Interval velocities obtained by adaptive hybrid inversion in multiscale approach.

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This paper was prepared for presentation during the 15th International Congress of the Brazilian Geophysical Society, held in Rio de Janeiro, Brazil, 31 July to 3 August, 2017.

Abstract

The understanding of the interior of the planet through the seismic method requires the mapping of the velocities with which the elastic waves propagate. The main objective of this work is the development of improved techniques to obtain interval velocities in time, by inversion of RMS velocities. In this article, the data inversion is performed with a combination of local and global optimization methods. In order to reduce the problems related to the large number of inversion parameters, a multiscale approach will be presented in the parameter domain. The proposed method is tested in subdetermined problems and with addition of noise. The results shown in the M3 model simulate how this methodology it can be applied applied to real data, which raises the relevance of this research.

Introduction

The seismic reflection method uses the propagation of the seismic wave in subsurface with the objective of delineating geological structures. It is the most common method used in the oil industry by being able to record reflections associated with reflectors located a lot miles deep. Much of what is known about the interior of the planet through seismic requires knowledge of the velocities with which elastic waves propagate in subsurface. This fact makes the study of the field of velocities one of the main objectives of this geophysical method, and for this reason the search for this one has been the object of study in many researches to the logo of the years. Many of the proposals used to solve this problem have limitations, such as: Need for a good initial field, overdetermined problem, noise free data and parallel plane models (Santana and Bassrei (2015); Rocha Junior and Porsani (2013); Stewart (1984); Schultz (1982)). The main objective of this paper is to present alternatives to obtain velocities in order to overcome the limitations described.

Even today, the velocity spectrum analysis is the most used methodology in the search of this field, but it is a big difference between the velocities obtained with such a procedure and the velocity with which a wave pulse travels in a range of rock (interval velocities V). Based on some considerations, velocity obtained with spectrum analysis approaches RMS velocity (Root Mean Square) V_{RMS}. When you know V_{RMS}, it can be, treating itself as data of an inverse problem, to obtain V as parameters of the model. The solution of this inverse problem is conventionally obtained by the Dix formula (Dix, 1955). However the use of this type of solution is restricted to a model in which the subsurface is formed by homogeneous layers without diving, and that the data is free of noise. When these premises are invalid, the obtained model distances itself from the true one often with abrupt and anomalous variations in relation to the real field. In this work, methods are combined in multiscale approach and using a priori information, in order to offer an alternative approach to solve the problem when the data is contaminated with noise or even when it is sub-determined problem.

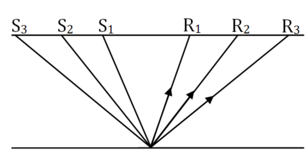
The inversion methods are divided into two groups, the first one dealing with the search for local solutions and the others seeking solutions in the global set of models. Both as methodologies of classification advantages and disadvantages, and a combination of the two types that allow one to have an algorithm of excellence. Chunduru et al. (1997) showed that hybrid algorithms are more efficient than global methods. In this work the inversion is performed by combining of the global method very fast simulated annealing (VFSA) with the local Gaus Newton (GN), in multiscale hybrid approach, and with the use of the reflectivity as a priori information.

Seismic Velocity

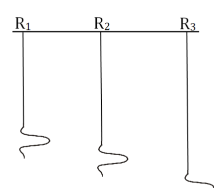
In the seismic method, there are several velocity definition, which depends on the processing step and the technique for obtaining it: velocity - interval, apparent, average, root mean square (RMS), instantaneous, phase, group, normal moveout (NMO), stacking, etc (Yilmaz, 1987).

Velocities: NMO, RMS e Intervalar

In CMP processing (Common Mid Point), meets in the same panel (Fig. 1b), all traces they have in common the midpoint between source and receiver (Fig. 1a). Starting



(a) Seismic reflection CMP .



(b) CMP family.

