Prestack seismic waveform inversion in the angle domian: the AVA – FWI method

Sergio Adriano M. Oliveira - LENEP/UENF, INCT-GP and Invision Geophysics
Anderson Wilson Peixoto de Franco - Grupo de Inferência de Reservatório GIR/LENEP

This paper was prepared for presentation during the 14th International Congress of the Brazilian Geophysical Society held in Rio de Janeiro, Brazil, August 3-6, 2015.

Abstract

We present a new seismic nonlinear waveform inversion method designed to determining the elastic parameters from amplitude variations with incidence angle (AVA). The method is based on the exact Zoeppritz reflection coefficients and considers the complete seismic response of a layered medium and therefore is not subject to various simplifications assumed in the conventional, primaries only linear AVA inversion. It is more appropriate to handle with the presence of thin layers, especially in the case of high impedance contrasts. Another advantage is that the range of angles that can be used for inversion is extended, what is very important for more accurate density estimation. The feasibility of the method is demonstrated by an example of application connected to the geophysical characterization of a gas reservoir. We also address the problem of the high computational cost by presenting an efficient strategy for calculation of differential angles gathers.

Introduction

Currently, the estimation of subsurface elastic parameters from seismic data is performed by linear inversion methods, using AVO (angle versus offset) or AVA (angle versus angle) amplitude information. These two close related approaches use approximations of Zoeppritz equation for modeling seismic amplitude as a function of angle of incidence and contrast of elastic parameters between layers. Such techniques have been able to provide reasonable results mainly for P wave impedance, provided that the incidence angle is restricted to an upper limit of about 30 degrees. However the conventional AVO/AVA based elastic inversion suffers many limitations. The approximations used are valid only for small contrasts between the elastic properties of the layers. Density estimation from AVO inversion is particularly difficult, as demonstrated by Lines (1999) that based his study in Fatti approximation (Fatti et all,1994). The problem is that, given small incidence angles, the approximated reflection coefficient *Rpp* has low sensitivity with respect to density variations and the use of larger incidence angles is problematic due to two key factors: the first is that the approximation is valid only until the critical angle. The second concerns the influence of locally converted S waves and internal multiples (Mallick, 2007). These events diverting the AVA curve from the behavior expected by the conventional method of inversion, since it considers only the primaries reflections. This effect tends to become more pronounced at larger incidence angles, particularly for thin layers and great elastic properties contrasts (Simmons and Backus,1994). More recently Hounie and Oliveira (2014) showed that the influence of locally converted waves in the result of elastic inversion can actually be quite significant, particularly with respect to the determination of density.

An alternative to linear AVO/AVA inversion is the waveform inversion. This nonlinear method determine a subsurface model that explain the seismic data taking into account all the events and propagation effects, so it is based on the wave equation. The inversion algorithm we present in this work is called AVA-FWI (Angle versus Angle Full waveform inversion) because has the angle gathers as input data. It uses the Gauss Newton method for minimizing the objective function where the most crucial step is the synthetic angle gathers modeling and the calculation of differential angles gathers. These are the heaviest computational task, therefore we develop and present an efficient way to compute these seismograms in the plane wave domain using the reflectivity method.

Method

The linear AVA inversion is based on approximations of Zoeppritz equations given in terms of elastic properties contrasts between layers (Aki and Richards, 2002). The approximated equation for the P wave reflection coefficient can be written in the following generic way:

$$R_{pp} \cong a\,r_\alpha + b r_\beta + c\,r_\rho \qquad (1)$$

Where $r_\alpha=(\alpha_2-\alpha_1)/(\alpha_2+\alpha_1)$, $r_\beta=(\beta_2-\beta_1)/(\beta_2+\beta_1)$, $r_\rho=(\rho_2-\rho_1)/(\rho_2+\rho_1)$, α *is the P wave velocity*, β *is the S wave velocity*, ρ *is the density and* the coefficients a, b and c depend on the incidence angle and the ratio between the S and P-wave velocity. This formula can be presented in the following matrix form to model the amplitude as function of incidence angle:

$$\mathbf{Mr} = \mathbf{d} \qquad (2)$$

Where the matrix $\mathbf{M}$ has three coluns and N lines whose elements are: $m_{i,1}=a_i$, $m_{i,2}=b_i$ e $m_{i,3}=c_i$. The column vector $\mathbf{r}$ contains the coefficients r_α, r_β, r_ρ and $\mathbf{d}$ is the vector whose elements are the amplitudes of the reflected waves with incidence angle θ_i.

