Velocity analysis in homogeneous VTI media using AB- and SVD-semblance

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Abstract

Velocity analysis in prestack reflection data on CMP gathers is traditionally performed using semblance coherence measure with hyperbolic moveout curves. However, due to the assumptions made in its development, this measure is inaccurate in estimating parameters in sections with long-spread, anisotropy, or in the presence of amplitude variation with offset (AVO), especially in the presence of polarity reversals. Nonhyperbolic moveout approximations are commonly used to approximate moveout on surveys with long offsets in anisotropic media. Among these, the shifted hyperbola approximation is considered quite valid to model traveltime reflections in CMP gather data with large offsets obtained in homogeneous VTI media. Amplitude variations with offset, in the presence of reverse polarity may be introduced in velocity analysis, modifying the traditional semblance algorithm. Singular value decomposition (SVD) applied to the semblance reduces the influence of AVO even in the presence of noise. In this paper we perform velocity analysis based on AB- and SVD-semblance with shifted hyperbole traveltime approximation in estimating parameters in anisotropic media VTI (transversely isotropic with a vertical symmetry axis) in data free and with noise. The results suggest the validity, accuracy and robustness with ones, to estimate parameters in such media.

Introduction

Velocity analysis in seismic sections is performed using coherence measures, in general semblance. However, in seismic data with high amplitude variation along of the seismic events and offset-depth ratio greater than one, the semblance loses precision in these estimations. This has lead to consider modifying the traditional semblance algorithm in order to consider seismic sections with anisotropy, long offsets, variations in amplitude with the offset and reverse polarization.

Several nonhyperbolic moveout curves in CMP sections with long offsets have been proposed to approximate traveltime in VTI media (Tsvankin and Thomsen (1994); Alkhalifah and Tsvankin (1995); Fomel (2004)). Methods to performing semblance-based velocity analysis using nonhyperbolic approximations, have been presented by Alkhalifah (1997) and Grechka and Tsvankin (1998). Using approximations by rational interpolation Douma and Calvert (2006) and Douma and Baan (2008) presented an accurate method for performing semblance-based velocity analysis in homogeneous VTI layered media.

Sarkar et al. (2001) and Sarkar et al. (2002) presented AB and AK-semblance algorithms in order to make the semblance, sensitive to variations in phase and amplitude with offset. Their experiments showed that the new measures were robust to deal with AVO anomalies and reverse polarity in isotropic media. Yan and Tsvankin (2008) extended the AK-semblance algorithm for VTI media and large offsets using nonhyperbolic moveout equation of Alkhalifah and Tsvankin (1995). Using singular value decomposition (SVD) of the matrix formed by the windowed data, Ursin et al. (2014) showed an adaptation to semblance, precise and robust enough to handle data with large amplitude variations with the offset.

In this paper, we present the velocity analysis based on semblance, AB and SVD-semblance algorithms with nonhyperbolic moveout aproximation shifted hyperbola in order to measure the accuracy and robustness in the parameter estimation in homogeneous VTI media.

Wave propagation in VTI media

TI media are characterized by five elastic parameters, density-normalized: $a_{11}, a_{13}, a_{33}, a_{44}$ and a_{66}. The P-wave propagation in VTI media has phase velocity equation, as a function of the phase angle θ given by Gassmann (1964); Thomsen (1986):

$$\frac{v^2(\theta)}{v_{Pz}^2} = 1+\varepsilon\sin^2\theta + \sqrt{1+\frac{4\sin^2\theta}{f}(2\delta\cos^2\theta-\varepsilon\cos 2\theta)+\frac{4\varepsilon^2\sin^4\theta}{f^2}}, \quad (1)$$

where ε and δ are dimensionless parameters Thomsen (1986) and $f = 1-\frac{v_{Sz}^2}{v_{Pz}^2}$, with $v_{Pz}=\sqrt{a_{33}}$ and $v_{S}z=\sqrt{a_{44}}$ the P- and S-wave vertical velocity, respectively.