Figure 1: CMP processing (Silva, 2004).

from the simplified Fig. (2), you can use the Eq. (1) to calculate the propagation time between source and receiver. Where X represents the offset, V velocity and

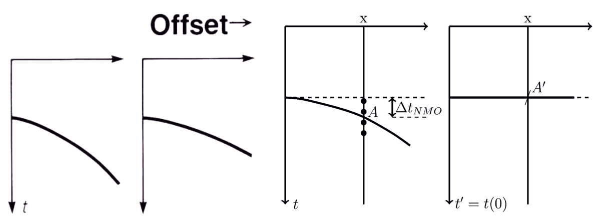
h the depth of the layer.

$$t^2(X) = (2h/V)^2 + \left(\frac{X}{V}\right)^2 \quad (1)$$

For a null offset, namely $X = 0$, $t(0) = (2h/V) = t_0$ where t_0 represents the double time in zero offset. The difference between t_0 and the time for another offset (Δt_{NMO}), is calculated by Eq. (2), and is called normal move-out (NMO) for flat reflectors and dip move-out for inclined reflectors (Yilmaz, 1987). Δt_{NMO}, it make, that the reflections due to the same midpoint in a CMP panel, form a curve hyperbolic, where a curvature is a function of the velocity of the layer (Fig2a).

$$\Delta t_{_{NMO}} = t(X) - t_0 = t_0 \left\{ \left[1 + \left(\frac{X}{V_{_{NMO}} t_0} \right)^2 \right]^{1/2} - 1 \right\} \quad (2)$$

NMO correction is the name of the process that removes Δt_{NMO} Of the traces, and causes the reflection hyperboles to horizontalize (Fig. 2b). The NMO correction is performed by estimating the NMO velocity (V_{NMO}). The equation



(a) CMP's with different velocities V. (b) Aplly NMO corection.

Figure 2: Paineis CMP's (Yilmaz, 1987; de Souza, 2014).

(2) compute $\Delta t_{_{NMO}}$ for the propagation of the wave in a single layer (Fig. 2), in cases where the wave moves through several layers with different velocities (Fig. 3), such equation becomes (3) (de Souza, 2014).

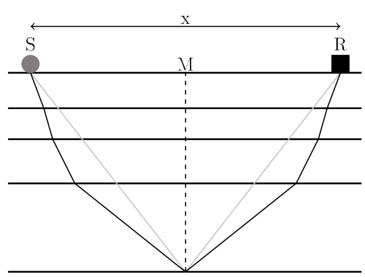


Figure 3: Multi-layer CMP (de Souza, 2014).

$$\Delta t_{_{n,NMO}} = t_{x,n} - t_{0,n} = \left[t_{0,n}^2 + \left(\frac{X}{V_{NMO}} \right)^2 \right]^{1/2} - t_{0,n} \quad (3)$$

where $t_{0,n}$ is double time in *zero-offset* from the reflection in the nth layer. Taner and Koehler (1969) showed that for small offset V_{NMO} can be approximated by the V_{RMS}, and this can be calculated by Eq. (4).

$$V_{RMS} = \sqrt{\frac{1}{t_{0,n}} \sum_{i=n}^{n} V_i^2 t_i} \quad (4)$$

where V_i interval velocity of each layer and t_i is double time for the wave to travel the distance between the top and bottom of it.

Direct Modeling and the Inverse Problem

The problems involved in geophysics are classified into two categories, the first being referred to as a direct problem, where the physical parameters of a model are known (**m**) (density, resistivity, velocity and etc.) with the use of a theory ($\mathscr{F}$) simulates the response of a supposed geophysical survey ($\mathbf{d}_{cal}$). The second class is the inverse problem, where data obtained by direct measures are known ($\mathbf{d}_{obs}$), and it is attempt, by means of a theoretical relation, to reconstruct the parameters of a model (**m**) for the sub-surface (Sen and Stoffa, 1995). The equation (5) shows a relation between **d** and **m**, and a figure 4 shows their role in the direct and inverse problems.

