Complexity analysis of nonhyperbolic approximations

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Abstract

The complex models of the Pre-salt from Santos Basin are a challenge for the seismic processing. These models and the technological seismic acquisition used during offshore surveys generate a nonhyperbolic wave reflection event which cannot be modeled by using the conventional hyperbolic mathematical proposition.

For this reason it is necessary the use of nonhyperbolic multiparametric approximations that can control this effect. So, we must understand which approximation must be used. For this, we must perform a complexity analysis by observing the topography of the objective function for each approximation.

With the results that we obtained here, it is possible to select which nonhyperbolic multiparametric approximation and which kind of optimization algorithm must be used to perform a more efficient velocity analysis.

Introduction

The Pre-salt geology can be very complex even when we study a conventional model from Santos Basin. For this reason to perform the velocity analysis, we cannot use the conventional hyperbolic approximation proposed by Dix (1955) due to the fact of the kinds of models studied here have characteristics which generate nonhyperbolic reflection events. Even a nonhyperbolic approximation can be ineffective to control the nonhyperbolicity of some models (Aleixo and Schleicher, 2010; Golikov and Stovas, 2012; Zuniga *et al*., 2015 and 2016).

The Pre-salt models analyzed here (Figure 1 and Figure 2) are from Santos Basin, and the reservoirs are at more than 5000 meters depth, and the water depth is more than 2000 meters depth (Figure 1 and Figure 2). To understand how we can perform a better velocity analysis for the wave reflection events of these models, the complexity of some approximations must be analyzed. We propose here a complexity analysis by observing the topography of the objective function for each nonhyperbolic approximation used here.

Aiming to understand which approximation can bring the best results and with the less processing time possible, the complexity analysis allows us to select the best option of approximation and which kind of optimization algorithm (unimodal or multimodal) must be used.

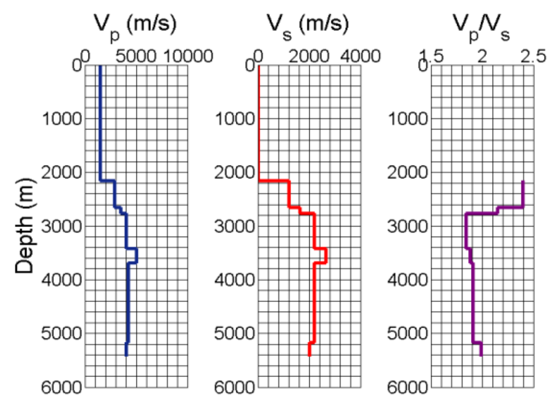

Figure 1: P wave velocity (Vp), S wave velocity (Vs) and Vp/Vs ratio of the first pre-salt model from Santos Basin studied here.

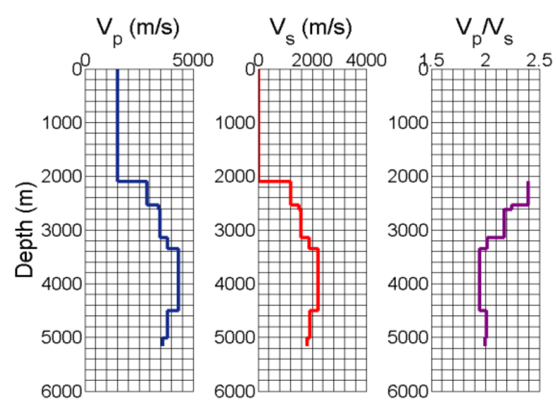

Figure 2: P wave velocity (Vp), S wave velocity (Vs) and Vp/Vs ratio of the second pre-salt model from Santos Basin studied here.

Method

To control the effects of the nonhyperbolicity it is necessary to use nonhyperbolic multiparametric approximations capable of minimize these effects.

The approximations used here were described and compared by Zuniga *et al*. (2015) to understand the applicability of each one. The specific applicability of these approximations was previously studied and defined

(Thomsen, 1986; Castle, 1988 and 1994; Tsvankin and Thomsen, 1994; Li and Yuan, 1999; Tsvankin and Grechka, 2000a and 2000b; Fomel and Grechka, 2000 and 2001; Tsvankin, 2001; Yuan and Li, 2002; Li, 2003).