In anisotropic media the group velocity equation and the group angle equation, are difficult to explain. However, its square magnitude, V, and the group angle Θ, can be obtained from the phase velocity, v, and phase angle, θ, respectively, as Berryman (1979):

$$V^2(\Theta) = v^2(\theta)+\left[\frac{dv(\theta)}{d\theta}\right]^2 \quad (2)$$

and

$$\tan(\Theta) = \frac{\tan(\theta)+\frac{1}{v(\theta)}\frac{dv(\theta)}{d\theta}}{1-\frac{\tan(\theta)}{v(\theta)}\frac{dv(\theta)}{d\theta}}, \quad (3)$$

Considering a flat reflective interface to a depth z, the wave propagation traveltime in CMP gather as a function of the offset x in homogeneous VTI media, is nonhyperbolic and it is given by:

$$t^2(x) = \frac{4z^2+x^2}{V^2(\Theta)}, \quad (4)$$

However, due to their representation depends explicitly of the group velocity, moveout approximations are often used on the steps of seismic processing. Moveout approximations based on Taylor series expansion are widely used in seismic processing. The *shifted hyperbola* approximation Malovichko (1978) is an accurate moveout curve of the fourth order Castle (1994), ie it is nonhyperbolic. Fomel (2004) explained, analytically, the shifted hyperbola approximation for homogeneous VTI media as:

$$t_P^2(x) \approx \frac{3+4\eta}{4(1+\eta)} t_{Ph}^2(x) + \frac{1}{4(1+\eta)} \sqrt{t_{Ph}^4(x) + \frac{16\eta(1+\eta)}{(1+2\eta)} \frac{t_{Pz}^2 x^2}{v_{Pn}^2}}. \quad (5)$$

where $t_{Ph}^2(x) = t_{Pz}^2 + \frac{x^2}{(1+2\eta)v_{Pn}^2}$, with: t_{Pz} the P-wave two-way vertical traveltime, $v_{Pn} = v_{Pn}(1+2\delta)$ is the normal moveout velocity, and $\eta = \frac{\varepsilon-\delta}{1+2\delta}$ the anelliptic parameter Alkhalifah and Tsvankin (1995).

Velocity analysis in VTI media

Seismic data in a CMP gather with N traces, based-semblance velocity analysis is performed to estimate t_z and v_n, maximizing the functional Taner and Koehler (1969):

$$S(t_z, v) = \frac{\sum_{\tau=t_z-T/2}^{t_z+T/2} [\sum_x \mathbf{D}(\tau,x)]^2}{N\sum_{\tau=t_z-T/2}^{t_z+T/2} \sum_x \mathbf{D}^2(\tau,x)}, \quad (6)$$

where τ is the two-way vertical traveltime in a time window, T, centered at time t_z and $\mathbf{D}(t,x)$ is a data window sampled by a moveout curve $t(\tau, v)$ with v being the stacking velocity. Generally semblance uses hyperbolic moveout curve and is accurate in estimating parameters for data without AVO and offset-depth ratio x/z values at most one.

However, current seismic processing techniques have used as a model of subsurface anisotropic media, especially anisotropic VTI media. This has required adjustments to processing steps in such a way that they can incorporate the wave propagation characteristics in such lithologic models. VTI media are widely used as a model of the subsurface seismic exploration because they describe more accurately the wave propagation in hydrocarbon reserves,for exemple, in shales and periodic thin layering (thin isotropic layers).

AB-Semblance in homogeneus VTI media

Sarkar et al. (2001) presented the coherence measures AB- and AK-semblance, in order to add to the traditional semblance in 6, the variation of the amplitude with the offset (AVO). AB-semblance is defined as:

$$S_{AB}(t_z, v) = 1 - \frac{1}{\|\mathbf{M}(\tau,x) - \mathbf{D}(\tau,x)\|^2} \|\mathbf{D}(\tau,x)\|^2, \quad (7)$$

where $\mathbf{M}(\tau,x)$ is is a parametrized model for variation of the amplitude with the offset, which, in the case in question, is the linearization of the Zoeppritz equation for the reflection coefficient obtained by Shuey (1985):

$$\mathbf{M}(\tau,x) = A(\tau) + B(\tau)\sin^2\theta_x, \quad (8)$$

with $A(\tau)$ and $B(\tau)$ the respective AVO intercept and gradient for vertical traveltime reflection events τ and θ_x is the incidence phase angle in the reflector, which in inverse problems in isotropic and homogeneous media can be estimated as:

$$\theta_x \approx \frac{x^2}{x^2 + v_n^2 t_z^2}. \quad (9)$$

However, Sarkar et al. (2001) observed that the dependence of two parameters on the Shuey approximation (8) increased the uncertainty in the inverse process making the estimates unstable. To reduce these effects, the relationship between the intercept and gradient were restricted. Thus, the Shuey approximation (8) can be rewritten as:

$$\mathbf{M}(\tau,x) = A(\tau)\left(1 + K\sin^2\theta_x\right), \quad (10)$$

where $K = B(\tau)/A(\tau)$ is fixed inside each window semblance, which means that the shape of the pulse does not change with the offset, ie, the amplitude variation along any moveout curve with vertical traveltime τ inside a pulse is a scaled version of the AVO along the moveout trajectory traced by the center of the pulse. In addition, it is assumed that the semblance window contains either a single event or several events with identical AVO behavior.

