation of the travelimes along each ray.

The expression (1) results in a non-linear system of four ordinary differential equations (ODEs) for the 2D case, and 6 ODE equations for the 3D case. These ODEs are called the ray kinematic equations (CERVENY, 2001). Parameterizing (1) using the traveltime, it takes the form:

$$\frac{dx_i}{d\tau} = v^2 p_i, \qquad i = 1{,}2 \qquad (2)$$

$$\frac{dp_i}{d\tau} = -\frac{1}{v}\frac{dv}{dx_i}, \qquad i = 1{,}2 \qquad (3)$$

wherein x_i represents the coordinates of the position vector $\mathbf{x}$ along the ray, and the parameter p_i represents the components of the slowness vector, defined by $\mathbf{p} = \nabla\tau(\mathbf{x})$, com $|\mathbf{p}| = 1/v$.

The normal ray is different from the image ray because of the initial parameter of ray p. This parameter determines the direction in which the ray propagates. The normal ray has p not constant, so that the ray can have any initial direction. Otherwise the ray image has p perpendicular to the surface. Both are traced until that the half of the traveltime is consumed. Figure 2 exemplifies the case of the normal ray while Figure 3 shows the image ray
.

The seismic data is typically mapped and acquired in the $x - t$ domain. If the data is mapped to the time intersection domain τ, the horizontal parameter is p (BENESSOVA et. al, 1974; CHAPMAN, 1978). Set each domain parameter $x - t$ and $\tau - p$, we can do the transformation from one domain to the other, by the sum along straight set the appropriate parameter. Thus, the intersection of time τ is given by:

$$\tau = t - px \qquad (4)$$

$$F(\tau, p) = \int f(\tau + px, x)dx \qquad (5)$$

wherein τ is the intersection time, t is the traveltime, p is the ray parameter and x is the coordinate where the ray starts.

The section in the $x - t$ domain is mapped through several straight lines with p directions and linear coefficient τ (intersection time). The amplitudes intercepted by these lines are summed and passed to the $\tau - p$ domain. Each amplitude in $\tau - p$ domain may be related to a parameter p, the time to be consumed in ray tracing t, and the initial coordinate where the ray starts x, obtained by (4). Thus, in the $\tau - p$ domain, the reverse transformation is done by straight lines with slopes x, with intersection point τ. These straight lines cut the $\tau - p$ section in various amplitudes, each with a differents p. With the initial conditions *x,p, t/2*, the normal ray is traced, converting the $\tau - p$ section into the depth domain. The ray parameter p will provide the initial direction of the normal ray through the expression

$$\theta_{emerg} = asin\left(\frac{p}{2V_{surf}}\right) = \theta_0 \quad (6)$$

where V_{surf} is the velocity in the surface coordinate, p is the ray parameter and θ_{emerg} is the inital direction angle.

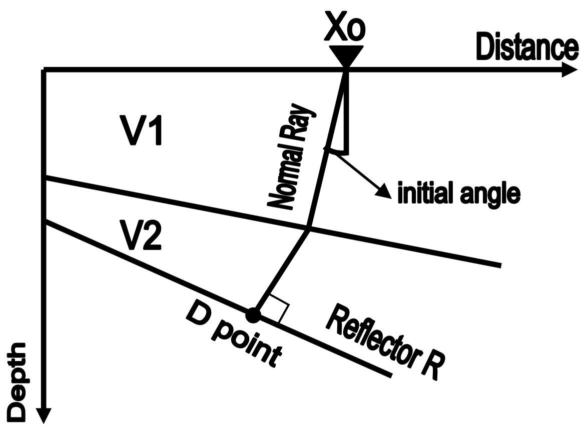

Figure 2 - Representation of a normal ray. In this experiment called zero offset, source and receiver are at the same point. This ray focuses perpendicular to the reflector R, at point D, from a given initial angle for that specific situation. (Source: Modified ROBEIN , 2003).

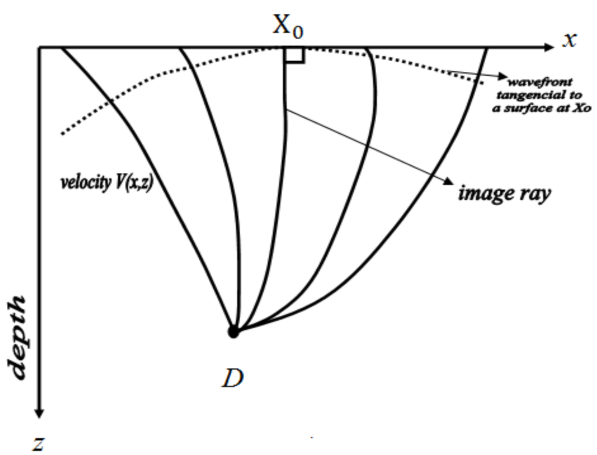

Figure 3 - Model with velocity varying in the directions of the x and z axes. Notice that the image ray is the only one which is perpendicular to the surface z=0, starting from the difractor point D in depth. This point generates several rays which creates several wavefronts (Source: modified from ROBEIN, 2003).

