Bayesian Inversion of Receiver Functions and Surface Wave Dispersion Data in the Brazilian Northeast.

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Abstract

We present crustal shear wave velocity models beneath 43 stations in the Northeast region of Brazil, using a Bayesian joint inversion of receiver functions and surface wave dispersion data. Our approach can be divided into two major stages: (i) estimating the number of layers in the earth model through a trans-dimensional optimization procedure, and (ii) performing a Bayesian sampling of the model space for estimation of model uncertainties. Data errors, which are required in stage (ii), are inferred from a combination of empirical and hierarchical approaches. The inversion procedure is tested on data collected in the Borborema Province of NE of Brazil. Our results are consistent with independently published results that demonstrate the crust in the Borborema Province is bimodal: a thicker crust (35-41km) observed in the high topography of the southern Borborema Plateau (south of the Patos Lineament); and a thinner crust (29-34km) in the low-lying topography of the Sertaneja Depression and northern Plateau. We thus confirm independent findings that the thickness of the crust correlates well with topography, except for the northern Borborema Plateau where elevated topography is underlain by thin crust. The uncertainties determined for each data set strongly influences the solutions of the inversion accounting for non-stationary and strongly correlated errors, and avoiding arbitrary use of large number of unknowns to parametrize the covariance matrix.

Introduction

Methods based on Bayesian inference are becoming increasingly common in the Geosciences to perform joint inversion of geophysical datasets, such as receiver functions (RFs) and surface wave dispersion curves (SWD). Despite this methodology being relatively recent (Bodin et al., 2012; Sambridge et al., 2013; Shen et al., 2013), the improvement in computational efficiency broadens the practical use of this type of approach. Ideally, in a fully Trans-dimensional Bayesian inversion, the data itself determines the complexity of the model constraining the optimum number of unknowns (model parameters) as well as the different levels of uncertainty brought by different data types. However, it is important to keep in mind that this kind of approach is extremely high cost computationally. Part of the computational burden can be avoided by selecting an optimal model parametrization through the use of a trans-dimensional optimization algorithm, to then sample the model space for uncertainty estimation with the optimal parametrization.

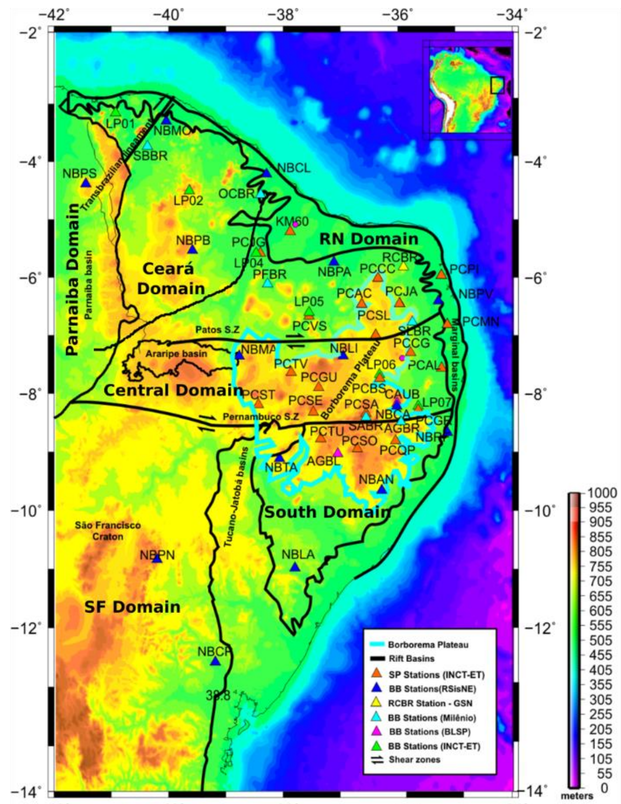

Figure 1. *Topographic map of the Borborema Province showing the principal tectonic domains and the location of the stations considered in this study (modified from Luz et al., 2015b).*

In this work, we test the Bayesian joint inversion of Rayleigh-wave group-velocity dispersion curves and receiver functions with data collected in the Borborema Province of NE Brazil (Figure1). The Province is located in the northeastern corner of the South American continent and is characterized by a complex tectonic framework consisting of up to five different tectonic domains separated by a network of shear zones oriented in the E-W and NE-SW directions (e.g. Almeida et al., 1981). The crustal thickness of the Borborema Province is relatively well understood from receiver function studies

(Luz et al., 2015a; Almeida et al. 2015), gravity surveys (Oliveira and Medeiros, 2012), magnetotelluric surveys (Santos et al., 2014), and one refraction line (de Lima et al., 2015). S-velocity models from the joint inversion of receiver functions and surface-wave dispersion were already developed in Luz et al. (2015b) from the iterative, linearized approach of Julià et al. (2003). The aim of this work is to apply a new, Bayesian-based joint inversion approach (Kim et al., 2016) to the same dataset to assess the performance of the non-linearized approach.

