A comparison of 2D Seismic data regularization with MWNI and MP

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Abstract

Regularization and interpolation of prestack seismic data have become an essential part of modern data preconditioning flows before imaging and inversion. In this paper, we present a comparative study of two techniques for seismic data regularization that operate in the Fourier domain: Matching Pursuit (MP) and Minimum Weighted Norm Interpolation (MWNI). We illustrate the performance of both methods on 1D harmonic functions and afterwards we apply them to the 2D prestack regularization of the Parnaíba basin dataset. Our numerical tests show that MWNI is faster than MP, but the MP produce slightly better results.

Introduction

Migration and inversion methods assume that prestack data are densely and regularly sampled in space. However, acquisition often fails in providing dense and regularly sampled data. The latter is due to operational constraints and the presence of obstacles. In marine acquisition, another type of irregularity occurs due to ocean currents that do not allow hydrophone cables to remain equally spaced. All these difficulties make the seismic data irregularly sampled or to have large empty gaps. Therefore, regularization is needed to represent prestack data in a regular complete grid.

Currently, there are many seismic regularization techniques. They can be grouped into three categories: 1) techniques for interpolating regularly sampled data, such as *f-x* interpolation (Spitz, 1991); 2) techniques that require the data to be on a regular grid, with optimal missing traces (Cabrera and Parks, 1991; Liu and Sacchi, 2004); and 3) techniques that use the exact input locations of the data (Xu et al., 2005; Mallat and Zhang, 1993, Schonewille et al., 2013). Nguyen and Winnet (2011) compare two methods of the third group (Xu et all., 2005, and Mallat and Zhang, 1993). This paper concentrate on the categories two and three of the methods and pays particular attention to two method of them (Liu and Sacchi, 2004; and a similar method to that proposed by Schonewille et al., 2013).

As it was already mentioned, we work with two interpolation methods that operate in the Fourier domain, Matching Pursuit (Mallat and Zhang, 1993) and MWNI (Liu and Sacchi, 2004). First, each trace is transformed into the frequency domain (from *t-x* domain to *f-x* domain). Then, for each temporal frequency *f*, an MP or MWNI algorithm can be used to estimate the spatial Fourier coefficients that synthetize the data. For 3D data, MP and MWNI are implemented the *f-x-y* domain.

MWNI (minimum weighted norm interpolation) is a Fourier reconstruction method that is based on regularized inversion. In other words, one estimates the Fourier coefficients that synthetize the spatial data via the solution an inverse problem (Liu and Sacchi, 2004). The procedure is an extension of the adaptive frequency-domain weighted norm scheme proposed by Cabrera and Parks (1991) to extrapolate time series. The Matching Pursuit (MP), on the other hand, uses a Greedy Algorithm (Cormen et al., 2009) to estimate the Fourier coefficients that synthetize the observations. Once the Fourier coefficients have been found via MWNI or MP one can use the inverse Fourier transform to reconstruct spatial data.

In this paper, we compare Matching Pursuit with MWNI method in two spatial dimensions. In other words, the regularization is carried simultaneously in receiver and source domain.

Minimum weighted norm interpolation

The MWNI method entails solving an inverse problem where a wavenumber-domain regularization term is included (Liu and Sacchi, 2004). In general, it minimizes a wavenumber weighted norm that lets us incorporate a prior spectral signature of the unknown wavefield. It is a fast and efficient method because it uses an iterative solver (conjugate gradients) in conjunction with fast matrix-vector multiplications that are implemented via the fast Fourier transform (FFT). The algorithm can be used to perform multidimensional reconstruction in any spatial domain.

Thus, for each temporal frequency the algorithm minimizes equation 1 in order to estimate the correct Fourier coefficients. We can call *J* our objective function:

$$J = \left|TFc - d^{dec}\right|_2^2 + \lambda \left|Wc\right|_2^2 \quad (1)$$

Where the operator *T* is the sampling matrix with size N_x-N_y. The elements of the sampling are 1 positions with data, and zero for data bins that are empty. The operator *F* symbolizes the inverse Fourier transform, it can be performed through the inverse fast Fourier transform

(IFFT), and c are the complex Fourier coefficients. The term d^{dec} is the binned data with missing samples, its size is N_x-N_y, with zeros in the empty positions. The scalar λ is the trade-off parameter of the problem, and W is a matrix of weights that updates the Fourier coefficients in each iteration.

