A Sparse Representation Technique for Interpolation and Denoising: Aplication on VSP data

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Abstract

This study focuses on an application of the Orthogonal Matching Pursuit (OMP) technique for interpolation and denoising of seismic signals.OMP is an algorithm for sparse coding based on orthogonal projections of the signal over an overcomplete dictionary. This overcomplete dictionary was designed with K-times Singular Values Decomposition (K-SVD). Developed algorithms were applied to VSP seismic data and results achieved, in restored lost traces and denoised signals, are presented.

Introduction

Many techniques have being proposed for improving signal quality through interpolation and noise reduction, most of them are classified into three main categories: methods based on physical wave propagation modeling, predictive modeling based on the linearity of seismic events and domain-transform methods based on the simplicity of the sparsity of seismic data in an auxiliary domain.

Domain-transform methods for interpolation and denoising of seismic data are generally processed using the following transform-domain: Curvelet, Pocs and Dreamlet; in addition to this, learning dictionaries for noise reduction were reported in (Rubinstein et al. (2008), Zhu et al. (2015) y Chen et al. (2016)). In a previous work Beckouche designed a learning dictionary that only made use of one datum, Beckouche and Ma (2014). However, despite of being a learning dictionary statistical processes need more than one datum for extracting important and accurate information from signals.

The purpose of this study is to examine a sparse representation technique for denoising and interpolation of seismic VSP data. In this study sparse representation uses the OMP algorithm and a dictionary trained with a 29 VSP data set for denoising seismic signals. After the signal is being denoised, the interpolation process takes place: it recognizes the array of training set by using adjacent traces. We presented good results for this study with synthetic traces generated by an attenuation model, as well as for real VSP seismic data.

Sparse Representation

Sparse representation using overcomplete dictionaries has been satisfactorily implemented in medical imaging, seismic and audio applications. Basically its principle consists of representing a signal with a linear combination of only a few atoms previously specified.

The signal is denoted by a vector $\mathbf{x} \in \Re^n$ and it can be represented in other domains by linear transformations using a dictionary denoted by a matrix $\mathbf{D} \in \Re^{nxn}$ with coefficients $\mathbf{a} \in \Re^n$.

$$\mathbf{x} = \mathbf{D}\mathbf{a} \tag{1}$$

Sparse representation can be accomplished through diverse techniques such as OMP, basis pursuit, FOCUSS and many others. In this work we use OMP and KSVD for the learning dictionary. When sparse representation is used for denoising it is assumed that the desired signal can be restored by using only a small number of atoms in the dictionary and then by taking each datum and transforming it to another domain. In addition to this, the denoising process is possible using OMP because sparse representation causes thresholding.

Orthogonal Matching Pursuit Algorithm

In each step the greedy OMP algorithm selects the atom with the highest correlation with respects the current residual. The OMP algorithm gives an approximate solution to equation 1 providing a solution to one of the following problems:

a) Sparsity-constrained coding problem, given by:

$$a = \text{argmin}||\mathbf{x} - \mathbf{D}\mathbf{a}||_2^2 \qquad \text{subject to } ||\mathbf{a}||_0 \le K \tag{2}$$

b) The error-constrained sparse coding problem, given by

$$a = argmin||\mathbf{a}||_0 \qquad \text{subject to } ||\mathbf{x} - \mathbf{D}\mathbf{a}||_2^2 \le \varepsilon. \tag{3}$$

The OMP algorithm can be stated as follows: Firstly initialize the residual $\mathbf{r} = \mathbf{x}$, then select in each step the atom $\mathbf{D_i}$ with the highest correlation followed by a comparison to the current residual. Once the atom $\mathbf{D}i$ is selected, the signal is orthogonally projected to the span of the selected atoms, the residual is recomputed, and the process repeats from the beginning (see Algorithm 1). The reader is encouraged to note that in line 5 it is presented the greedy selection step, and in line 6 it is shown is the orthogonalization step. Aharon et al. (2006) and Rubinstein et al. (2008).

