Seismic tomography inversion of velocity field parameterized by Haar wavelet series

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Abstract

The representation of compressional seismic velocities fields, originated from geological models, by means of numerical parameters has a basic geophysical importance, because it makes possible to quantify such qualitative models, allowing mathematical manipulation. The parameterization by Haar wavelet series can be seen as an attractive alternative to quantify such geological models. The pyramidal algorithm can be used to obtain a multi-scale wavelet series and, also, it helps in the application of filters and techniques for coefficients reduction that assures an optimized parameterization of the model, with substantial parameter reduction without significant loss in the model representation. In this work, it is accomplished the parameterization of a mathematical function (with some degree of complexity) and, at same time, a geologic models already known (schematic synclinal) in order to verify the capacity of the wavelet series (linear combination of simpler functions) to represent them, in an acceptable way, using coefficients provided by the pyramidal algorithm. After certification of the possibility to parameterize models with a small number of coefficients, it is done synthetic traveltime data modeling on the current model, using ray-tracing techniques, and, then it is applied an inversion process defined by the Metropolis method. It aims convergence to a target model previously proposed. In this way, coefficients of the 1D Haar wavelet series are used as parameters of the model to be estimated by a tomographic inversion procedure.

Key-Words: Haar Wavelet Series, Seismic Velocity Field, Traveltime, Metropolis Method, Parameterization, Pyramidal Algorithm.

Introduction

A mathematical problem of great interest, not only theoretically, but also with respect to applications, is that relating to the representation (or decomposition) of functions with a high degree of complexity as a linear combination (series) of functions that are simpler or easier Manipulated (Cerqueira & Figueiró, 2012), (dos Santos & Figueiró, 2006) and (Bastos, 2013). Like the already known series, the wavelet series are defined (Morettin, 1999) as mathematical functions capable of decomposing, representing or describing other functions. In order to allow the analysis of these functions at different scales, such series are used. With respect to the observed data, they are used: in their representation, in their compression and in the attenuation of noises present in them (Misiti et al., 2007). In addition, it is applied constantly in areas of science and technology such as: physics, electrical engineering, geophysics, and etc. (Lee & Yamamoto, 1994).

In the 1930s, many researchers independently developed papers that served as the basis for more recent work. The first author to mention the term wavelet was Alfréd Haar in his thesis (Haar, 1910) and the formal concept of wavelets, which is currently known, was first proposed by Jean Morlet with the help of A. Grossman (Polikar, 1999). The idea resembles that proposed by Fourier, which consists of the decomposition of periodic and piecewise continuous functions into a sum of coefficient-weighted sine-functions.

The wavelet series represents continuous and discontinuous functions by a linear combination of simpler functions belonging to a base. In the multi-scale version of the wavelet series it is possible to obtain its coefficients from the pyramidal algorithm (Cunha, 2009), or, also, known as the discrete wavelet transform. The parameterization of a compressional seismic velocities field, corresponding to the schematic model of a synclinal, using the wavelet Haar series with coefficients obtained using the pyramidal algorithm is carried out in this work. This allows to classify the coefficients according to different levels of scale, aiming to control on which (and how) such coefficients can be eliminated without significant loss in the quality of the representation of the field, with a view to reducing to the number of coefficients used. Coefficient reduction techniques are applied in conjunction with the pyramidal algorithm.

In a later step, the synthetic data modeling is performed. The ray tracing technique proposed by Cervený (2001) was implemented in a way that allows the calculation of the time that waves spend to traverse the path that connects the positions of sources and receivers (Perin & Figueiró, 2010), generating a profile of traveltimes and, thus, partially simulating a real seismic acquisition. By, initially, treating the velocity field in a discrete mode, and not as continuous functions, it was necessary to implement interpolation techniques for the calculation of derivatives required in ray tracing.

Current models parameterized by the Haar wavelet series are submitted to an inversion process that aims to obtain a model that minimizes the difference between the observed and calculated data. A global scope inversion method, known as Metropolis, was used to obtain such a model, taking into account a stopping criterion. This method uses traveltime data having as parameters of the model the coefficients of the Haar wavelet series that represent a velocity field.