The conjugate gradient method is adopted for the minimization of (1). However, in MWNI the matrix of weights W is a function of the unknowns c. Therefore, we adopt the method of Iterative Reweighted least-squares (Zhou et al., 2014) to update the weights. To summarize it, we have two iteration loops: an internal loop to solve minimization problem (1) via conjugate gradients for fixed W and an external loop to update the weights.

The method can be summarized as follows:

1. Initialize the Fourier c coefficients and the weights W;
2. Use conjugate gradients to find c that minimizes (1);
3. Use current Fourier coefficients c to update the weighs W;
4. Go to 2 and repeat until convergence;
5. Use the estimated Fourier coefficients to synthetize spatial data via the inverse Fourier transform.

Matching Pursuit

The MP is a greedy algorithm that can be used to estimate the Fourier coefficients of irregular sampled data. The process works recursively by finding first the most energetic Fourier coefficient. The DFT of data on an irregular 1D grid can be expressed via,

$$f(k) = \sum_{l=1}^{N_1} d(x_l) e^{-2\pi i k_x x_l} \qquad (2)$$

The MP algorithm estimates the Fourier coefficients for all the wavenumbers (k) first. Then, it selects the ones with maximum energy (f_{kp}), and uses them to synthetize spatial data via an inverse DFT. The synthetized data is removed from original data and the process continues after fitting the original data.

$$d^k(x_l) = f_{kp} e^{2\pi i k_p x_l} \qquad (3)$$

$$d^{s+1}(x_l) = d^s(x_l) - d^k(x_l) \qquad (4)$$

Where s represents the steps, N_1 is the sample number of x; d^s is the input data at the step s; d^{s+1} is the residue on the step s and the input for next iteration; d^k is the transformed Fourier component to the input location; k_p represents the wavenumber in x to the maximum coefficient.

The MP algorithm can be implemented in the following steps:

1. Choose the threshold ε;
2. Calculate $f(k)$ according to (2) and find the Fourier component with maximum energy (f_{kp});
3. Fit and transform this Fourier component back to the input locations according to (3);
4. Subtract the result from step 3 from the input data for this iteration according to (4);
5. Repeat steps 2-4 until reaching a pre-defined threshold error iteration or when $||d^{s+1}||$ becomes sufficiently small;
6. Saves all the selected coefficients to realize an inverse Fourier transform and reconstruct the signal at the desired output location.

Note that when the algorithm terminates at iteration M, we do not have $f(k)$ that satisfy the inverse of (2) exactly, but only approximately. Also the norm of the residual vector (d^{s+1}) cannot increase after each iteration, because the residual is projected into two components that orthogonal to each other.

Numerical Experiments

For both methods, the regularization is done for each time-frequency separately of a seismic data. Therefore, our numerical example can be used to represent only a time-frequency slice in a signal 1D. The Figure 1(a) shows our function as a signal 1D ($d(x) = 5\sin(x\pi/16) + 4\sin(x\pi/8+1)$), where the x position has 128 samples. We decimate this function randomly. Only 88 samples are kept in x (Figure 1(b)).

Both methods reconstructed the decimated data efficiently. Figure 1(c) shows the result of the MWNI reconstruction method and Figure 1(d) shows the result of the MP reconstruction method. One important point needs to be mentioned. While in the MWNI the input data were binned to a regular grid, the MP method honors the true positions of the traces.

The MWNI method needed about 30 iterations to converge, while the MP required 12 iterations to converge. In the MWNI method the cost of each iteration is dominated by the FFT. On the other hand, in the MP method the cost of each iteration is dominated by the DFT. This explains the difference in running time for both methods. For this example running time for MP was 0.0187 seconds and 0.0148 seconds for MWNI (although MWNI has more iterations). The MP presented a relative error of 0.93%, while MWNI presented 3.75%. We can see the absolute error of each method in the Figures 1(e) and (f).