In AVA inversion, the vector $\mathbf{d}$ contains the amplitudes of reflections at many incidence angles and the over determined system represented by equation (2) can be solved in the least squares sense in order to determine $\mathbf{r}$, which leads to the following normal equation:

$$\mathbf{M}^{\mathbf{T}}\mathbf{M}\mathbf{r} = \mathbf{M}^{\mathbf{T}}\mathbf{d} \qquad (3)$$

This is the most direct and simplest type of elastic inversion. It determines r_α, r_β and r_ρ at each time sample over the angle gather. However, such inversion scheme is based on approximated formulas for reflection coefficient R_{pp} and it also assumes that the amplitudes are influenced only by the primaries reflections and also does not consider transmission effects. In the nonlinear waveform inversion all these effects are taken into account. In this case the mathematical relationship between the data and the model parameters is given by:

$$\mathbf{d} = \mathbf{S}(\mathbf{m}) \qquad (4)$$

Where $\boldsymbol{S}$ is a mathematical operator that links the data and the model parameter vector $\mathbf{m}$. Note that equation (2) is a particular case of equation (2), where $\mathbf{S(m)=Mr}$. In the present work the subsurface geology is represented as 1-D media and is discretized into elementary layers of the same width. The parameters to be determined are the P wave velocity, S-wave velocity and density for each layer. The inversion is done by finding a solution vector $\mathbf{m}$ that minimizes an objective function which measures the distance between the observed and calculated data:

$$E(\mathbf{m}) = \Delta\mathbf{d}^{\mathbf{T}}\Delta\mathbf{d} + \lambda\,\Delta\mathbf{m}^{\mathbf{T}}\Delta\mathbf{m} \qquad (5)$$

Where $\mathbf{\Delta d}$ is a vector whose elements are the difference between the calculated and observed data: $\mathbf{\Delta d=(S(m)-d)}$ and the vector $\mathbf{\Delta m}$ is the difference between $\mathbf{m}$ and the initial model parameter vector: $\mathbf{\Delta m=(m-m_0)}$. This function uses the standard L2 norm for measuring the distance between the calculated and observed data and the distance between the solution and the initial model (Menke, 1989). The inverse problem in question is ill-posed and admits several solutions that meet some tolerance criterion to minimize the data error. Because of this, the second term of the equation (5) is essential to regularize the problem, since it penalizes solutions that deviate much of the initial model (Thikonov and Arsenin, 1977). This initial model should contain the solution components that belongs to the null space of the model space, ie the components that, when disturbed, do not generate changes in the data. The variable λ controls the regularization weight, higher values of λ favor solutions that do not deviate much from the initial model.

Here the classic Gauss Newton method (Fletcher, 1987) will be adopted. In this method the vector $\mathbf{m}$ is updated to approach the solution that minimizes the objective function (5), for this the following linear system have to be solved at each iteration (Aster et all, 2005):

$$(\mathbf{J}_k^{\mathbf{T}}\mathbf{J}_k + \lambda\mathbf{I})\boldsymbol{\delta}_k = -\mathbf{J}_k^{\mathbf{T}}\Delta\mathbf{d}_k - \lambda\,\Delta\mathbf{m}_k \qquad (6)$$

The solution is then updated as: $\mathbf{m}_{k+1} = \mathbf{m}_k + \boldsymbol{\delta}_k$. The Jacobian $\mathbf{J}$, also known as sensitivity matrix, has as elements the derivatives of the data with respect to the model parameters: $J_{o,p}=\partial S_o/\partial m_p$. Note that the Jacobian, the data difference and model difference vectors must be updated at each iteration.

In the present inversion method the subsurface is considered a 1-D layered medium and the reflectivety method (Müller, 1985) is used in order to calculate the sinthetic angle gathers and the differential angle gathers. In the reflectivety method, the response of the layered medium to an harmonic elastic plane wave with angular frequency ω and ray parameter p is given by the function $r(p,\omega)$. This method is based on a recursive formula. The transmission effect, internal multiple and converted waves occurring in each layer is added to the solution from the last layer until it reaches the top of the package and the full response of the medium is obtained. In the particular case we are interested in the response of the medium to a P wave and $r(p,\omega)$ corresponds to the reflectivity at the top of the first layer: $r(p,\omega) = r_{pp}^0$.