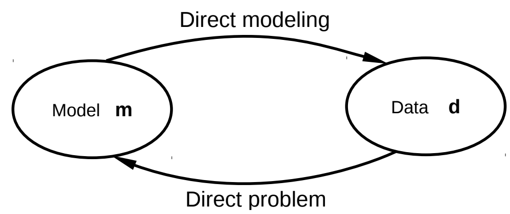


Figure 4: Relation of **d** and **m** to direct and inverse geophysical problems (Rodrigues and Bassrei, 2015).

$$\mathbf{d} = \mathscr{F}(\mathbf{m}) \quad \text{or} \quad \mathbf{d} = \mathbf{G}\mathbf{m} \quad \text{if } \mathbf{m} \text{ and } \mathbf{d} \text{ is LD} \quad (5)$$

where $\mathbf{G}$ is a linear operation in $\mathbf{m}$ to obtain $\mathbf{d}$. When **m** and **d** are LD (linearly dependent), the problem is said to be linear. Solving the inverse problem means finding $\mathbf{m}$ when known $\mathbf{d}$. In this work, we try to find the interval velocities (V) as a solution of a nonlinear inverse problem, which has as input data the velocities RMS (V_{RMS}).

Inversion methods

The solution of the inverse problem is based on the minimization (or maximization) of an objective function Q(**m**), where such a function usually measures the energy of the errors between observed data ($\mathbf{d}_{obs}$) and calculated ($\mathbf{d}_{cal}$) as shows Eq. (6) (Wright and Nocedal, 1999).

$$Q(\mathbf{m}) = \mathbf{e}^T \mathbf{e} \quad \text{where} \quad \mathbf{e} = \mathbf{d}_{obs} - \mathbf{d}_{cal} \quad (6)$$

In the case of linear problems Q (**m**) represents a parabolic surface, the minimum of which is located in its vertex, can be found by the Gradient Conjugate (CG). The CG part of the initial model $\mathbf{m}_o$, and seeks a solution $\mathbf{m}_{min}$ that minimizes Q(**m**), in a subspace containing the gradient of the current iteration and the previous iterations (Hestenes and Stiefel, 1952; VanDecar and Snieder, 1994). In the problems where **m** and **d** are LI (linearly independent), Q (**m**) represents a rough surface with various minimum (Fig. 5), and for this type of problem the inversion methods are divided into local and global, and each has advantages and disadvantages. The local methods are sensitive to the initial model $\mathbf{m}_o$ and converge to the nearest minimum of this. Unlike local, global methods allow the global minimum to be found independent of the initial model.

Gaus Newton Method - GN

For a strictly nonlinear problem (Fig. 5), the Gaus Newton (GN) method creates a quadratic approximation of Q(**m**) which defines a paraboloid $\tilde{Q}(\mathbf{m}_k)$ so that it is tangent to

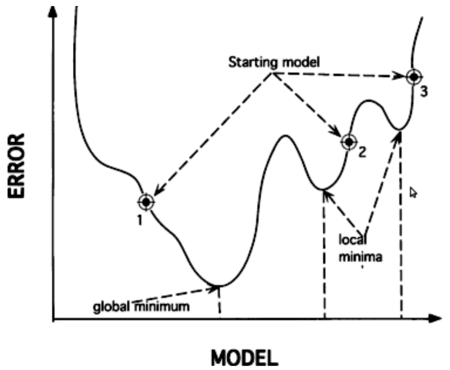


Figure 5: $Q(\mathbf{m})$ nonlinear problem (Sen and Stoffa, 1995)

Q(**m**) at the current point. In this way the current model is updated by Eq. (7):

$$\mathbf{m}^* = \mathbf{m_k} + \Delta\mathbf{m_k} \tag{7}$$

where $\Delta\mathbf{m_k}$ is the perturbation performed in the model parameters, and this is found as a solution of the linear equation (Eq. 8), which can be solved by GC for example.

$$\mathbf{G_k^T G_k} \Delta\mathbf{m_k} = \mathbf{G_k^T e_k} \tag{8}$$

where $\mathbf{e}_k$ is a linear approximation of the noise and $\mathbf{G}_k$ is the matrix of the derivatives of equation (5) in relation to the parameters of the model, known as sensitivity matrix. **m*** is updated until it represents a local minimum, and for this to be the global of the problem, $\mathbf{m}_o$ must be close to $\mathbf{m}_{min}$, and this characterizes GN as local search method.