The Bayesian joint inversion of different data types requires a reliable estimation of the uncertainty for each data set, as different data sets have different levels of noise and their relative contribution to the misfit function is expected to be defined by the corresponding uncertainties (Bodin et al., 2012). For this reason, well-quantified uncertainties are critical for a reasonable assessment of data misfit. As part of our inversion, we have rigorously estimated the level of uncertainty of each data set during the Bayesian sampling phase, thus reducing the influence of noise in the final models while avoiding the use of arbitrary weighing. The data infers the appropriate level of data misfit by considering both the empirical and hierarchical estimations for the uncertainty and the inversion computes the ensemble of models correspondent to the lower noise distribution. This approach avoids assumptions on setting the starting model, arbitrary weighing factors to tune different datasets and, regularization procedures.

Methodology

This section briefly summarizes the Bayesian approach to the joint inversion of receiver functions and surface-wave dispersion. A full description of the inversion procedure, as well as numerical experiments, can be found in Kim et al. (2016).

Bayesian inference means that all information is represented statistically in terms of a probability distribution. The solution of the Bayesian inversion is thus an ensemble of models that represent a posterior conditional probability distribution based on the observed data and on *a priory* information about the model. Mathematically, this is formulated as

(1) $$\mathbf{p}(\mathbf{m}|\mathbf{d},\mathbf{k}) = \frac{\mathbf{p}(\mathbf{d}|\ \mathbf{m},\mathbf{k})\ \mathbf{p}(\mathbf{m}|\ \mathbf{k})}{\mathbf{p}(\mathbf{d}|\ \mathbf{k})} \quad ,$$

where **m** is the model vector of unknown parameters and **d** is the vector containing the observations. The posterior probability density (PPD) of the model parameters **p(m|d, k)** is the ensemble of candidate models that might represent the structure of the earth underneath the receiver. The likelihood function **p(d|m,k**) is interpreted as the probability of the observations given a particular member of the model ensemble.

The prior probability density function **p(m|k)** offers *a priory* constraints about the model, and is generally chosen as a simple uniform PDF within predefined bounds. The choice of the prior reflects directly on the results of the inversion. The vertical bar is a standard notation that implies that the terms to the right are fixed and that the probability function is conditional to those values. Therefore, **k** to the right of the vertical bar in each term of eq, (1) denotes that the number of unknowns is fixed (standard Bayesian inference). The difference between the posterior and prior probabilities represents the influence of the data set in the inversion (Sambridge et al, 2013). Thus the importance of using a reliable ensemble of observed data during the inversion.

Finally, the Bayesian evidence **p(d|k)**, also known as the likelihood of the model parametrization, is intrinsically related to the parametrization of the model (Sambridge et al 2006b). In spite of its importance in the quantification of data for an optimal parameterization (e.g. number of layers), some studies disregard the estimate of this parameter because of its high computational costs.

The inversion code used in this work is run in two major steps. The first one sets up an optimal parametrization through a non-linear algorithm. This approach avoids the computational burden inherent to a fully Bayesian inference and helps reduce the burn-in period of the corresponding Bayesian sampling. The selection of the model parametrization is performed prior to the Bayesian inference through a Trans-dimensional optimization procedure. The term Trans-dimensional means that the number of unknowns in the model (e.g. number of layers) is relaxed instead of being fixed in advance. This kind of non-linear optimization allows the search for the most probable set of parameters in a large multidimensional model space. For such optimization, the likelihood function is replaced by an objective function denoted Bayesian Information Criteria (BIC) that provides constraints on model complexity (Molnar et al., 2010). Once the optimal parameterization (eg. number of layers of the model) is found, the Bayesian sampling is carried out for a fixed dimension.