In their experiments, restricted to isotropic media, Sarkar et al. (2001) showed that the AB- and AK-semblance measurements are accurate for estimating data parameters with AVO anomalies and polarity reversals. However, they also concluded that in the absence of reversals polarity the traditional semblance is more accurate. They also showed that the AB- and AK-semblance operators are sensitive to noise, which was later demonstrated by Fomel (2009).

SVD-Semblance in homogeneus VTI media

Consider now the $N_t \times N_x$ data matrix $\mathbf{D}(\tau,x)$:

$$\mathbf{D} = \mathbf{W} + \mathbf{N}, \quad (11)$$

where $\mathbf{W}$ is the $N_t \times N_x$ signal matriz and $\mathbf{N}$ is the $N_t \times N_x$ noise matriz. In this way, the semblance 6 is rewrite as:

$$S = \frac{\|\mathbf{W}\|_F^2}{\|\mathbf{D}\|_F^2}, \quad (12)$$

where $\|\mathbf{D}\|_F$ is the Frobenius norm, defined as:

$$\|\mathbf{D}\|_F = \sum_{i=1}^{N}\sum_{j=1}^{N} \|\mathbf{D}\|^2, \quad (13)$$

and $\mathbf{W}$ is the signal matrix:

$$\mathbf{W} = \mathbf{s}\mathbf{e}^T, \quad (14)$$

which is the product between the signal vetor $\mathbf{s}$ and the transposed of $N_x \times 1$ scattered vector: $\mathbf{e}^T = [1, 1, \ldots, 1]$.

Calculating the SVD (*singular value decomposition*) of the data matrix Golub and Van Loan (1996):

$$\mathbf{D} = \mathbf{U}\Sigma\mathbf{V}^T = \sum_{i=1}^{N_t} \mathbf{u}_k \sigma_k \mathbf{v}_k^T, \quad (15)$$

where $\mathbf{u}_k$ and $\mathbf{v}_k$ are the $N_t \times 1$ and $N_x \times 1$ ortonormal vectors, respectivaly, and σ its singular values chosen such that: $\sigma_1 \geq \sigma_2 \geq, \ldots, \geq \sigma_{N_t}$, to $N_t \leq N_x$. The semblance can then be rewritten asUrsin et al. (2014):

$$S = \frac{\sum_{k=1}^{N_t} \sigma_k^2 \|\mathbf{D}\mathbf{v}_k\|^2}{N_x \sum_{k=1}^{N_t} \sigma_k^2}. \quad (16)$$

However, assuming that the shape signal remains the same in all traces, but with the amplitude varying with the offset, the signal matrix can take the form:

$$\mathbf{W} = \mathbf{s}\mathbf{a}^T, \quad (17)$$

where $\mathbf{a}^T = [a_1, a_2, \ldots, a_{Nx}]$, and $\mathbf{a}$ a vector with amplitudes in each channel. Golub and Van Loan (1996) showed that a least squares estimation of an array, is a first self-image. Thus, the signal matrix can be decomposed as $\mathbf{W} = \mathbf{u}_1 \sigma_1 \mathbf{v}_1^t$, and semblance operator modified as Gersztenkorn and Marfurt (1999); Ursin et al. (2014):

$$S_{SVD} = \frac{\sigma_1^2}{\sum_{k=1}^{N_t} \sigma_k^2}, \tag{18}$$

hereinafter called SVD semblance.