Equation 1 - Malovichko (1978).

$$t = t_0^2\left(1 - \frac{1}{S}\right) + \frac{1}{S}\sqrt{t_0^2 + \frac{Sx^2}{v^2}} \tag{1}$$

Equation 2 - Alkhalifah and Tsvankin (1995).

$$t = \sqrt{t_0^2 + \frac{x^2}{v^2} - \frac{2\eta x^4}{v^2\left[t_0^2 v^2 + (1+2\eta)x^2\right]}} \tag{2}$$

Equation 3 - Ursin and Stovas (2006).

$$t = \sqrt{t_0^2 + \frac{x^2}{v^2} - \frac{(S-1)x^4}{4v^4\left(t_0^2 + \frac{(S-1)}{2}\frac{x^2}{v^2}\right)}} \tag{3}$$

Equation 4 (Blias, 2009).

$$t = \frac{1}{2}\sqrt{t_0^2 + \frac{1-\sqrt{S-1}}{v^2}x^2} + \frac{1}{2}\sqrt{t_0^2 + \frac{1+\sqrt{S-1}}{v^2}x^2} \tag{4}$$

Equation 5 - Muir and Dellinger (1985).

$$t = \sqrt{t_0^2 + \frac{x^2}{v^2} - \frac{f(1-f)x^4}{v^2\left(v^2 t_0^2 + fx^2\right)}} \tag{5}$$

Equation 6 – Li and Yuan (2001).

$$t = \sqrt{t_0^2 + \frac{x^2}{v^2} - \frac{(\gamma-1)}{\gamma v^2}\frac{(\gamma-1)x^4}{4t_0^2 v^2 + (\gamma-1)x^2}} \tag{6}$$

Where t is the travel-time, x is the offsets, t_0 is the time for zero-offset and v is the velocity of reflected wave. The S parameter is the heterogeneity parameter. The η parameter is the one which quantifies the nonhyperbolicity concerning the anisotropy. The f parameter is the anellipitical parameter. The γ parameter considers the effects of wave conversion, anisotropy and heterogeneity.

The analysis of the complexity of each approximation is an important step to select which one is the most appropriate to be used in a specific model. The analysis here was performed studying the topography of the objective function for each approximation and it was based on the observation of the residual function maps (Larsen, 1999; Kurt, 2007). Then, it is possible to determine which approximation and which kind of optimization algorithm (global search algorithm or local search algorithm) must be used.

Results

Figure 3 shown the unimodality of the Equation 1 and how simple is its topography. For the Model 1 the topography is very stable concerning the correlation of the PP and PS wave reflection event. However, the Model 2 presents a strong variation between the conventional and the converted wave event.

It can be seen in Figure 4 that the Equation 2 is very stable concerning to its topographic structure. However, the slim structure can generate a distortion in the global minimum region which can brings a significant associated error even with this approximation being unimodal.

Equation 3, which results can be found in Figure 5, presents a stable topography for the two models and the two wave reflection events. However, in previous works (Zuniga *et al.*, 2015, and Zuniga *et al.*, 2016) this approximation shown an idiosyncratic characteristic, as its condition of multimodality and unimodality varies with the model. Therefore, this approximation must be used carefully and only in models which have some *a priori* informations known.

Figure 6 also presents few variations between the models and between the wave reflection events. However, the Equation 4 has many similarities with the Equation 3, which includes the characteristic of being sometimes multimodal and sometimes unimodal. So, it can bring the same problems of the Equation 3.

Equation 5 has the same problem of the Equation 2, where the slim topographic structure can distort the global minimum region and generate a significant associated error during the inversion (Figure 7). Though the topography is strongly stable for both wave reflection events and both models, it is multimodal what brings more processing time.

In Figure 8, it can be observed that the Equation 6 is multimodal. However, the topographic structure is very well defined and has few variations from the conventional event to the converted event, and it can be also seen that the complexity of the topography has few variations between the two models.