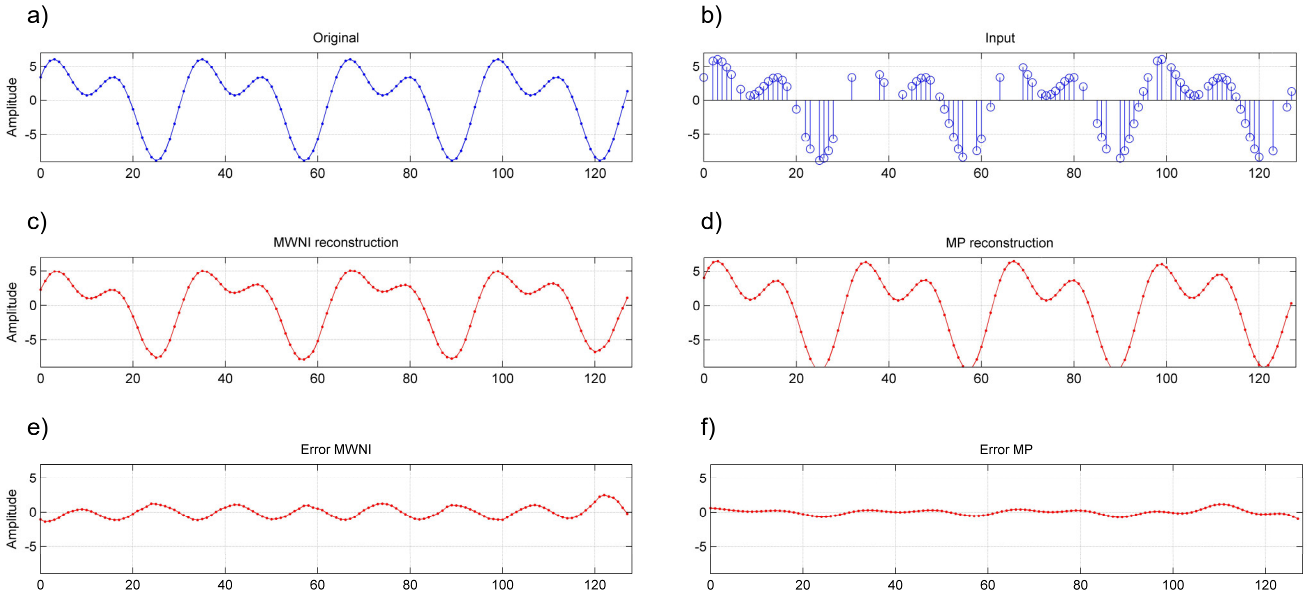

Figure 1 *- (a) Original data, with 128 samples in x direction. (b) The data was decimated randomly, with 40 missing samples in x direction. (c) Data reconstruction using MWNI. (d) Data reconstruction using MP. (e) Absolute error of the MWNI (subtraction of the real data with the reconstructed data). (f) Absolute error of the MP.*

Seismic Example

In our real land data example, we used a subset of a 2D seismic line data of the Parnaíba basin, Northeast region Brazil, which were authorized for use in research work by Parnaiba Gas Natural SA. These data was acquired using explosive sources and the following processing flow was applied before submitting for regularization: 1) field static corrections, 2) spherical divergence compensation, 3) coherent noise attenuation, 4) deconvolution, 5) velocity analysis, 6) residual static correction. The spatial coordinates to interpolate both methods (MWNI and MP) were shot and offset, but similar results could be obtained by interpolating in midpoint-offset or source-receiver, this is criteria of the user.

We extracted 7 shots from that data set, with 160 traces per shot, so it has a total of 1120 traces (we can see three shots of this data in Figure 2(a)). The receivers are sampled every 50 m, and the shot-receiver intervals are of 50 m. We decimated this data subset randomly, removing some traces in each shot, resulting in a data with a total of 704 traces (we can see the same three shots on the decimated data in the Figure 2(b)).