Velocity Analisy using AB- and SVD-semblance

In this experiment are performed velocity analysis based on AB- and SVD-semblance, with the *shifted hyperbola* nonhyperbolic approximation (5) considering as a model of subsurface the VTI shale Greenhorn Jones and Wang (1981), whose elastic parameters normalized by the density are: $a_{11} = 14.47 km^2/s^2, a_{13} = 4.51 km^2/s^2, a_{33} = 9.57 km^2/s^2$ and $a_{55} = 2.28 km^2/s^2$. The seismic sections was obtained considering a reflective interface located at a depth $z = 1,0\ km$, two way traveltime $t_z = 0,6464\ s$, velocities $v_{Pn} = 2,9336\ km/s, v_{Px} = 3,8045\ km/s$ and parameter $\eta = 0,3409$. The Figure 1 present the synthetic sismograms where the traveltime was the sampled at $\Delta t = 4ms$ and the signature source is the Gabor wavelet with frequency $f = 40\ Hz$. In Figure 1 (a), (b) and (c) there are variation in the noise level.

First, the velocity analysis is performed on data for AVO variation ans free noise. The Figure 2 illustrate the v_{Pn} *versus* v_{Px} (a) Traditional semblance, (b) AB-semblance and (c) SVD-semblance maps and the results are shown in Table 1. What can be seen is that the velocity analysis with SVD semblance is most accurate in parameter estimates than other ones, because the relative error in estimating the velocity v_{Pn} does not exceed 0.4% and the relative error of the η parameter is closed 1%. It is noted also that the value of maximum SVD semblance is 1.

Table 1: relative error in estimated velocities v_{Pn}, v_{Px} and η parameter, and maximum semblance; obtained from velocity analysis in data with AVO and free noise, in the VTI Greenhorn shale to $x/z = 4,0$.

Semb.	v_{Pn} err. (%)	v_{Px} err. (%)	η err. (%)	S_{max}
Trad.	9.62	2.23	68.99	0.76
AB	1.20	0.75	9.85	0.92
SVD	0.35	0.12	1.10	1.00

Then, experiments are performed on data with AVO and variation in the values of s/n. The Figure **??** illustrate the v_{Pn} *versus* v_{Px} (a) Traditional semblance, (b) AB-semblance and (c) SVD-semblance maps obtained using shifted hyperbola nonhyperbolic approximation to $s/n = 15$. The results are shown in Table **??**. What can be observed, again is that the velocity analysis based SVD-semblance is quite accurate in parameter estimates, because the relative error in estimating the velocity v_{Pn} does not exceed 0.4%, the relative error of the η parameter is less than 2% and the value of maximum SVD semblance is approximately 1.

Finally, the Figure **??** illustrate the v_{Pn} *versus* v_{Px} (a) Traditional semblance, (b) AB-semblance and (c) SVD-semblance maps obtained using shifted hyperbola

Figure 1: CMP sintético (a) sem ruído, (b) com razão sinal-ruído igual a 15 e (c) com razão sinal-ruído igual a 5, obtido no folhelho VTI Greenhorn para $x/z = 4,0$.

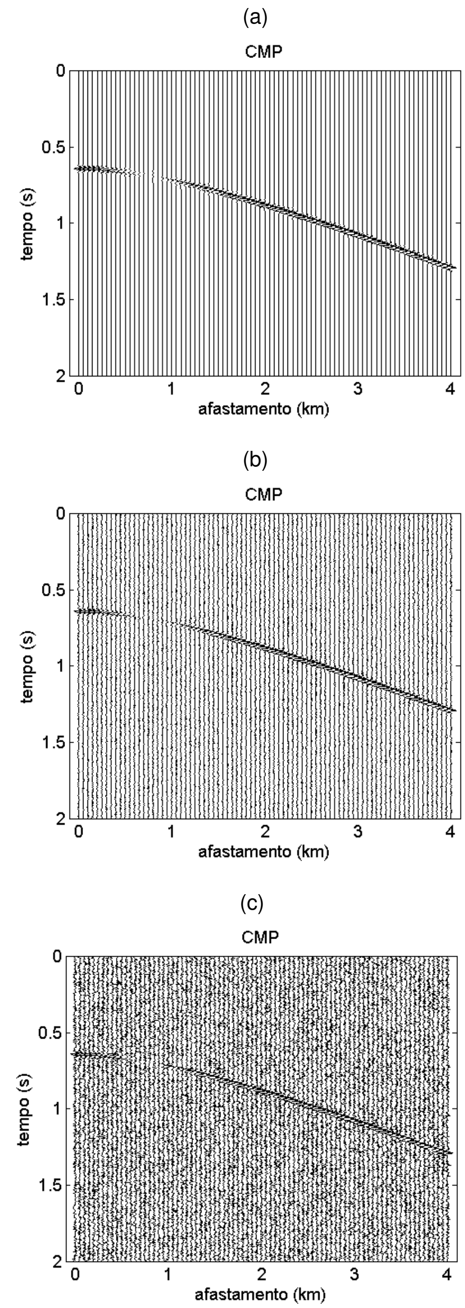
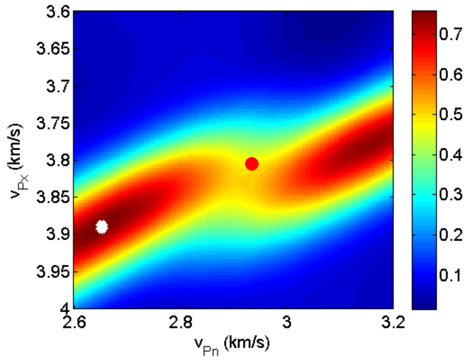