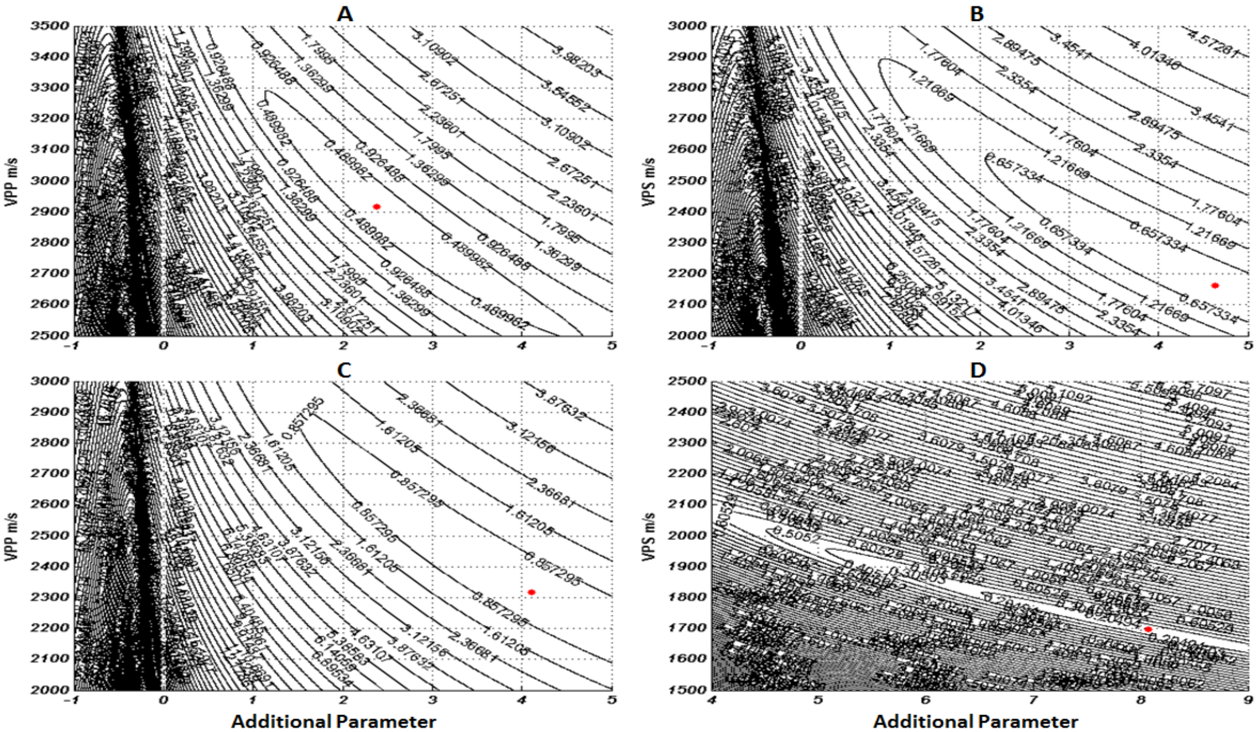

Figure 3: Residual function maps demonstrating the complexity of Equation 1 (Malovichko, 1978) for the (A) PP and (B) PS wave reflection event of the Model 1, and for the (C) PP and (D) PS wave reflection event of the Model 2. Red dispersions represent the global minimum region.