The time-depth conversion using image ray converts a time-domain migrated section. For each point of coordinates $x - t$ of the migrated section, an image ray is traced from the corresponding coordinate space x until half of the double traveltime of the migrated section is consumed. At the end point of the ray tracing is placed the amplitude of the respective point of the migrated section. After conversion of all points of the migrated section in time for depth, we have the seismic section in depth. Migration was used Kirchhoff time migration (YILMAZ, 1987 and CLAERBOUT, 1993).

Examples

In this study we used the synthetic velocity model shown in Figure 4. It is composed of six layers with constant velocities and smooth interfaces. The velocities are v_1 = 1508m/s, v_2 = 1581m/s, v_3 = 1690m/s, v_4 = 1826 m/s, v_5 = 2000 m/s and v_6 = 2236m/s.

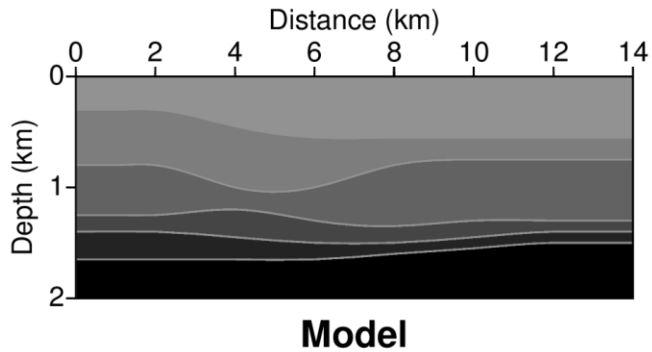

Figure 4 – Synthetic velocity model for six layers.

The Table 1 below shows the parameters used in the synthetic data acquisition using the software *Seismic Unix.*

Parameters of the data acquisiton	
Survey extention	14km
Shots	12000
Sources	200
Receivers	60
Interval among receivers	50m
Interval among shots	50m
Sampling rate	0.002s
Time window	3s
Samples per trace	1502
Coordinate of the first/last receiver	525m/13425m
CMPs	458
Peak frequency of the wavelet	0.1/*dt* = 0.1/0.002

Table 1 - Table with the parameters used in the synthetic survey.

Results

The results of this paper were obtained from the model of Figure 4 generated in Seismic Unix software. The algorithms for normal and image ray tracing were written in Fortran and Matlab. The layers that compose the model have constant velocity and are bounded by smooth curves and interfaces. The geometry of the survey was split spread type.

According to the flowchart of Figure 1, Figure 5 shows the stacked section from the NMO correction.

The Figure 6 shows the result of Kirchhoff time-migration applied to the stacked section of Figure 5.

The Figure 7 shows the result of the transformation of the stacked section from Figure 5 to the $\tau\text{−}\rho$ domain.

The Figure 8 shows the result of time-depth conversion applied to the section of Figure 6. This conversion was performed by the image ray tracing. Figure 9 shows the result of time-depth conversion applied to the section of Figure 7. This conversion was performed by tracing normal rays with the parameters obtained by the proposed methodology.

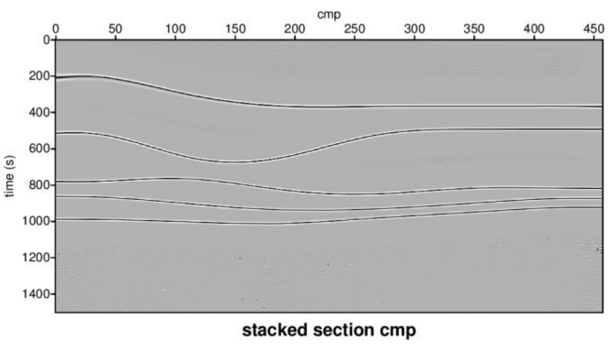

Figure 5 – NMO-corrected stacked section from figure 4.