Table 2: relative error in estimated velocities v_{Pn}, v_{Px} and η parameter, and maximum semblance; obtained from velocity analysis in data with AVO and $s/n = 15$, in the VTI Greenhorn shale to $x/z = 4,0$.

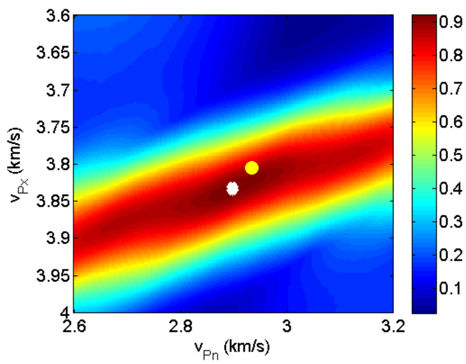
Semb.	v_{Pn} err. (%)	v_{Px} err. (%)	η err. (%)	S_{max}
Trad.	9.62	2.23	68.99	0.76
AB	1.10	0.70	9.04	0.91
SVD	0.24	0.17	0.33	0.99

Figure 2: v_{Pn} *versus* v_{Px} semblances map, using *Shifted hyperbola* aproximation (5) to estimating velocities and η parameter in datas with AVO and free noise, obtained in VTI Greenhorn shale, to $x/z = 4,0$. Yellow cirle - exact value and white losango - approximate value.

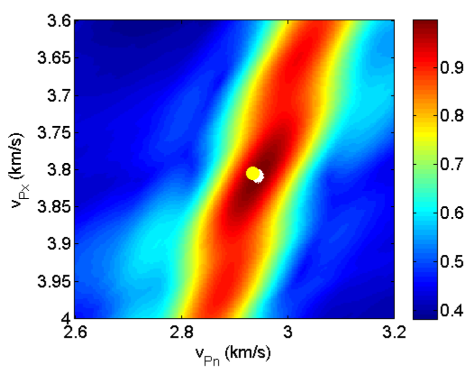
(a)



(b)



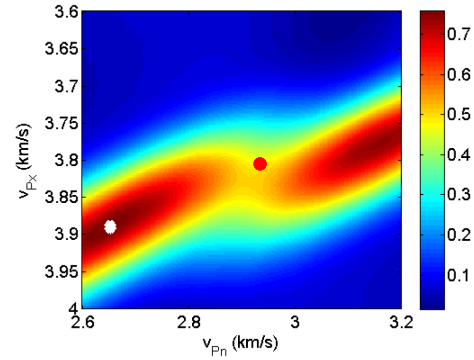
(c)



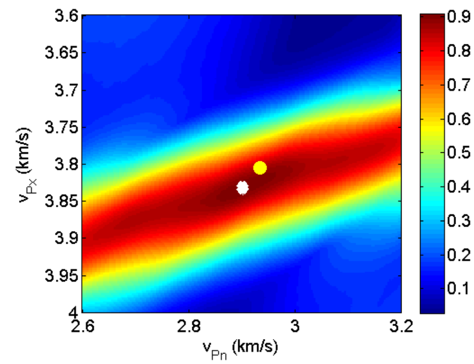
nonhyperbolic approximation to $s/n = 5$. The results are shown in Table **??**. What can be observed, again is that the velocity analysis usin SVD-semblance is quite accurate in parameter estimates, because the relative error in estimating the velocity v_{Pn} does not exceed 0.7%, the relative error of the η parameter is approximately 4% and the value of maximum SVD-semblance is 1.