A plane wave seismogram can be obtained in the τ-p domain, to this we must take the solution $r(p,\omega)$ to the time domain, taking into account the influence of the seismic pulse:

$$S(p,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\; r(p,\omega)\; e^{i\omega\tau} d\omega \qquad (7)$$

Where $F(\omega)$ is the Fourier transform of the seismic pulse function $f(t)$. For obtaining differential seismograms, the seismogram $S(p,\tau)$ should be derived with respect to the model parameters:

$$\frac{\partial S(p,\tau)}{\partial m_k} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\frac{\partial r(p,\omega)}{\partial m_k} e^{i\omega\tau} d\omega \qquad (8)$$

The model parameter m_k refers to the P-wave velocity, S-wave velocity or density. An analytical procedure for calculation of $\partial r/\partial m$ is given in (Hounie and Oliveira, 2014). Differential seismograms can also be obtained from schemes based on direct perturbation of the parameters, see, for example, Sen and Roy (2003).

In the plane wave seismograms the reflections have ellipsoidal moveout (Clearbout, 1985), so a τ-p nmo correction should be applied in order to align the pulses.

The ray parameter is constant along each trace of the plane wave seismogram in τ-p domain, but the angle of incidence with which the wave reaches the interface varies, so it is necessary to map the seismograms from τ-p to the τ-θ domain, so the angle of incidence will be the same for every reflection along the traces. This mapping is a simple procedure based on Snell's law and is done by collecting the samples along the curve $p=sin\ (\theta)\ /\ \alpha\ (\tau)$ in the NMO corrected τ-p seismogram, where $\alpha(\tau)$ is the P wave velocity given in function of two way travel tame.

Examples

In this section we present an application example of the AVA-WFI method in the caracterization of a siliciclastic, land gas reservoir. The stacked seismic section is shown in figure (4a). The angles gather contain nine traces ranging from 3 to 27 degrees and a classe III AVO anomaly can be clearly seen associated to the reservoir top (figure 1). The sonic and neutron log shows a decrese of P wave and density in the reservoir layer (figure 2). The S wave velocity was not measured, but it was inferred by castagna relation (Castagna et all, 1993), with fluid substitition correction. Note that an increase in the S wave velocit in the reservoir is predicted by this procedure. Before undergoing inversion, the angle gathers were preconditioned for residual normal moveout correction and frequency enhacement by the application of an inverse Q filter. The initial low frequency models were obtained by extrapolating the Vp, density and calculated Vs from well logs, following some seismic horizons. An unique wavelet *f(t)* was derived from the seismic to well tye and it was used for inverting all angles simultaneously.

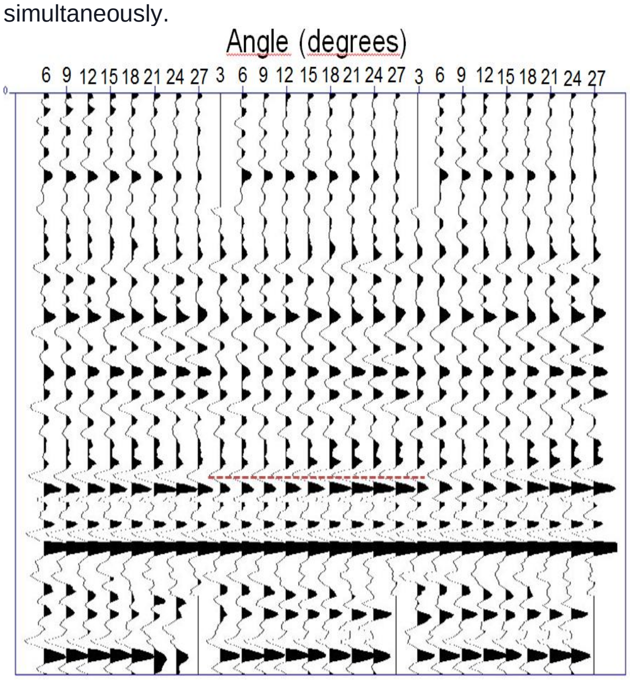

Figure 1 – Angle gathers after preconditioning. The red line identifies a class III AVO anomaly that occurs at the top of the gas reservoir.