Very Fast Simulated Annealing - VFSA

The VFSA method is an alternative to find the global minimum of the objective function, and is described as a variant of Simulated Annealing (SA) (Sen and Stoffa, 1995), which allows narrowing the search interval in the iteration, which results in faster convergence. From an initial model $\mathbf{m}_k$, the updating of the same occurs through Eq. (9), where each parameter m_k^i is disturbed by a factor y_i, generated randomly by equation (11) (Sen and Stoffa, 1995).

$$m_i^{k+1} = m_i^k + y_i\left(m_i^{max} - m_i^{min}\right), \tag{9}$$

The parameter y_i is generated from the following distribution

$$g_T(y) = \prod_{i=1}^{NM} \frac{1}{2\left(|y_i| + T_i\right) ln\left(1 + \frac{1}{T_i}\right)} = \prod_{i=1}^{NM} g_{T_i}(y_i), \tag{10}$$

Thus a random number u_i drawn from a uniform distribution u[0,1] can be mapped into the above distribution with the formula

$$y_i = sgn(u_i - \frac{1}{2}) T_i \left[\left(1 + \frac{1}{T_i}\right)^{|2u_i - 1|} - 1\right]. \tag{11}$$

with m_i^{max} and m_i^{min} Represents the bounds of the model, and u_i generated randomly. Ingber (1989) showed that the overall minimum is obtained statistically using the cooling criterion shown in Eq. (12), Where the temperature T_i is reduced at each iteration.

$$T_i(k) = T_{0i} e^{-C_i k^{1/NM}}, \tag{12}$$

The model generated by Eq. (9) will be taken as the current model based on the criterion of metropolis (Metropolis et al., 1953). The statistical character of generation and acceptance allows the algorithm to escape from local minimum.

Hybrid optimization methods

Local and global optimization algorithms are used commonly in geophysical data inversion. Each type of algorithm has unique advantages and disadvantages (Chunduru et al., 1997). In this work the idea is to use a combination of the two types, in order to extract The advantages of each of them, and discard their disadvantages. Chunduru et al. (1997) showed that hybrid algorithms to be computationally more efficient than conventional methods of global optimization. In particular, we combine the local GN method with a global VFSA approach to solve problems of geophysical interest. The method structure is GN at the end of the VFSA, Where the VFSA gets the starting model for GN.

Hybrid search algorithms have the potential to make use of the important features of both global and local algorithms they do not require a good starting solution, they are computationally less expensive compared to global algorithms, and they obtain good models with poor starting solutions (Chunduru et al., 1997).

Multiscale approach

It is known that the higher the number of inversion parameters, the greater the difficulty of convergence. This work applies a multiscale approach, which consists in solving the problem at different scales, where it is changed at each step, and the input model is given as the one obtained in the previous step, until the problem is completely solved. Different multiscale approaches are applied in different domains (Zun-Ze et al., 1998; Shi et al., 2000), in this work, multiscale is used in the parameter domain, which allows a significant reduction in the number of inversion parameters, and this quantity is increased at each iteration.

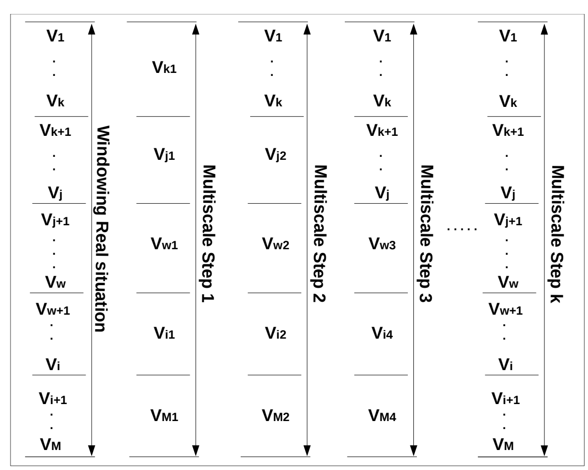


Figure 6: Multiscale scheme in the domain of parameters

The approach is based on two steps, where in the first is created a windowing in the parameters of the model, so that the velocities are separated in blocks, and within this the set of velocities are described by a single parameter, hence the problem is solved by using the VFSA method (Fig. 6). The model obtained in the initial step is given as input model to be optimized in the following steps by the GN method. At each step of GN the solution is being improved by gradually increasing the inversion parameters, until the moment the problem is completely solved (Fig. 6).