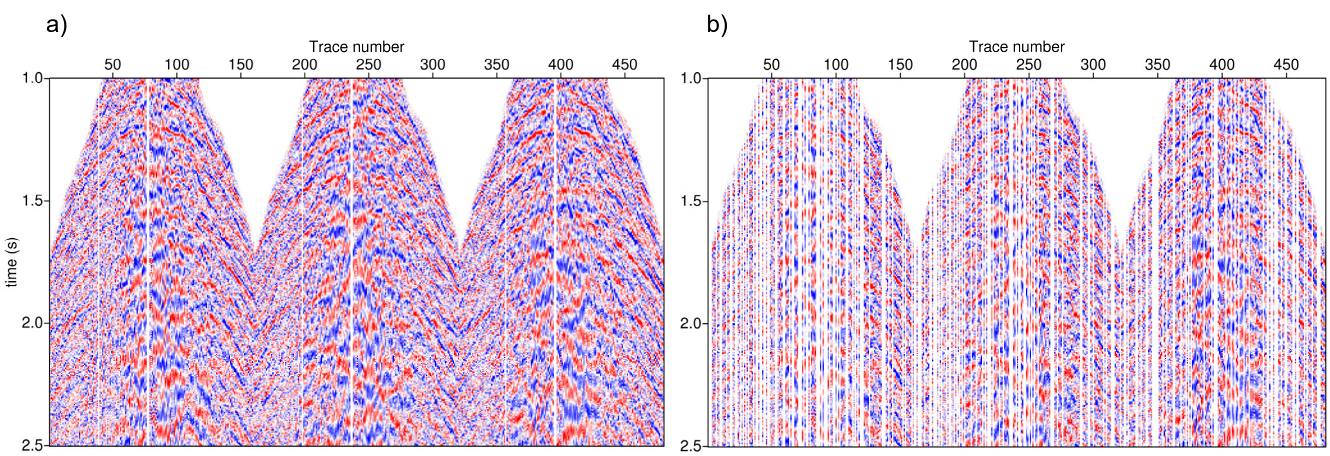

Figure 2 *- (a) Three shots extracted from the Parnaíba basin data set. (b) Decimated data from original Parnaíba basin data set at the same intervals.*

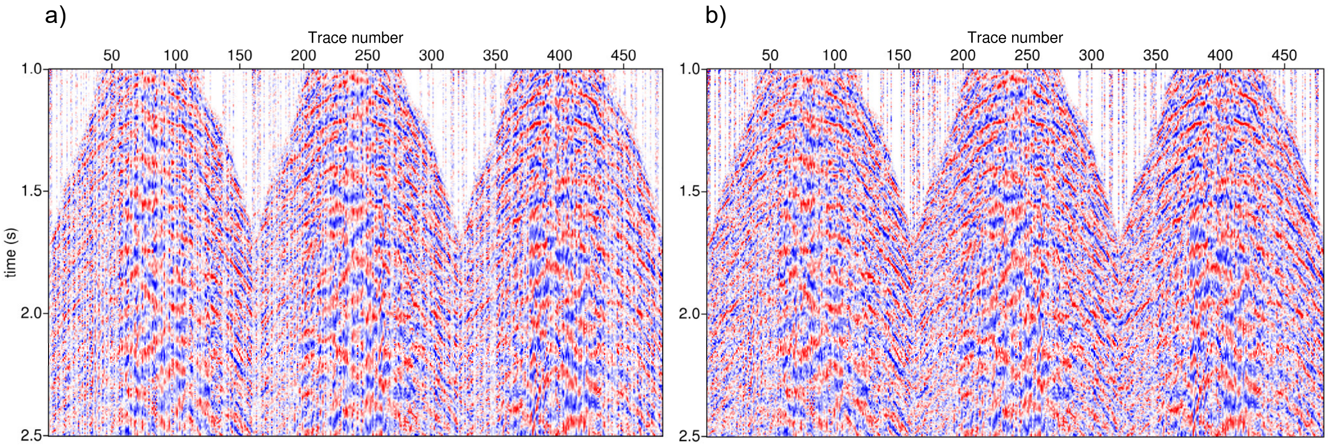

Figure 3 *- (a) Reconstructed data using the 2D MWNI. (b) Reconstructed data using the 2D MP.*

To perform a better reconstruction, we apply an NMO correction and then inverse NMO to compare with original data. We used the same conditions to perform both methods. The Figures 3(a) and (b) show the results of the MWNI and MP reconstructions, respectively.