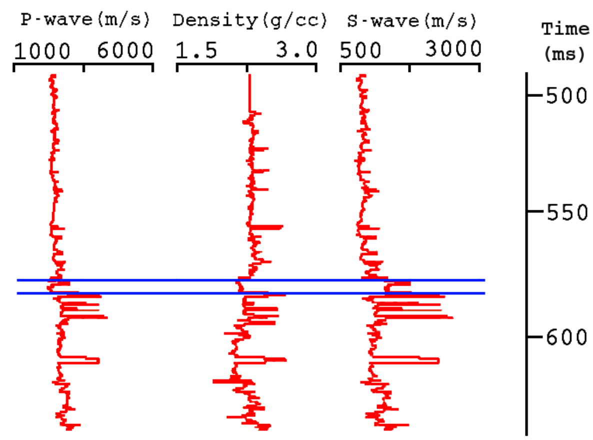

Figure 2 – The P wave velocity, density and calculated S wave vetocity logs. The blue lines identifies the top and base of the reservoir layer.

Results

The inversion generated results with a good correlation with the filtered well logs curves in the seismic bandwidth, and correctly reproduced the decrease in Vp and Density in the reservoir layer and also the expeted increase in Vs (figure 3c). Other elastic parameters were obtained from Vp, Vs and density by direct calculations. The imcompressibility and Poison ratio maps are shown in figure (4). The reservoir layer was very well delineated on the imcompressibility map whyle the Poison ratio map revealed the gas accumulation as a well defined anomaly located on top of reservoir (figure 4c).

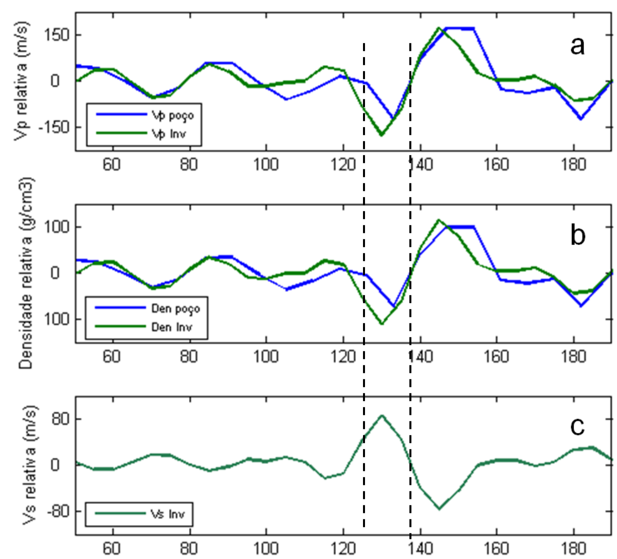

Figure 3 – Inversion result (green) versus filtered well log curves (blue) for (a) P wave velocity (b) Density and (c) S wave velocity. Dashed lines refers to reservoir top and base.