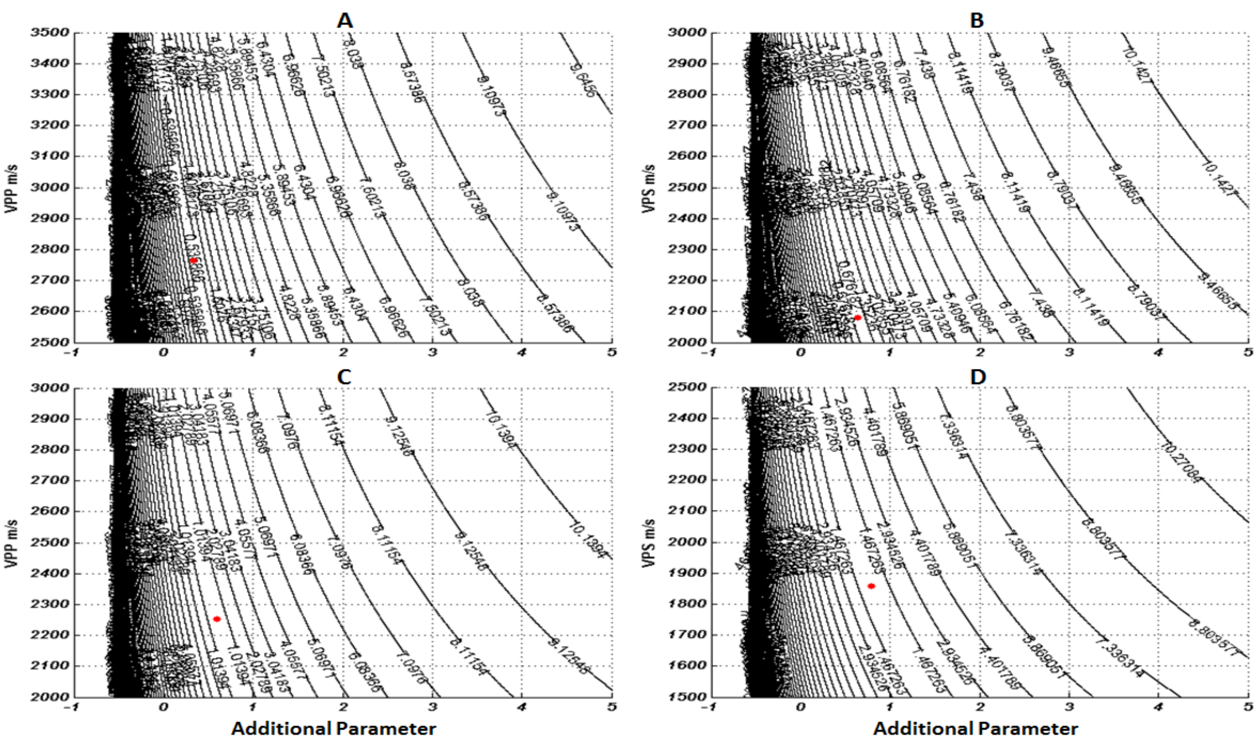

Figure 4: Residual function maps demonstrating the complexity of Equation 2 (Alkhalifah and Tsvankin, 1995) for the (A) PP and (B) PS wave reflection event of the Model 1, and for the (C) PP and (D) PS wave reflection event of the Model 2. Red dispersions represent the global minimum region.