Figure 3: v_{Pn} *versus* v_{Px} semblance map, using *Shifted hyperbola* aproximation (5) to estimating velocities and η parameter in datas with AVO and $s/n = 15$, obtained in VTI Greenhorn shale, to $x/z = 4,0$. Yellow cirle - exact value and white losango - approximate value.

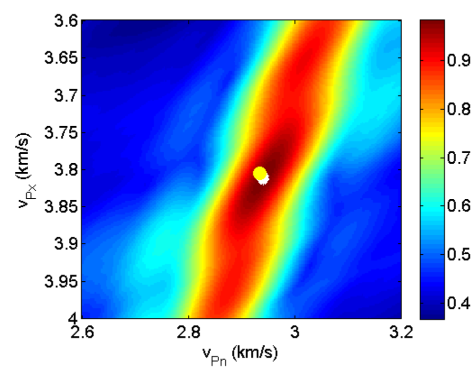
(a)



(b)



(c)

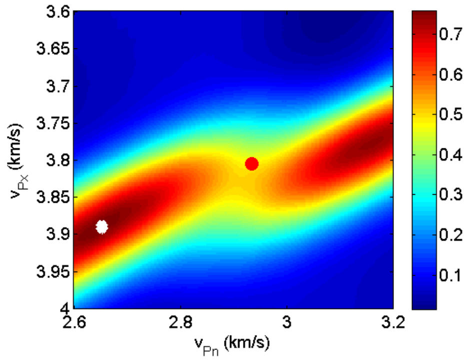


Results and Conclusions

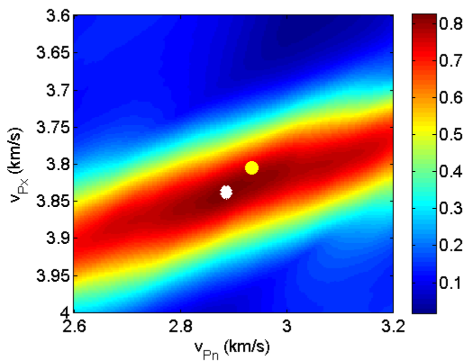
We have presented an adaptation of AB- and SVD-semblance operator to estimate NMO velocity and η parameter in VTI media. We have showed that, in general, the SVD-semblance with *Shifted hyperbola* (5) nonhyperbolic approximation is a good option in velocity analysis to estimate velocyties: v_{Pn} and v_{Px}, and η parameter in VTI data with AVO variation and polarity reversed. Experiments with and without noise and offset-

Figure 4: v_{Pn} *versus* v_{Px} semblance map, using *Shifted hyperbola* aproximation (5) to estimating velocities and η parameter in datas with AVO and $s/n = 5$, obtained in VTI Greenhorn shale, to $x/z = 4,0$. Yellow cirle - exact value and white losango - approximate value.

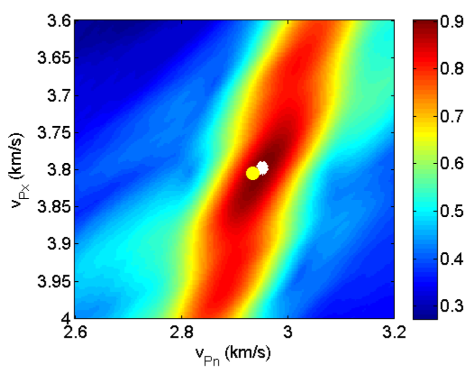
(a)



(b)



(c)



depth ratio $x/z = 4,0$, showed that the SVD-semblance most accurate and robust in parameter estimates than traditional semblance and AB- semblance. Thus, we conclude that the SVD-semblance is valid, accurate and robust estimate parameter mitigating the effects of AVO. It also acts as a noise filter.

Acknowledgments

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Table 3: relative error in estimated velocities v_{Pn}, v_{Px} and η parameter, and maximum semblance; obtained from velocity analysis in data with AVO and $s/n = 5$, in the VTI Greenhorn shale to $x/z = 4,0$.

Semb.	v_{Pn} err. (%)	v_{Px} err. (%)	η err. (%)	S_{max}
Trad.	9.62	2.23	68.99	0.76
AB	3.25	4.03	38.54	0.74
SVD	0.24	0.17	0.33	0.99
Error v_{Pn} (%)	Error v_{Px} (%)	Error η (%)	S_{max}	

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