Results

The determination of interval velocities can be expressed as an inverse problem using Eq. (5), where using (4) we have that:

$$\mathbf{m} = [V_1 \dots V_n]^T \qquad \mathbf{d} = [V_{RMS,1} \dots V_{RMS,n}]^T \tag{13}$$

RMS Velocity profiles are used in time as input data (**d**), and through the proposed methodology, interval velocity profiles V are also obtained in time as parameters of the model (**m**)). In order to satisfy Eq. (13), both profiles are discretized in vectors according to figure (7).

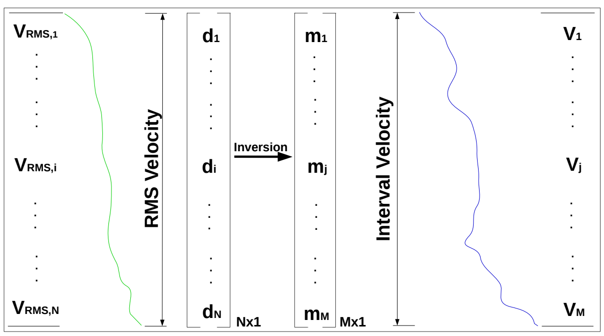


Figure 7: Relationship between interval and RMS velocity in the inversion problem

The velocity search methodology is applied in both one-dimensional (1d) and two-dimensional (2D) models .

Model M1 - 1D

In 1D models (V_{RMS}) are modeled directly by equation (4), and the problem is undetermined type, where the parameter number (M = 300) of the model is greater than the number of data of the problem (N = 100). The inversion was performed with the hybrid method in multiscale having as the initial model profiles with constant velocities $V_0 = 4200$ft/s. For each 1D model, the result of the inversion obtained with the first step (**VFSA**) and the resulting improvement in the other steps (**Hybrido GN**), besides the real (**initial**) and initial (**initial**) models, are shown in a single figure for possible comparisons.

The Model M1 is based on the idea that within a same layer the velocity is constant. The results are shown in figure 8.

It is interesting that in the multiscale approach presented, the effectiveness of the method is directly related to the size and quantity of windows for each profile, and in model M1 the windowing was done in a random way (Fig. 8). Seeking

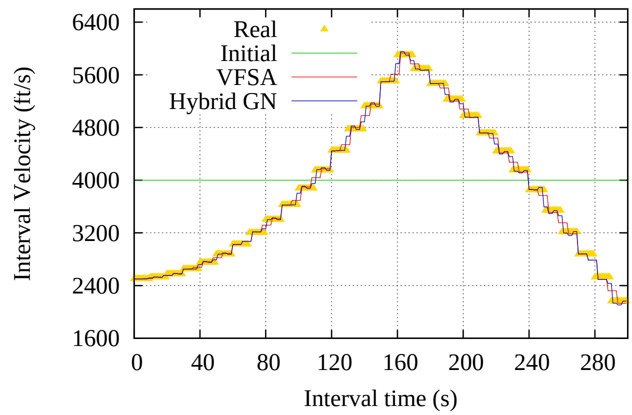


Figure 8: Profile of interval velocities as a function of time, model M1.

to improve the efficiency of the method was created a criterion of windowing that is based on using the reflectivity of the model as a priori information in the search of the ideal window, so that the set of velocities separated by two primary reflections were windowed (Fig. 9) The M1

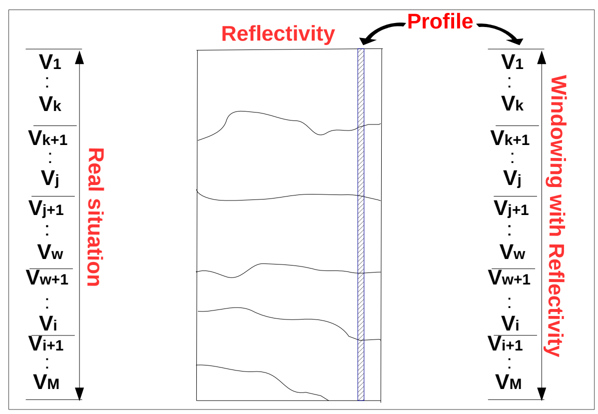


Figure 9: Application of reflectivity in the windowing of the parameters in the multiscale approach.

model was reversed again, now using reflectivity as a priori information (Fig. 10).