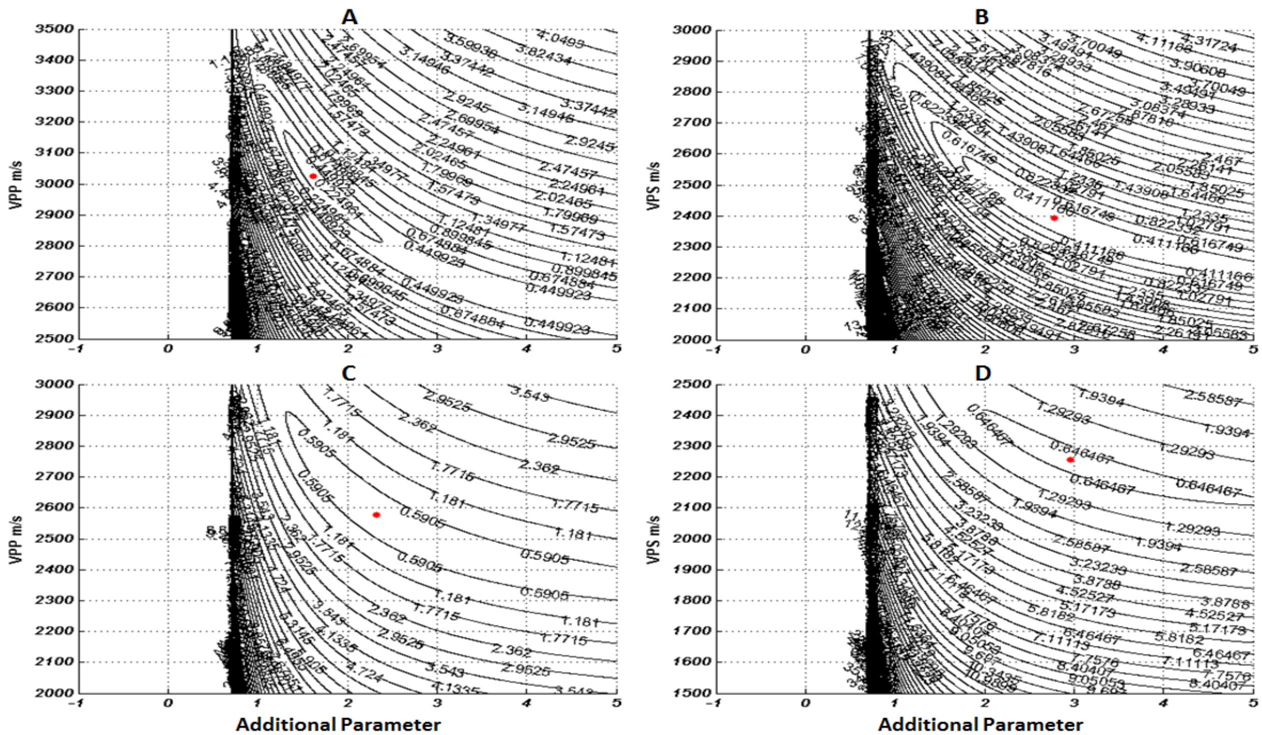

Figure 5: Residual function maps demonstrating the complexity of Equation 3 (Ursin and Stovas, 2006) for the (A) PP and (B) PS wave reflection event of the Model 1, and for the (C) PP and (D) PS wave reflection event of the Model 2. Red dispersions represent the global minimum region.