Theoretical Aspects

Wavelet Series

From a function ψ, of zero mean and finite energy, called wavelet function, a wavelet basis is defined as that constituted by functions of the type $\psi_{j,k}$, called daughter wavelets. These functions are generated by a binary dilatation 2^j and a dyadic translation $k.2^{-j}$ of the function ψ, where the indices j and $k \in \mathbb{Z}$. The wavelet series representation considers amplifications, dilatations (or contractions), and translations of this unique function, ψ, in order to obtain an approximation, as exact as possible, of the function to be represented. This orthogonal basis of functions obeys the form given by Eq. (1) (Morettin, 1999):

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \text{where } j \text{ and } k \in \mathbb{Z}. \quad (1)$$

If the system $(\psi_{j,k})$ forms an orthonormal basis of $L_2(\mathbb{R})$, space of real functions of real variable and square integrable; then, there exist coefficients $c_{j,k}$ that make possible to represent a function $f(t) \in L_2(\mathbb{R})$, defined in a limited interval and of zero average, as follows:

$$f(t) = \sum_{j=-\infty}^{+\infty}\sum_{k=-\infty}^{+\infty} c_{j,k}.\psi_{j,k}(t). \quad (2)$$

Eq. (2) is called the wavelet series and its coefficients are given by:

$$c_{j,k} = \langle f, \psi_{j,k}\rangle = \int_{-\infty}^{+\infty} f(t)\,\psi_{j,k}(t)dt. \quad (3)$$

The Haar Scale Function

The Haar scale function is defined by:

$$\phi(t) = \begin{cases} 0, & \text{se } -\infty < t < 0 \\ 1, & \text{se } 0 \le t < 1 \\ 0, & \text{se } 1 \le t < +\infty, \end{cases} \quad (4)$$

whose graph is shown in Figure 1.

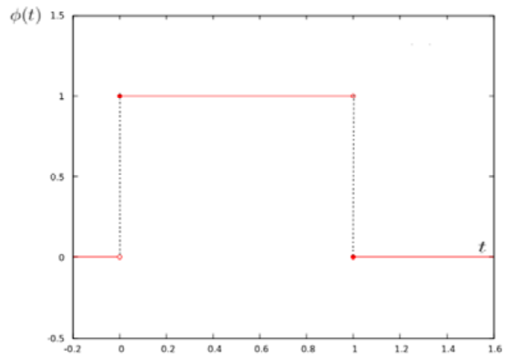

Figure 1: The Haar scale function (Cerqueira, 2015).

The Haar scale function can produces a function basis in the same way as indicated by Eq. (1), generating the following orthogonal family:

$$\boldsymbol{\phi}_{j,k}(t) = 2^{j/2}\boldsymbol{\phi}(2^j t - k), \forall \boldsymbol{t}: k\boldsymbol{2^{-j}} \le \boldsymbol{t} \le (\boldsymbol{k}+\boldsymbol{1})\boldsymbol{2^{-j}}. \quad \textbf{(5)}$$

It is easy to see that ϕ has the following property:

$$\phi(t) = \phi(2t) + \phi(2t-1). \quad (6)$$

The Haar Wavelet Function

The Haar wavelet function is defined by:

$$\psi(t) = \begin{cases} 0, & \text{se } -\infty < t < 0 \\ 1, & \text{se } 0 \le t < 1/2 \\ -1, & \text{se } 1/2 \le t < 1 \\ 0, & \text{se } 1 \le t < +\infty, \end{cases} \quad (7)$$

whose graph is shown in Figure 2.

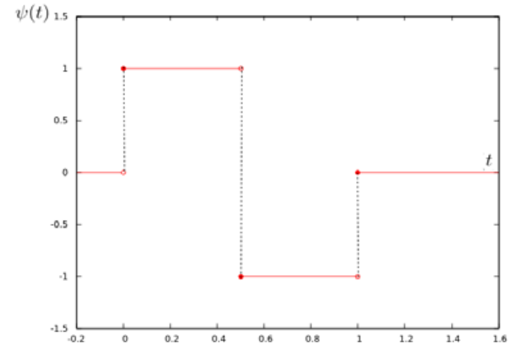

Figure 2: The Haar wavelet function (Cerqueira, 2015).