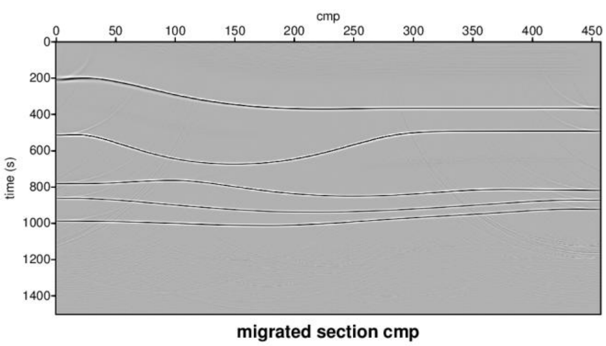

Figure 6 – Migrated section from the stacked section of Figure 5. These data were migrated using the Kirchhoff method through *Seismic Unix* software.

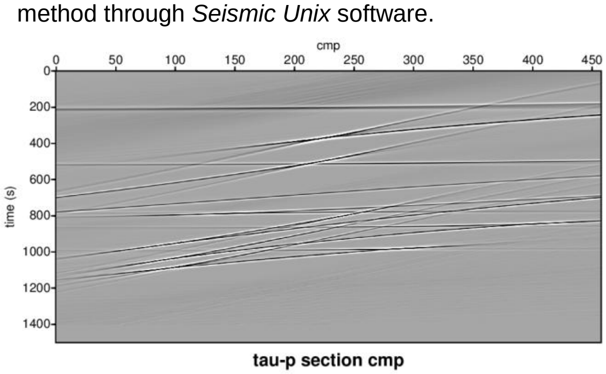

Figure 7 – $\tau\text{−}\rho$ section obtained from the stacked section of Figure 5.

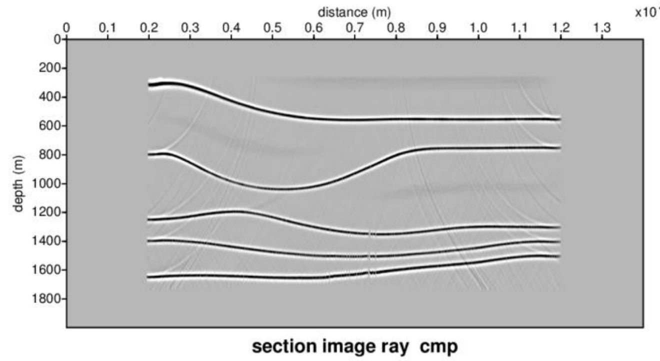

Figure 8 – Depth section obtained by the image ray method from the migrated section of Figure 6.

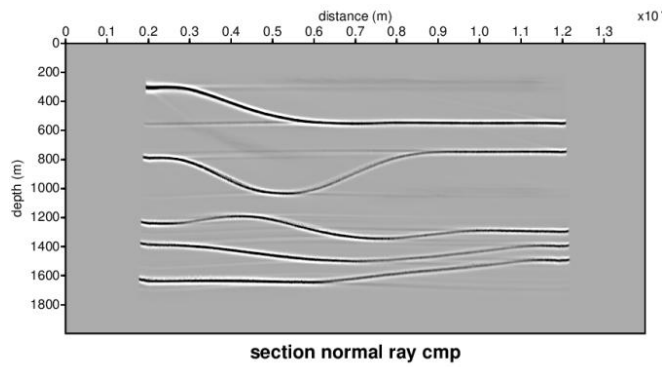

Figure 9 – Depth section obtained by the normal ray method from the $\tau - \rho$ section of Figure 7.

Conclusions

The conversion using ray image shows a satisfactory recovery of the reflectors' features as well as their depths. However, the reflectors presented failures in some areas due to lack of information converted (only one image ray is traced for each information in time). Furthermore, artifacts inherent of time migration are also converted to depth.

The section in depth, obtained from the $\tau - p$ seismogram, shows that there is a better recovery of the reflectors´ continuity because the number of normal ray tracings is larger than in the image ray method. However, these sections suffer from the inherent artifacts of $\tau - p$ conversion. These artifacts exist due to the aliasing of the conversion to $\tau - p$ domain (STOFFA, 1981), and the same data can be migrated to the depth domain more than once in a constructive way.

Note that the time-depth conversion using image rays trace only a ray for each data of the time-migrated section. In the conversion of normal ray traces a set of rays with initial directions is obtained by the method explained. These initial parameters p, provide the initial direction for each data to be converted from the section in the $\tau - p$ domain. This increases the computer time to time - depth conversion, because few image rays are traced when compared to the amount of normal rays to be traced.

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