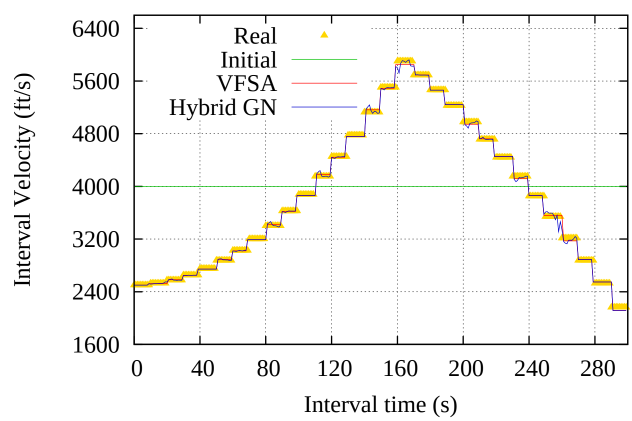


Figure 10: Profile of interval velocities as a function of time, model M1 with reflectivity as prior information.

When we compare the figures 8 and 10, we can see a real improvement resulting from the use of reflectivity with a priori, for this reason this information has been inserted in all other models.

Model M1 - 1D with noise

The data vector of the M1 model (V_{RMS}) was disturbed with random noise of $\pm 1.7\%$ and the inversion results are shown in figure 11.

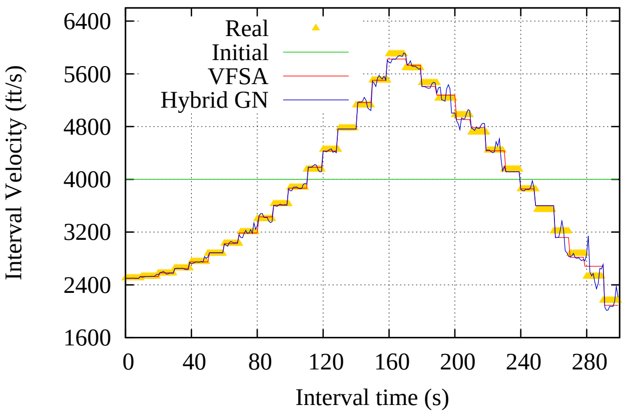


Figure 11: Profile of interval velocities as a function of time, model M1 with reflectivity as prior information, noisy data.

Model M2 -1D

The model M2 describes a situation similar to M1, but in M2 the velocity inside the layers is not constant as M1, and is subjected to a variation by use of velocity gradient, and this differs from one layer to another. The figure 12 summarizes the results of this model.

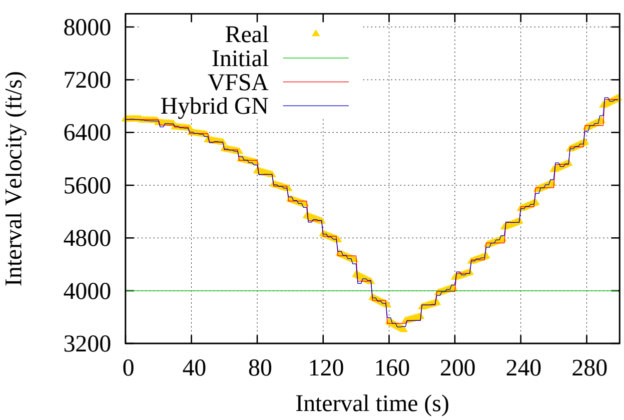


Figure 12: Profile of interval velocities as a function of time, model M2 with reflectivity as prior information.

Based on the high quality results of the 1D approach presented in the figures (8) , (11) e (12), we can see the autonomy and efficiency of the method, since in all situations, even if it was an undertermined problem and in the presence of noise, convergence occurs for models that are highly representative of true ones, even though the initial model was constant.