It is easy to see that ϕ and ψ have the following property:

$$\psi(t) = \phi(2t) - \phi(2t-1). \quad (8)$$

The system

$$\{\phi_{j_0,k},\ \psi_{j,k};\ \text{where } j_0,\ j \ge j_0 \text{ and } k \in \mathbb{Z}\} \quad (9)$$

is orthonormal. Then, it allows to rewrite $f(t)$**,** that appears, in Eq. (2) as:

$$f(t) = \sum_{\boldsymbol{k=-\infty}}^{+\infty} \boldsymbol{d_{j_0,k}}.\boldsymbol{\phi_{j_0,k}(t)} + \sum_{\boldsymbol{j\ge j_0}}^{+\infty}\sum_{k=-\infty}^{+\infty} c_{j,k}.\psi_{j,k}(t), \quad (10)$$

where

$$d_{j_0,k} = \langle f, \phi_{j_0,k}\rangle = \int_{-\infty}^{+\infty} f(t)\,\phi_{j_0,k}(t)dt. \quad (11)$$

The daughter wavelets of ϕ and ψ are given, respectively, by:

$$\phi_{j,k} = \begin{cases} 2^{j/2}, & \text{se } k2^{-j} \le t < (k+1)2^{-j} \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

and

$$\psi_{j,k}(t) = \begin{cases} 2^{j/2}, & \text{se } k2^{-j} < t < (k+1/2)2^{-j} \\ -2^{j/2}, & \text{se } (k+1/2) \le t < (k+1)2^{-j} \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

Pyramidal Algorithm

The Haar pyramidal algorithm (also known as the discrete wavelet transform) calculates, recursively, the coefficients of expansion given by Eq. (10), represented by Eqs. (3) and (11), using coefficients of scale $l = 0$ of the basis $\{\phi_{j,k}\}$ to get coefficients of the more refined scales by means of the following equations:

$$\phi_{l,k}(t) = \frac{1}{\sqrt{2}}[\phi_{l-1,2k}(t) + \phi_{l-1,2k+1}(t)], \quad (14)$$

as this relation is valid for all $l \in \mathbb{Z}$, we can rewrite it by moving the index l of a unit, like this:

$$\phi_{l+1,k}(t) = \frac{1}{\sqrt{2}}\left[\phi_{l,2k}(t) + \phi_{l,2k+1}(t)\right]. \quad (15)$$

Using Eq. (8), it is possible to write:

$$\psi_{l,k}(t) = \frac{1}{\sqrt{2}}\left[\phi_{l-1,2k}(t) - \phi_{l-1,2k+1}(t)\right] \quad (16)$$

and

$$\psi_{l+1,k}(t) = \frac{1}{\sqrt{2}}\left[\phi_{l,2k}(t) - \phi_{l,2k+1}(t)\right]. \quad (17)$$

Applying the operator $\langle f(t),\cdot\rangle$ to all terms of Eqs. (15) and (17), and using Eqs. (3) and (11), we can write:

$$d_{l+1,k} = \frac{1}{\sqrt{2}}\left(d_{l,2k} + d_{l,2k+1}\right) \quad (18)$$

and

$$c_{l+1,k} = \frac{1}{\sqrt{2}}\left(d_{l,2k} - d_{l,2k+1}\right). \quad (19)$$

Figure 3 represents the pyramidal algorithm used to obtain the coefficients of a function $f(t)$ from the scale $l = 0$ until $l = 3$ ($N_{max} = 7 = 2^{l=3} - 1$).

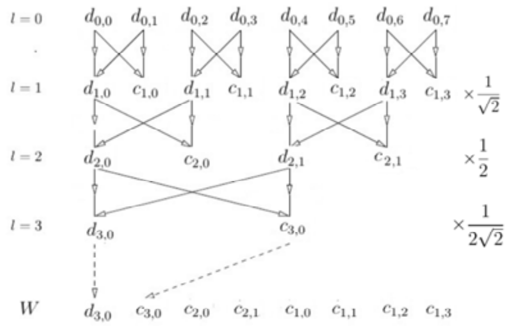

Figure 3: Diagram illustrating the pyramidal algorithm scheme to obtain coefficients of a wavelet series (Cerqueira, 2015).

Some Considerations

Some auxiliary topics necessary for the realization of the seismic inversion are, in this abstract, omitted. All can be found, in detail, in Cerqueira (2015). Such topics are related to the: seismic ray-tracing, traveltimes calculation, Metropolis method, numerical models generation, technique of previous reduction of model parameters, numerical differentiation, interpolation technique, calculation of traveltimes relative differences and also between models, acquisition system, and other very specific auxiliary tools.

Results

The Synclinal Model

The schematic geological model of the syncline, M, is represented by Figure 4, it represents a very common situation in geology in regions where converging efforts occur. The numerical model, M_N, (Figure 5) expresses like a matrix the seismic velocity field model M used for parameterization. As M is a model with little geological complexity, it was possible to generate it from a mathematical function.