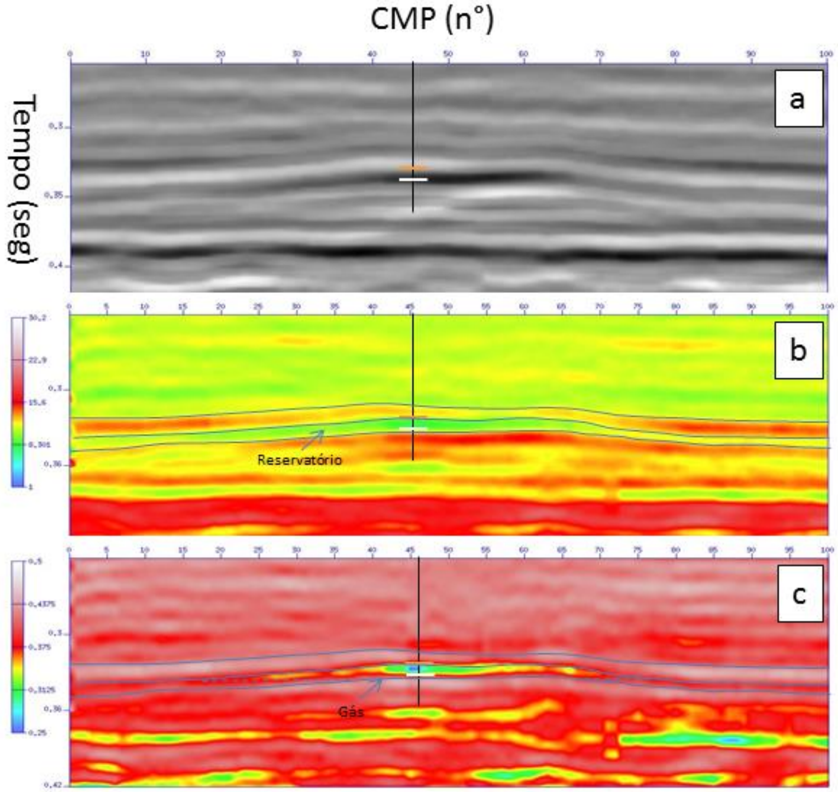

Figure 4 – (a) Stacked section with the top and base markers of the reservoir shown in the well (b) incompressibility section with the identification of the reservoir layer (c) Poison ration section with an anomaly related with the presence of gas.

Conclusions

We presented a new seismic waveform inversion method, named AVA-FWI that was designed to be a practical tool for quantitative reservoir characterization. The method considers the earth to be locally 1-D and should be applied in the angles gathers derived from prestack migration data. This method differs from conventional elastic inversion by considering the influence of converted waves and internal multiples in the AVA response of a layer. It was tested with success in the characterization of a gas reservoir, where it generated elastic parameters (Vp and density) that correlated very well with the available well log values and it also generated a Poison ratio map that confirmed the expected fluid distribution in the reservoir. The computational cost of the proposed method is very high when compared with the conventional linear AVA inversion. Efficient calculation of differential angles gathers is essential, since this is a numerically intensive task that is highly demanded by the Gauss Newton method during the minimization of the objective function. In the case of inversion of a large data volume or in the case where the number of elementary layers belonging to the inversion window is high, it is recommended to implement the method for parallel computation.

Acknowledgments

The authors thanks the Reservoir Inference Group for the use of their computing facilities and seismic data.

References

Aki K. and Richards P. G., 2002. Quantitative Seismology. University Science Books 2ed.

Aster R. G., Borchers B. and Thurber C. H., 2005. Parameter estimation and inverse problems. Elsevier Academic Press.

Castagna J. P., Batzle M. L. and Kan T. K., 1993. Rock physics – the link between rock properties and AVO. In offset – dependent reflectivity – theory and practice of AVO analysis. Society of Exploration Geophysicists.

Clearbout, J. F., 1985. Imaging the earth interior. Blackwell Scientific Publications ISBN 0-86542-304-0

Fatti J. L., Smith G. C., Vail P. J., Strauss P. J. and Levitt P. R., 1994. Detection of gas in sandstone reservoirs using AVO analisys. Geophysics Vol. 59 1362 – 1376.

Fletcher R., 1987. Practical methods of optimization. Wiley Inter. Science Publications.

Hounie J. N. and Oliveira S. A. M., 2014. The importance of locally converted waves on elastic inversion of compressional data. Brasilian Geophysical Journal Vol. 32 N 2 323-346.

Lines L., 1999. Density and AVO. Canadian Journal of Exploration Geophysics Vol. 35 N 1/2. 32- 35.

Mallick S., 2007. Amplitude-variation-with-offset, elastic impedance, and wave-equation synthetics – A modeling study. Geophysics Vol.72 N 1 C1 – C7.

Menke W., 1989. Geophysical data analysis: discrete inverse theory. Academic Press, San Diego.

Müller G., 1985. Reflectivity method: a tutorial. J. Geophys. Vol. 58 153-174.

Sen M. K. and Roy I. G., 2003. Computation of differential seismograms and iteration adaptive regularitation in prestack waveform inversion. Geophysics VOL. 98 N 6.

Simmons L. J. and Backus M., 1994. AVO modeling and the locally converted shear wave. Geophysics Vol. 59 N 9 1237 -1248.

Tikhonov, A. N. and Arsenin V. Y., 1977. Solution of ill-posed problems. Halsted Press, New York.