Algorithm 1 OMP

Input Dictionary $\mathbf{D}$, Signal $\mathbf{x}$, Sparsity K o error ε
Output Sparse coding $\mathbf{a}$ subject to $\mathbf{x} \approx \mathbf{Da}$
$\mathbf{r} \leftarrow \mathbf{x}$
while stop criterion **do**
  $i = \max \left|\mathbf{D}^T r\right|$
  $a_i = (D_i)^{+}\mathbf{x}$
  $\mathbf{r} = \mathbf{x} - D_i a_i$
end while

K-SVD

Sparse representation intrinsically implies that the signal can be reconstructed by using only a few number of atoms from a dictionary. This sparse coding can be easily obtained by designing a dictionary from a training data set. The learning dictionary shows a local structure from seismic data and a sparse coding with a fixed dictionary. A fundamental question in the above formulation is choosing an appropriate dictionary; for this purpose, a K- SVD algorithm is executed in order to design such a dictionary.

The K-SVD algorithm requests for an initial dictionary $\mathbf{D_0}$, iterations k and a data set arranged in an array $\mathbf{X}$. This algorithm searches for a good dictionary that best reproduces the signals $\mathbf{X}$, this problem is formulated as follows:

$$\min_{D} ||\mathbf{X} - \mathbf{DA}||_F^2 \qquad \text{Subject to } \forall_i \; ||A_i||_0 \leq K \tag{4}$$

The K-SVD algorithm initially calculates coefficients for sparse representation in a matrix $\mathbf{A}$ followed by an update of the atoms in the dictionary (see algorithm 2). In line 5 K-SVD uses OMP for sparse coding and the dictionary update is performed one atom at a time, thus optimizing the target function for each atom individually while keeping the rest fixed, Aharon et al. (2006) and Rubinstein et al. (2008).

Letting I denote the indices of the signals in $\mathbf{X}$ which use the j-th atom, the update is obtained by optimizing the target function

$$||X_i - \mathbf{D}A_i||_F^2 \tag{5}$$

over both the atom and its related coefficients in row $\mathbf{A_i}$. The resulting problem is a simple rank-1 approximation task given by

$$\{D,g\} = \min_{D,g} ||\mathbf{E} - \mathbf{D}^T||_F^2 \qquad \text{Subject to } ||D||_2 = 1 \tag{6}$$

Where $\mathbf{E} = X_i - \sum_{i \neq j} D_i A_{i,j}$ is the error matrix without the j-th atom, D_i is the updated atom and g^T is the new coefficient in row A_i. This problem can be solved directly via an SVD decomposition of the matrix $E_j X_j^T = U \Lambda V^T$, the update of the orthonormal base is given by $D_j = U$.

Synthetic and real data examples

The Learning Dictionary

The set of training data for designing the dictionary is composed of 29 checkshot VSP datasets. The matrix of

Algorithm 2 K-SVD

1: **Input Initial dictionary $\mathbf{D_0}$, Signals $\mathbf{X}$, Number of coefficients K o error ε, iteration number k**
2: **Output Dictionary $\mathbf{D}$, Sparse coding $\mathbf{A}$ subject to $\mathbf{X} \approx \mathbf{DA}$**
3: $D \leftarrow D_0$
4: **for** $n = 1...k$ **do**
5:   $A_i = \min_{A} ||x_i - D\gamma||_2^2$ Subject to $||\gamma||_0 \leq K$
6:   **for** $j = 1...L$ **do**
7:     $D_j = 0$
8:     $i =$ indices of signals in X sparse coefficient D_j
9:     $\mathbf{E} = X_i - \mathbf{D}A_i$
10:     $\{d,g\} = \min_{d,g} ||\mathbf{E} - D_i g^T||_F^2$
11:     $D_j = d$
12:     $A_{j,i} = g^T$
13:   **end for**
14: **end for**

training signals $\mathbf{X}$ is a non-overlapping array containing samples of size 100. In order to build an initial dictionary of 160 atoms; it is used random samples of the training data. The dictionary shown in Figure 1 was trained with the following parameters: $k = 15$ iterations and $K = 5$ number of coefficients for OMP sparse coding.