Inversion Results

The stations considered in this study are located in several tectonic regions within the Borborema Province. To illustrate the performance of the Bayesian approach, we present S-wave velocity model for one station in each of the five different tectonic domainsMoho is inferred with a cut-off S-velocity of 4.4 km/s. Comparing our results to previous studies, Moho depths inferred from our models are consistent with those inferred from linearized inverted models (Luz et al., 2015b), and from H-κ stacking analysis (Luz et al., 2015a).

Rio Grande do Norte Domain

The Rio Grande do Norte Domain is located in the northeasternmost corner of the Borborema Province (Figure 1), being sampled by up to 11 seismographic stations. Most of the stations display sharp Moho depths in a range of 30-34 km depth (LP04, NBPA, NBPV, PCJG, PCVS, PCAC, PCCC, and PCSL). Figure 2 displays the S-wave velocity profile for station NBPA, which reveals a Moho at 31.5 km depth, and variance reduction of 88.5% and 94.7% for receiver function and surface wave, respectively. All stations in this domain are located in a low-lying topography, with the exception of PCAC, PCCC, PCJA and PCSL, which are located at the northern portion of the Borborema Plateau. In this region, Moho depths are at 33.5 km and crustal velocities are lower than 4 km/s.

Sharp discontinuities are observed for all S-velocity models at 12.5-15 km depth, as reported in Almeida et al. (2015) and Luz et al. (2015b). This discontinuity was interpreted as a sub-horizontal detachment zone resulting from stretching of the crust in Mesozoic times.

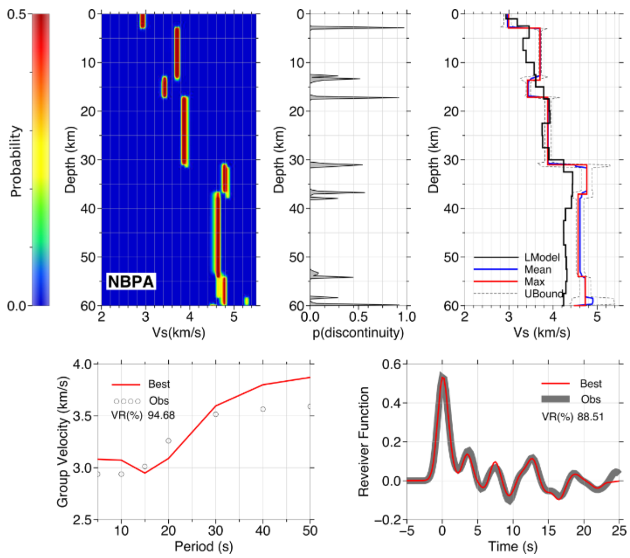

Figure 2. *Detailed inversion results for station NBPA. The top left panel is the posterior distribution for Vs at different depths; the top middle panel represents the interface probability; and the panel to the top right is a comparison of the marginal mean model (blue) with uncertainty bounds (dashed gray) to previous results based on a linearized inversion (black). In this panel, the red color represents the maximum value for the posterior probability on the left. Bottom panels display a comparison between observation and best prediction for GV (left) and RF (right) ,respectively.*

Ceará Domain

The Ceará Domain is the northwesternmost tip of the Borborema Province (Figure 1), and is sampled by up to 6 stations. The three stations to the East (LP02, OCBR, and NBCL) are close to the Rio Grande do Norte Domain and display crustal thicknesses in the 29-30 km depth range. Station OCBR, also located in the east portion, shows a depth of 33 km for the Moho, but surface wave dispersion is not matched by the velocity model. Stations located to the West of the domain could not be modeled through the Bayesian inversion procedure, with the exception of station SBBR. This station displays a thick crust of 37 km (Figure 3) with a gradational increase in shear velocity in right above the upper mantle from 4.2 to 4.4 km/s.

Other features in the velocity model include a high velocity layer in the upper crust, and an intra-crustal discontinuity at 12-15 km, marking the boundary between upper and lower crust. Above 30 km depth, crustal velocity is lower than 4 km/s.