Knowing that $v(r) = \frac{3r\sqrt{2}}{8} + 2$ with $r = \sqrt{(x-4)^2 + z^2}$ and $0 \le r \le 4\sqrt{2}$, we create the auxiliary variable $\xi = 2r\sqrt{2}$. Then, $\xi \in [0, 16] = [0, 2^4]$ and $v(\xi) = \frac{3\xi}{16} + 2$. Therefore, as $\xi = \sqrt{8[(x-4)^2 + z^2]}$, in terms of a function of two variables, v is given by:

$$v(x,z) = v[\xi(x,z)] = v\left[2\sqrt{2}\sqrt{(x-4)^2 + z^2}\right] = \frac{3}{8}\sqrt{2[(x-4)^2 + z^2]} + 2. \quad (20)$$

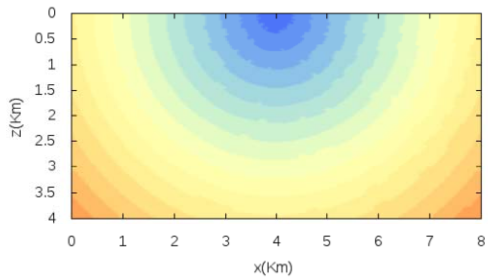

Figure 4: Schematic synclinal model (Cerqueira, 2015).

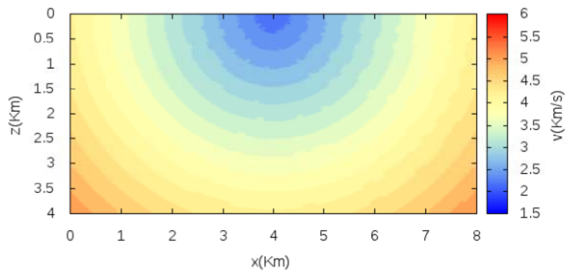

Figure 5: Numerical model M_N referring to model M. It was use 16 terms in the wavelet series that parameterizes such velocity field as a function that describes radially the syncline (Cerqueira, 2015).

Model Parameterization

The parameterization of the numerical velocity field, which resulted in the M_P model, was as a function of the variation of a radius $r \in [0, 4\sqrt{2}]$ centered on $(4, 0)$. The velocity $v(r)$ has a behavior as shown in Figure 6, which resulted in the model shown in Figure 7. The parameterization carried out using the pyramidal algorithm to obtain the coefficients $d_{j,k}$ and $c_{j,k}$ and so the velocity field is represented as the linear combination of functions of the Haar basis. Nevertheless, because it is a function of only one variable, it was possible to represent this model by the wavelet series with only five parameters.

Ray Tracing on the Model

The ray tracing technique is applied in the parameterized model M_P (Figure 7) with the objective to generate the ray field (Figure 8) and the traveltime profile (Figure 9) which is used as a synthetic observed data in the Metropolis inversion algorithm. The simulation is done with the source located in the zero position of the model ($x = 0, z = 0$.

The numerical value for the ray parameter, used in the ray tracing on this model, was $0.015\ km^2.s^{-1}$.

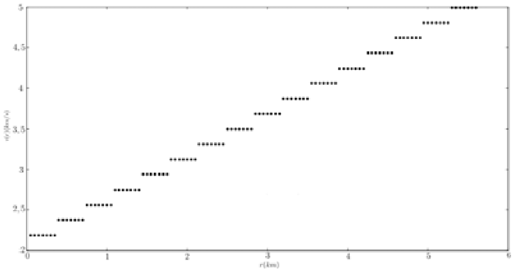

Figure 6: Variation of the velocity field M_N as a function of the radial direction r centered at the point $(4, 0)$, Cerqueira (2015).

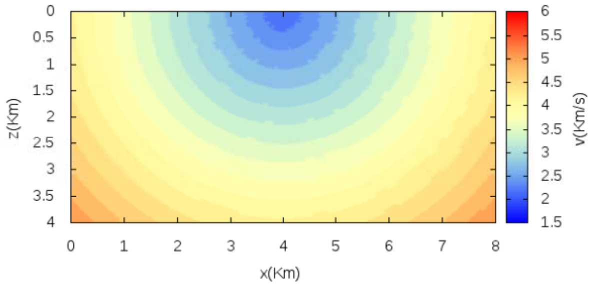

Figure 7: Parameterized model M_P indices l and k ranging from: $l_0 = 1$ to $l_f = 4$, and $k_0 = 0$ to $k_f = 7$, using 5 coefficients, since coefficients $c_{j,k}$ were made equal to c_j for all $k \in \{1,2,3\}$. This model is renamed to M_T, since it is the target model of the inversion performed (Cerqueira, 2015).