Figures 4(a) and (b) show the third shots of the Figures 2(a) and (b) in the wiggle form. Figures 5(a) and (b) show the reconstruction using MWNI and MP, respectively. With this Figure, we can see that both methods worked well, considering that a land data contains a lot of noise. However, the MP preserves better the events than MWNI (we can see in the red border zoom of Figures 4 and 5), comparing with the original shot.

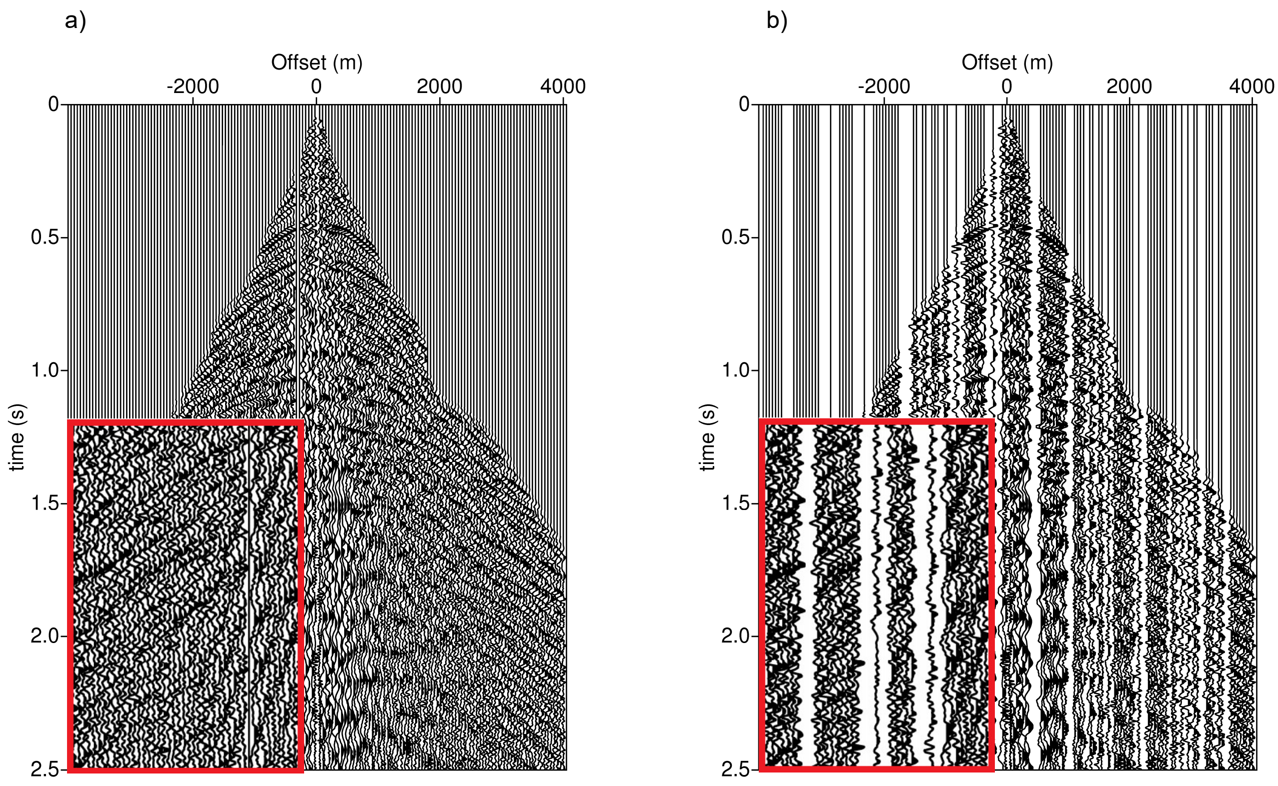

Figure 4 *- First shot of the Figure 2 showed in wiggle form. (a) Original shot. (b) Decimated shot.*