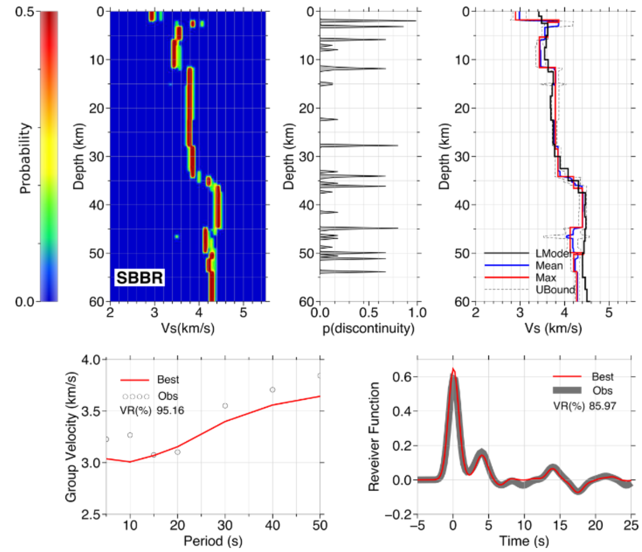

Figure 3. *Detailed inversion results for station SBBR. Panels are arranged as in Figure 2.*

Central Domain

This domain is densely sampled by up to 15 stations (Figure 1). Stations located to the East (low-lying topography) display crustal depths ranging between 31 and 34 km. Figure 4 shows the detailed inversion results for station PCAL, which is characterized by a 31 km thick crust and a sharp intra-crustal discontinuity at 12.5 km. S-velocity in the upper mantle is around 4.5 km/s. Variance reduction is 78.6% and 97% for receiver functions and dispersion velocities, respectively.

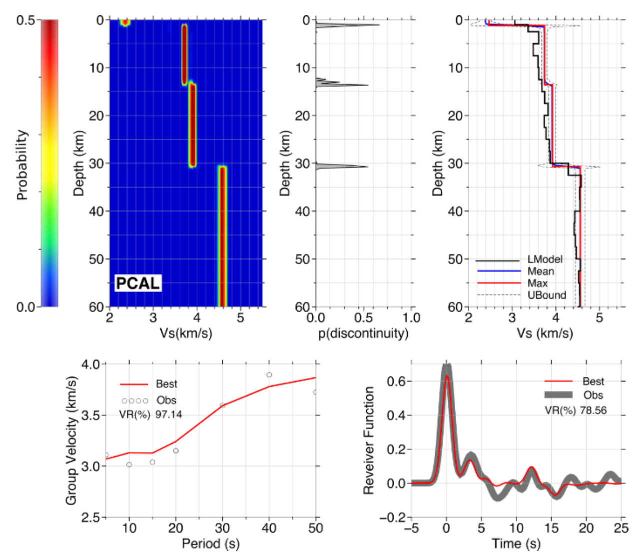

Figure 4. *Detailed inversion results for station PCAL. Panels are arranged as in Figure 2.*

Intra-crustal discontinuities are observed in some of the profiles, with depths varying between 5 and 15km depth. A high-velocity layer (> 4 km/s) is observed in most of the models at the bottom of the crust; velocities in the remaining of the crustal section are lower than 4km/s. Stations located in the southern portion of the plateau (PCTV, PCSA, NBCA, PCST, SABR and NBLI) are much thicker and Moho depths range between 36 and 37 km.

PCMN and NBMA profiles display gradational transitions of velocity at 34 km and 37 km depth, respectively, where S-velocity goes from 4.2 to 4.4 km/s.

South Domain

The South Domain is sampled by up to 7 stations, and is located between the Central Domain and the São Francisco domain (Figure 1). Five of the stations sit in the high-standing topography of the Borborema Plateau and display crustal thicknesses between 35 and 41 km. For some of the velocity profiles, a gradational increase in shear-wave velocity is observed in the lower crust. For NBAN and AGBR, a discontinuity at 16 km depth separates the upper crust (3.4-3.5 km/s) from the lower crust (~4 km/s).

Stations NBRF and NBLA are off the southern plateau and display crustal thicknesses of 30.5 km and 33.5 km, respectively. Results of other 3 stations hasn't offered reliable results.

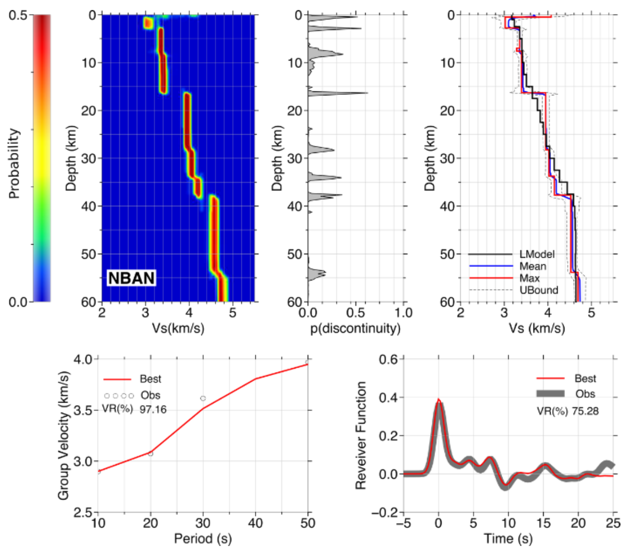

Figure 5. *Detailed inversion results for station NBAN. Panels are arranged as in Figure 2.*