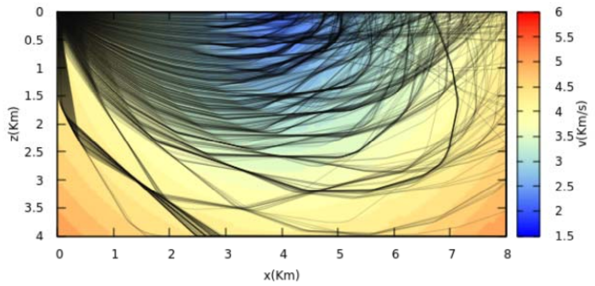

Figure 8: Ray-tracing in the parameterized model M_P, with 1,500 rays leaving the source located on the surface at position (0, 0).

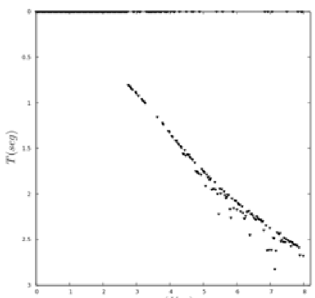

Figure 9: Profile of synthetic times obtained through the tracing of 1,500 rays on the parameterized model M_P.

Seismic Inversion

An efficient parameterization of the numerical model M_N is carried out and M_P is considered as the target model, M_T, of the inversion process, whose initial model (M_O) is presented in the Figure 10 and with coefficients shown in Table 1. The profile traveltime relative difference for models M_T and M_O is 136,15 %. The inverted model, M_I, is presented in Figure 11.

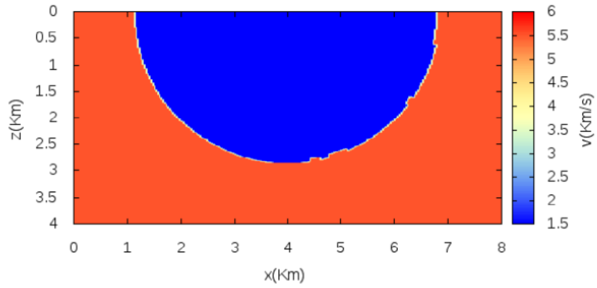

Figure 10: Initial model, M_O, used in the inversion process using the Metropolis algorithm.

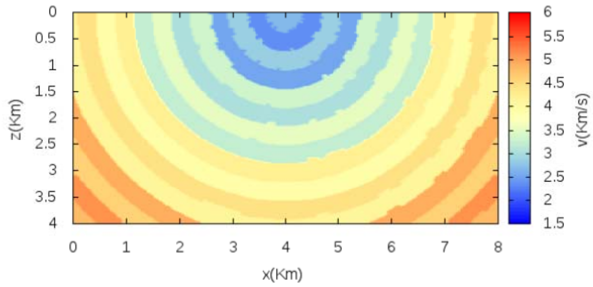

Figure 11: Inverted model, M_I, parameterized with 5 coefficients, obtained by Metropolis inversion method.

Model	$d_{4,0}$	$c_{4,0}$	c_3	c_2	c_1
M_O	14.000	−8.000	0.000	0.000	0.000
M_T	14.365	−2.995	−1.059	−0.374	−0.132
M_I	14.934	−3.166	−0.975	−0.196	0.220

Table 1: Coefficients used in the representation of the model: M_O, M_T, and M_I.

The Metropolis inversion process required 2,772 iterations for convergence, with many models rejected because they presented velocities much higher or lower than the limits of the range of valid seismic velocities. The stopping criterion is the relative error between observed and calculated data less than 10 %. The traveltime relative difference between profiles of M_I and M_T was approximately 9,97 %, showing again the good success of the procedure using the global scope inversion technique.