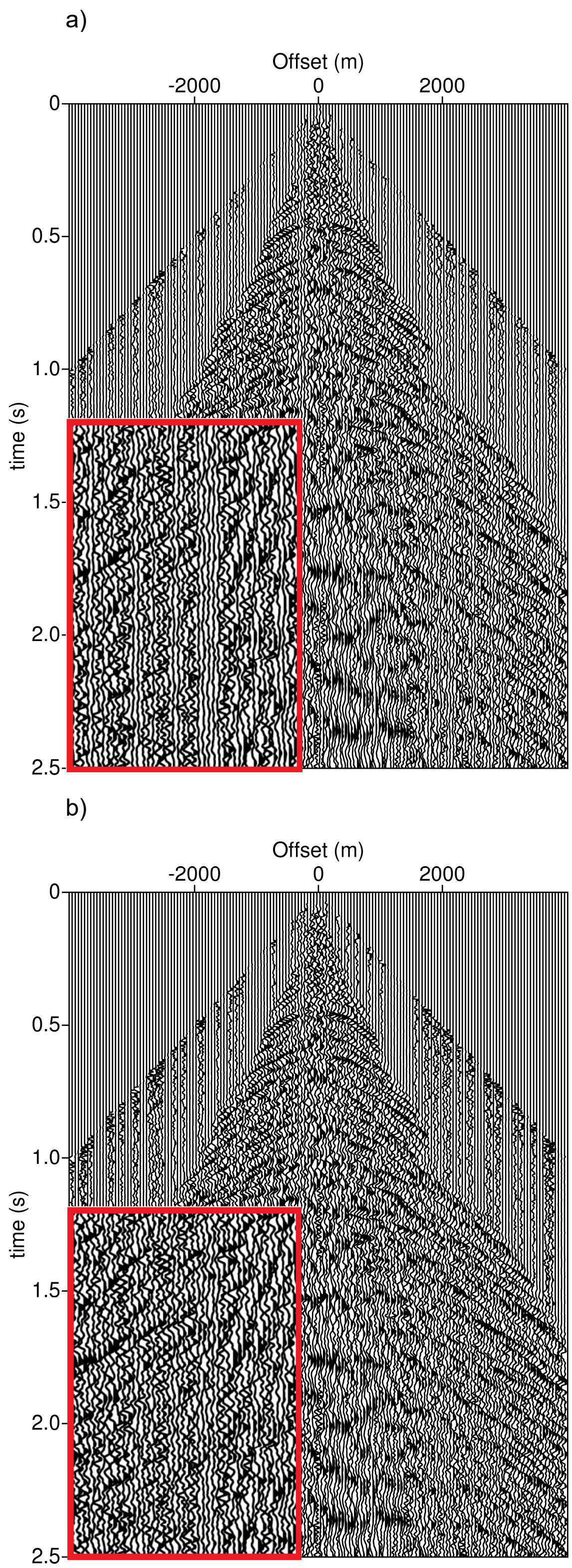

***Figure 5** - Third shot of the Figure 3 showed in wiggle form. (a) Reconstructed traces using MWNI. (b) Reconstructed traces using MP.*

Conclusions

We compared the MWNI and MP methods. We have concluded that both interpolation algorithms are capable of recovering missing traces. In particular, both method can cope with the reconstruction of large gaps.

MWNI works with fast multiplications between vectors and matrices in conjunction with the fast Fourier transform. The computational cost of the MP is proportional to $N_p N_k$ (with N_p input samples and N_k frequencies). The MWNI estimates the coefficients through an FFT at a cost of $M log_2 M$ (for M frequencies). We are working to improve code optimization and, in the future, make a comparison of the computational cost of both methods.

Although MWNI apparently presents a lower computational cost, this method has a slight disadvantage when it comes to reconstruction quality. This is because MWNI adopts a binning strategy in order to use the FFT. Binning is a problem when one wants to preserve the original positon of traces. The MP, on the other hand, does not have this problem, because in the DFT calculation it uses the true spatial positions.

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