Model M3 - 2D

The M3 model, shown in the figure (Fig 13), simulates a plane-paralele layer medium that has failed. In this model, the Seymic Unix package, which is based on ray tracing, was used to model 200 shots in an end-on arrangement (Fig 13). Seismic modeling, carried out in M3, generated a total of 497 CMPs ranging from 1 to 497, separated by 25 feet. In the maximum fold CMPs, from 100 to 400, a velocity analysis was performed, which resulted in the profiles of V_{RMS}, in addition, spectrum picking *time X Velocity*, a pseudo-reflectivity was generated to be used with a priori in the windowing of the multiscale.

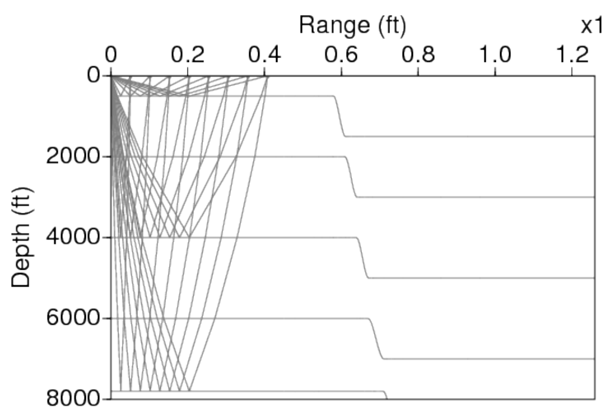


Figure 13: M4 Seismic modeling Cshot ray tracing.

The figure 14 shows the inversion of the resulting profile of the CMP 150. In this figure it can be seen that the use of GN, in search of resolution improvement, is strongly influenced by data noise, and that noise comes from two factors (i) V_{RMS} is best represented by V_{NMO} when the medium is free of lateral velocity variation, which is not obeyed by M3, (ii) The velocity analysis is a manual step and is subject to observer errors.

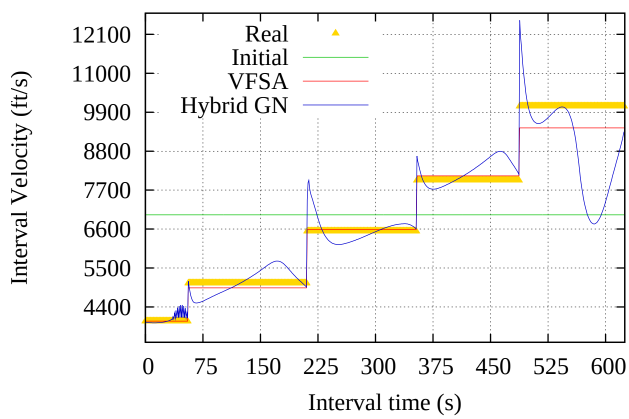


Figure 14: Profile of interval velocities as a function of time, model M3 with reflectivity as prior information, noisy data.

The M3 inversion is complete when all the CMP's of the seismic data are inverted and the interpolation of the generating a 2D model in time (Fig. 15).

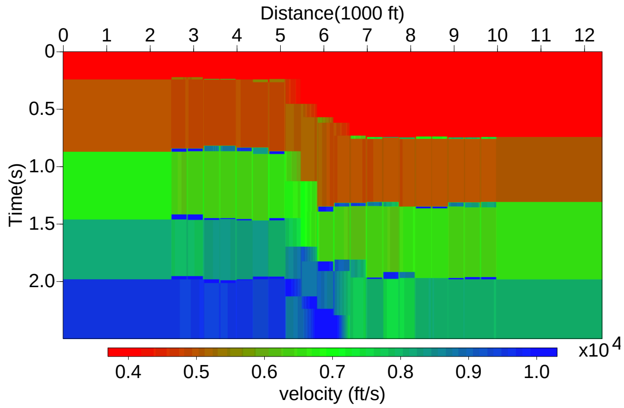


Figure 15: 2D session of inverted M3 model using reflectivity as prior information.