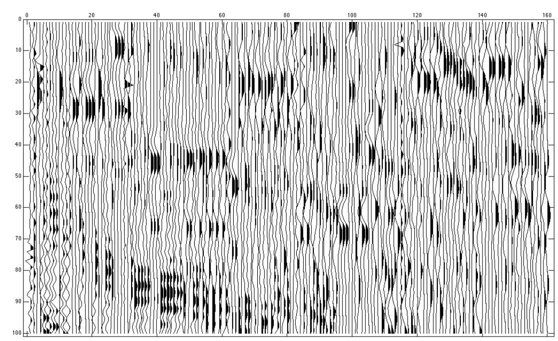

Figure 1: Dictionary of 160 atoms.

Denoising Synthetic VSP Data

Synthetic data was modelled with a visco-acoustic attenuation model, the source was modelled using a Ricker wavelet 25 Hz dominant frequency, a quality factor of 50, and a wave velocity of 2000 m/s, geophones were placed in separate locations. Signals were contaminated by additive gaussian noise with a SNR of 5 dB. For noise reduction was implemented an OMP algorithm with an overlapped window of 100 ms and one sample shifted at a time. Additionally, five coefficients were used for signal denoising in sparse coding OMP. The Figure 2 shows the seismic traces before and after being denoised. Synthetic traces were contaminated by white noise with Signal-to-noise ratio from 5 dB to 16 dB after the denoising process.

The Figure 3 shows the seismic trace 1 from the synthetic data before and after being denoised by using the algorithm proposed. The spectrum analysis shows that the signal content of the trace is preserved and noise in low and high

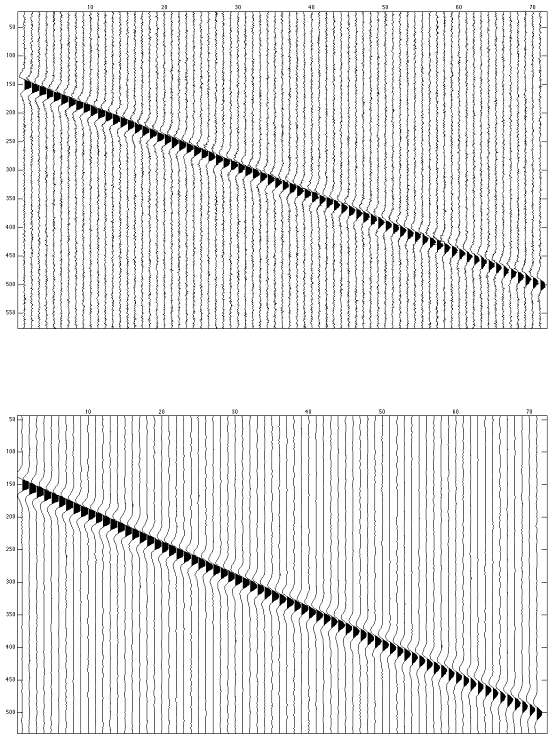

Figure 2: (Up) Synthetic traces contaminated with white noise. (Down) Synthetic traces after the denoising process.

frequencies are attenuated.

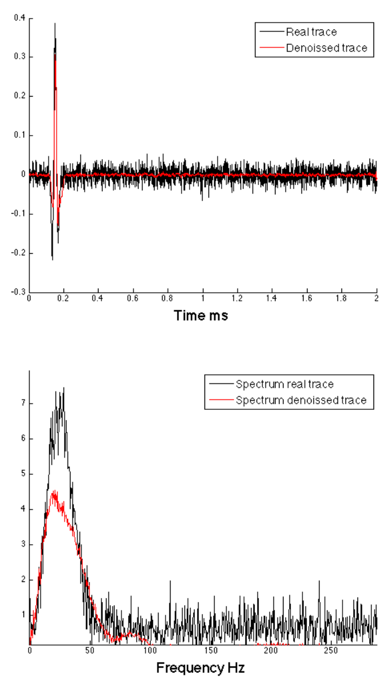

Figure 3: (Up) Original and denoised synthetic trace. (Down) Original and denoised spectrum of the trace.