Parnaíba Basin and São Francisco Craton

These two physiographic provinces are located in cratonic areas bounding the Borborema Province. They are characterized by old crystalline basement rocks and generally present a crust thicker than the continental average. The Parnaíba Basin is sampled by only one station (NBPS) and the profile displays Moho depths around 41 km. S-velocities in the upper mantle are 4.5-4.6 km/s. Similarly, the São Francisco Craton is covered by only 3 stations, displaying depths to the crust-mantle boundary as thick as 45 km under station NBIT.

Figure 6 displays one of the velocity models in the São Francisco craton, revealing a 38 km thick crust and an upper mantle S-velocity of 4.8-5.0 km/s. This profile (station NBCP) shows some intra-crustal discontinuities at 17.5 km depth that separates the crust into the upper crust (3.6 km/s) and lower crust (4 km/s). Again, a shallow high-velocity layer (~3 km thick) is observed at shallow depths. A slow shear-wave velocity for the upper mantle is observed at station NBIT, ranging from 4.2 to 4.4 km/s.

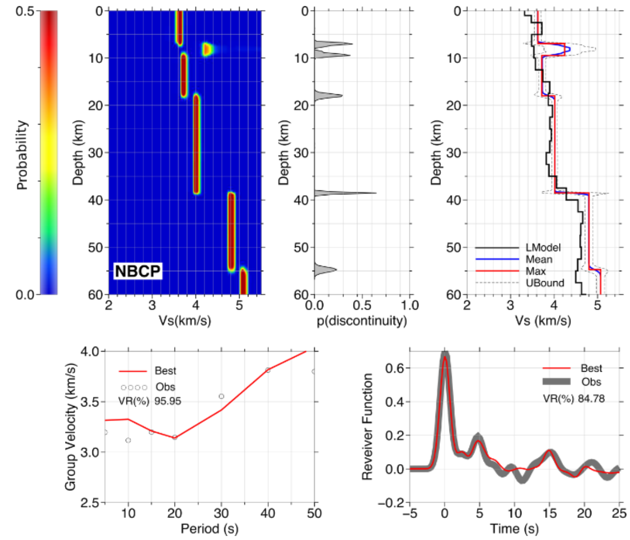

Figure 6. *Detailed inversion results for station NBCP. Panels are arranged as in Figure 2.*

Discussion and Conclusions

In general, Moho depths are successfully recovered from the Bayesian approach, with values fully consistent with those based on receiver functions alone (Almeida et al.,2015; Luz et al, 2015a) and the linearized approach (Luz et al., 2015b), although they are mostly sharper. The velocity models display intra-crustal discontinuities and shallow high-velocity layers that the linearized models do not see as we assume low frequencies RFs and do not apply a smoothness constraint in the velocity profiles.

A few stations (LP05, PFBR, PCJA, PCCG, LP07, PCGR, PCSE and PCGU) displayed optimal S-velocity profiles that did not match surface wave dispersion velocities. For these stations, constraints on absolute S-velocity and Moho depth are unreliable. We suspect that the relative influence of each dataset on the inverted model still keeps some dependence on the relative number of data points. The inversion code (Kim et al., 2016) is being implemented to more precisely estimate the level of uncertainty for SWD. The solution of the inversion strongly depends on the choice of the covariance matrix, and not to consider correlation between points of the SWD data, for instance, might help the covariance matrix of data errors to assume a more Gaussian distribution shape. That may increase the degree of reliability for the uncertainty estimation for SWD data, and therefore, offer a more precise weight for the SWD in the misfit function.

Although our geological interpretation is much limited, the importance of this work was to test a new methodology of inversion based on a Bayesian inference, considering confidence bounds which is a really novel contribution from this approach, and to concentrate our goal on the Moho structure resolution that came to be, in most cases, in excellent agreement with results from independent studies.

Acknowledgments

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