Conclusion

In this paper, the problem of obtaining V, was solved by the hybrid inversion of V_{RMS} in multiscale approach. The proposal presented an alternative to overcoming some existing limitations in previous approaches that deal with this problem, such as: need for a good initial field, overdetermined problem, noise-free data and parallel plane models. In all situations the convergence occurred starting from a constant model, which showed autonomy of the method in relation to the initial model. In the noisy situations, the method remained stable, satisfactorily solving the problem, but in these cases the increase in resolution reduced the accuracy of the results. The use of reflectivity proved to be a useful alternative to the multiscale giving automation and steering in the windowing phase. The high acuracy of the results, allows to conclude that the inversion hybrid multi-scalar approach was efficient in the solution of the proposed problem.

In furturos works it is intended to use the seismic modeling with complete equation of the wave by finite differences. It is also intended to combine other methods (global + local), in addition to applying such a methodology as an initial model estimator to be used in FWI. Finally, it is intended to apply this methodology to real data.

References

Chunduru, R. K., M. K. Sen, and P. L. Stoffa, 1997, Hybrid optimization methods for geophysical inversion: Geophysics, **62**, 1196–1207.

de Souza, M. S., 2014, Determinação automática das velocidades de empilhamento para obtenção da seção sísmica zero-offset: Trabalho de graduação, Universidade Federal da Bahia, Salvador, Brasil.

Dix, C. H., 1955, Seismic velocities from surface measurements: Geophysics, **20**, 68–86.

Hestenes, M. R. and E. Stiefel, 1952, Methods of conjugate gradients for solving linear systems, volume **49**: NBS.

Ingber, L., 1989, Very fast simulated re-annealing: Mathematical and computer modelling, **12**, 967–973.

Metropolis, N., A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller, 1953, Equation of state calculations by fast computing machines: The journal of chemical physics, **21**, 1087–1092.

Rocha Junior, D. C. and M. J. Porsani, 2013, Non-linear inversion of interval velocities: 13th International Congress of the Brazilian Geophysical Society & EXPOGEF, Rio de Janeiro, Brazil, 26–29 August 2013, 1242–1246.

Rodrigues, V. H. S. R. and A. Bassrei, 2015, Aplicação da tomografia de tempos de trânsito a dados do campo de miranga, bacia do recôncavo: 14th International Congress of the Brazilian Geophysical Society & EXPOGEF, Rio de Janeiro, Brazil, 3-6 August 2015, 1185–1190.

Santana, T. G. and A. Bassrei, 2015, Inversion of interval velocities: Application to a geological model from a pre-salt area: Revista Brasileira de Geofísica, **33**, 19–27.

Schultz, P. S., 1982, A method for direct estimation of interval velocities: Geophysics, **47**, 1657–1671.

Sen, M. K. and P. L. Stoffa, 1995, Global optimization methods in geophysical inversion: Cambridge University Press.

Shi, X., J. Wang, S. Zhang, and X. Hu, 2000, Multiscale genetic algorithm and its application in magnetotelluric sounding data inversion: CHINESE JOURNAL OF GEOPHYSICS-CHINESE EDITION, **43**, 122–130.

Silva, M. G. d., 2004, Processamento de dados sísmicos da bacia do tacutu: Dissert. de mestrado, Universidade Federal da Bahia, Salvador, Brasil.

Stewart, R., 1984, Vsp interval velocities from traveltime inversion: Geophysical Prospecting, **32**, 608–628.

Taner, M. T. and F. Koehler, 1969, Velocity spectra-digital computer derivation applications of velocity functions: Geophysics, **34**, 859–881.

VanDecar, J. C. and R. Snieder, 1994, Obtaining smooth solutions to large, linear, inverse problems: Geophysics, **59**, 818–829.

Wright, S. and J. Nocedal, 1999, Numerical optimization: Springer Science, **35**.

Yilmaz, O., 1987, Seismic data processing: SEG, Tulsa.

Zun-Ze, H., Y. Wen-Cai, and L. JIA-QI, 1998, Multiscale inversion of the density contrast within the crust of china [j]: Chinese Journal of Geophysics, **5**, 007.

Acknowledgements

The authors wish to express their gratitude to brazilian agencies, (INCT-GP/CNPq/MCT, CNPq, CAPES, FINEP) for financial support.