Figure 12 shows the relative error between inverted model M_I and the parameterized target model M_T. It was considered the following relations between parameters:

$$c_{3,0} = c_{3,1} = c_3;\ c_{2,0} = c_{2,1} = c_{2,2} = c_{2,3} = c_2; \text{and } c_{1,0} = c_{1,1} = c_{1,2} = c_{1,3} = c_{1,4} = c_{1,5} = c_{1,6} = c_{1,7} = c_1. \quad (21)$$

In terms of wavelet series:

$$v_O(\xi) = 20 \times 2^{-4/2}\phi(2^{-4}\xi) + (-6) \times 2^{-4/2}\psi(2^{-4}\xi) = 5 \times \phi(2^{-4}\xi) - \frac{3}{2}\psi(2^{-4}\xi), \quad (22)$$

$$v_T(\xi) = 14.3650 \times 2^{-4/2}\phi(2^{-4}\xi) - 2.9953 \times 2^{-4/2}\psi(2^{-4}\xi) - 1.0590 \times 2^{-3/2}[\psi(2^{-3}\xi) + \psi(2^{-3} - 1)] - 0.3744 \times 2^{-2/2}\sum_{k=0}^{3}\psi(2^{-2}\xi - k) - 0.1323 \times 2^{-1/2}\sum_{k=0}^{7}\psi(2^{-1}\xi -$$

$$k) = 3.5913\,\phi(2^{-4}\xi) - 0.7488\psi(2^{-4}\xi) - 0.3744\sum_{k=0}^{1}\psi(2^{-3}\xi - k) - 0.1872\sum_{k=0}^{3}\psi(2^{-2}\xi - k) - 0.0936\sum_{k=0}^{7}\psi(2^{-1}\xi - k), \quad (23)$$

$$v_I(\xi) = 14.9339 \times 2^{-4/2}\phi(2^{-4}\xi) - 3.1658 \times 2^{-4/2}\psi(2^{-4}\xi) - 0.9740 \times 2^{-3/2}\sum_{k=0}^{1}\psi(2^{-3} - k) - 0.1959 \times 2^{-2/2}\sum_{k=0}^{3}\psi(2^{-2}\xi - k) + 0.2201 \times 2^{-1/2}\sum_{k=0}^{7}\psi(2^{-1}\xi - k=3.7335\phi 2-4\xi-0.7915$$
$$\psi(2^{-4}\xi) - 0.3444\sum_{k=0}^{1}\psi(2^{-3}\xi - k) - 0.0980\sum_{k=0}^{3}\psi(2^{-2}\xi - k) + 0.1556k=07\psi 2-1\xi-k \quad (24)$$

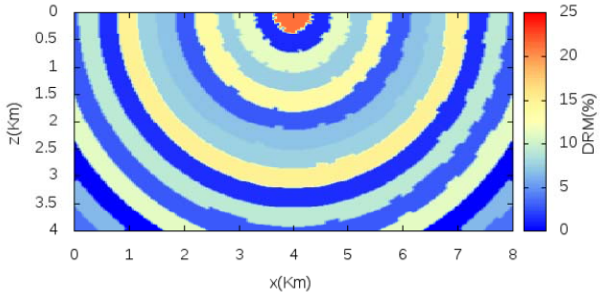

Figure 12: Relative Difference between the inverted M_I and the target M_T velocity field model (Cerqueira, 2015).

Conclusions

In the context of the parameterization, the use of the Haar pyramidal algorithm to obtain the coefficients proved to be extremely important, as was already expected in theory. Another positive factor in the use of this algorithm is that, even if it represents a discrete $2D$ model through the $1D$ model, it is possible to use a considerably small number of coefficients in the model representation with the application of the coefficient reduction method by average.

Another factor about the pyramidal algorithm is that it allowed to select the ideal values of the indexes j and k, leaving them in relative values to the samples of seismic velocities of the numerical model.

As for the aspects of the modeling, the ray tracing technique was shown efficient, presenting results of coherent traveltimes, considering the dimensions of the used model. There was a need to adapt a numerical derivative algorithm to make possible to carry out the ray tracing and to obtain the traveltimes in the model proposed in this work. The execution time of the algorithm became highly dependent on the number of samples used in the numerical model, increasing reasonably the processing time of this data for a well refined mesh.

The ray tracing technique used to model traveltimes in the target model was also used for the ray tracing in the current models, thus allowing the comparison of the time vectors obtained in each one of these models by means of the relative error between them.

The global scope algorithm, Metropolis, was efficient for the syncline model, considered simple and could also be represented through a mathematical function. An important factor that allowed the realization of these inversions was a small number of coefficients in the parameterization of such model.

The seismic tomography of velocity fields parameterized by Haar wavelet series using the Metropolis method, was not very accurate. The use of local search methods such as: Newton, Gauss Newton and others, may be used in future works in order to improve the resolution of inverted models.

Acknowledgments

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