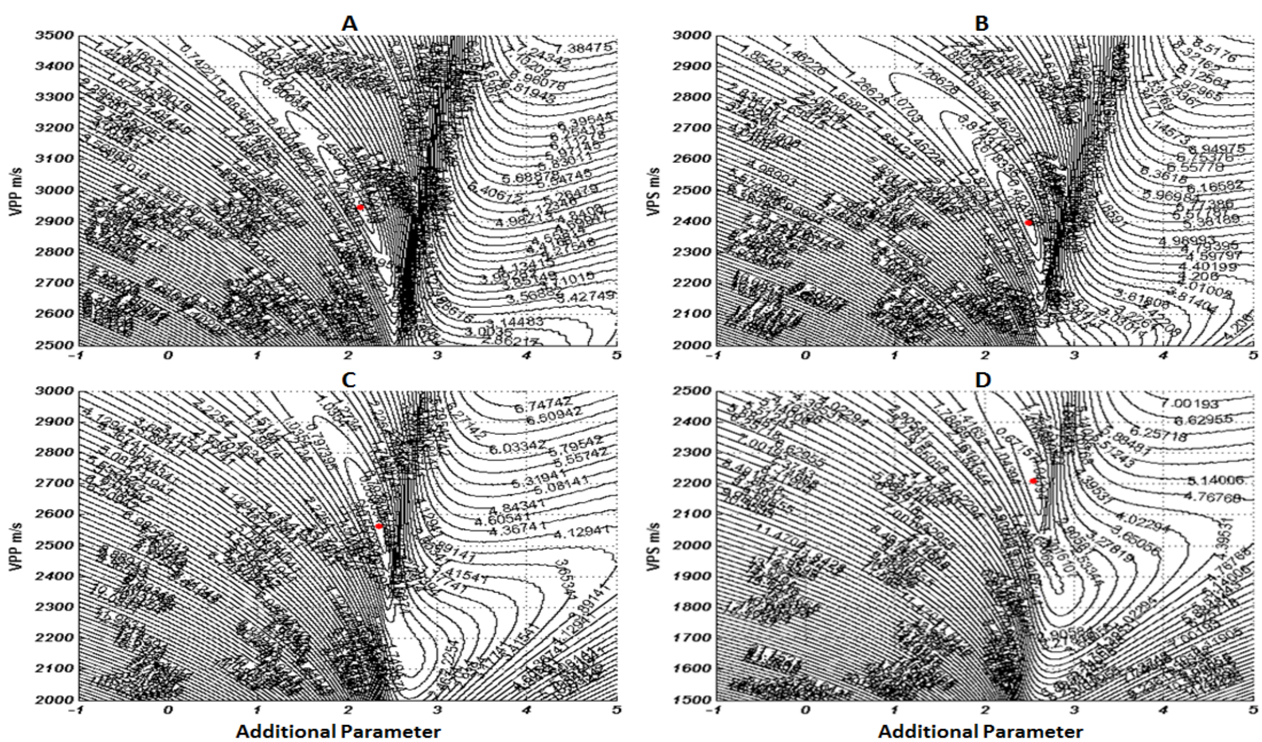

Figure 6: Residual function maps demonstrating the complexity of Equation 4 (Blias, 2009) for the (A) PP and (B) PS wave reflection event of the Model 1, and for the (C) PP and (D) PS wave reflection event of the Model 2. Red dispersions represent the global minimum region.

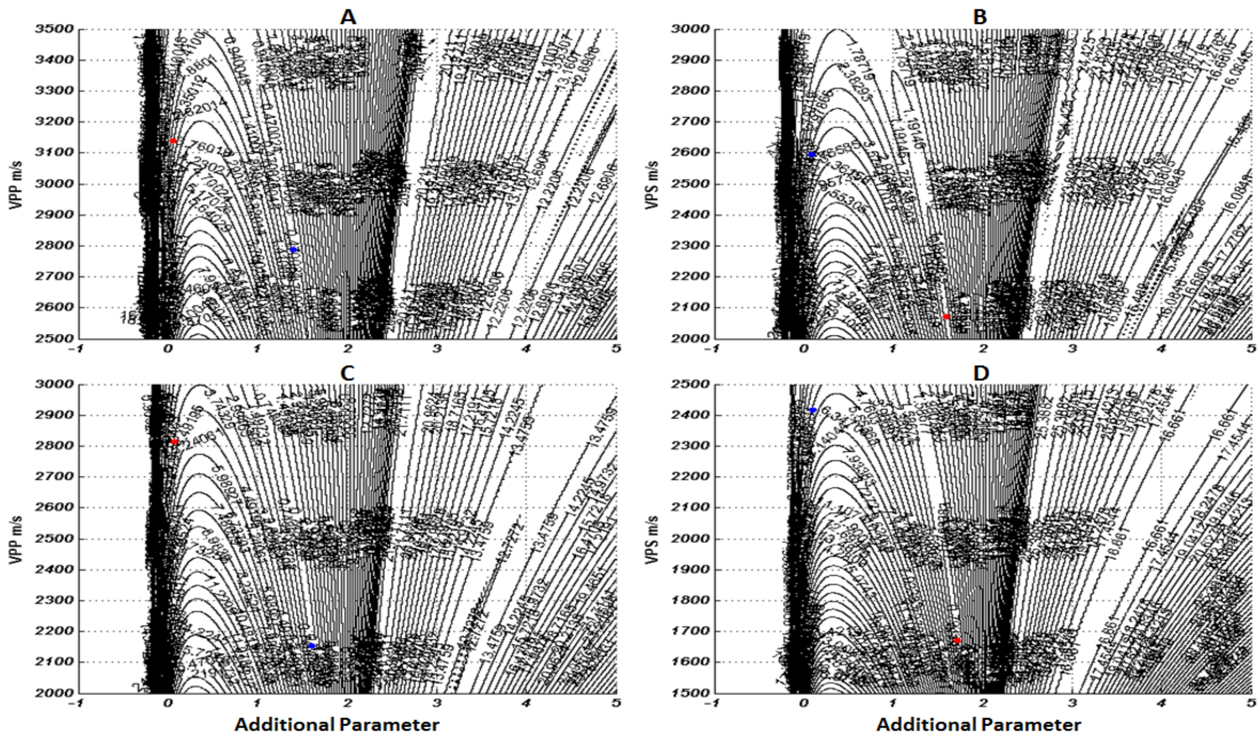

Figure 7: Residual function maps demonstrating the complexity of Equation 5 (Muir and Dellinger, 1985) for the (A) PP and (B) PS wave reflection event of the Model 1, and for the (C) PP and (D) PS wave reflection event of the Model 2. Red dispersions represent the global minimum region and blue dispersions represents local minimum region.