Interpolation and Denoising of Real Seismic VSP Data

Lost traces, in general, are interpolated using adjacent traces; for this purpose, it was executed the sparse coding process with the OMP algorithm. In the algorithm a patch of data (size 10 x 10) arranged in columns was taken for interpolating adjacent traces as it is depicted in Figure 4.

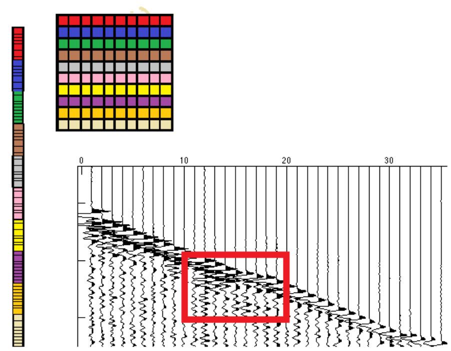

Figure 4: Patch of trained and denoised data using the OMP algorithm. The colors in the patch shows the arrangement of the matrix in columns.

In this experiment lost traces were simulated by removing ten random traces from original data as shown in Figure 5. Results for denoising and interpolation are shown in Figure 6, the wavelet spectrum were recovered in the interpolation process without noise frequencies components, attenuation in amplitude is relative to the original.

Conclusions

The main contribution to this work is the implementation of an algorithm for an interpolation and denoising process of VSP seismic data. In this work was also demonstrated that it was possible to accurately recover seismic synthetic signals contaminated by noise with a low signal to noise ratio by eliminating high frequencies through overlapping windows with only a few coefficients from its representation. An asset to this techniques relies on the fact that interpolation preserves the shape of the signal despite modifying signals relative magnitude. In addition to this the technique can also be applied to seismic reflection data. In a future work this technique will be compared to the conventional process used in seismic processing industrial applications.

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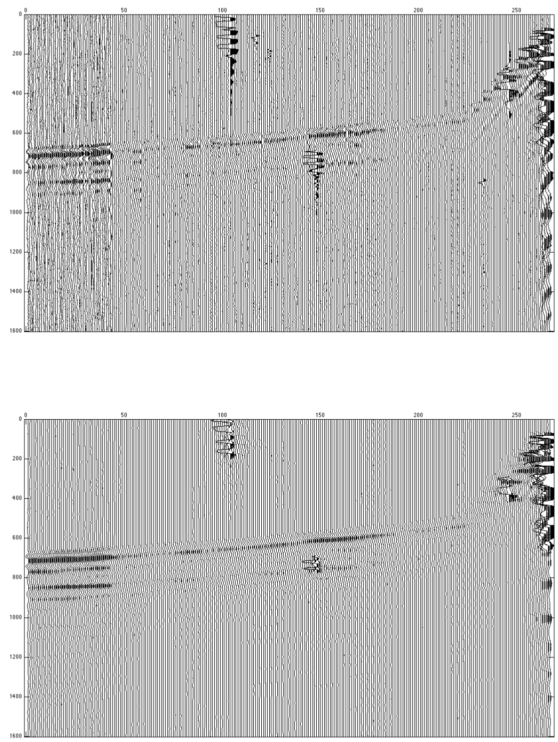

Figure 5: (Up) Real trace with 10 traces removed. (Down) Interpolated and denoised data.

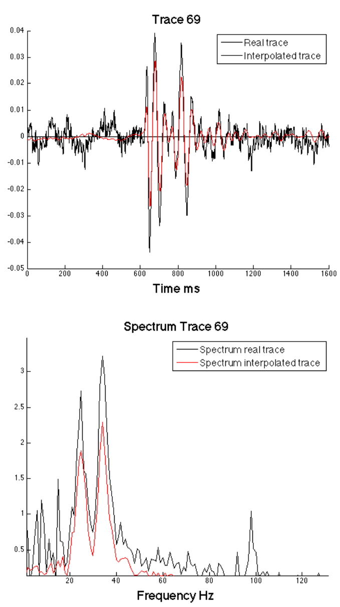

Figure 6: (Up) Real and interpolated trace 69. (Down) Original and denoised spectrum of trace.

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