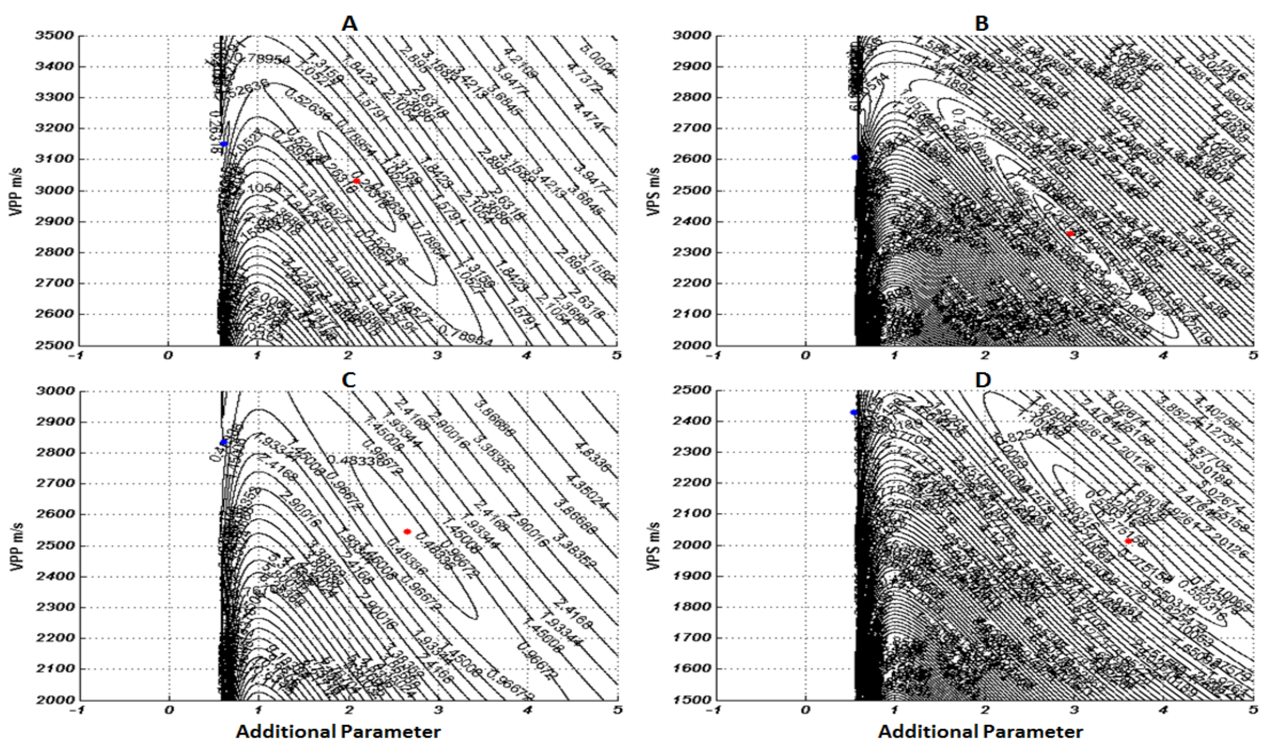

Figure 8: Residual function maps demonstrating the complexity of Equation 6 (Li and Yuan, 2001) for the (A) PP and (B) PS wave reflection event of the Model 1, and for the (C) PP and (D) PS wave reflection event of the Model 2. Red dispersions represent the global minimum region and blue dispersions represents local minimum region.

Conclusions

The Equation 6 (Li and Yuan, 2001) shown better stability than the others, and even being multimodal it shown a very well defined topography that makes easier the inversion procedure easier and can even present a low processing time. Based on the results here obtained, we conclude that Li and Yuan (2001) approximation is the most reliable one to be used in these kind of models of the Pre-salt from Santos Basin.

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