P-wave velocity estimation from well log data through least-squares inversion using Bayesian regularization

Odette C. A. de Aragão (IGEO/UFBA), Amin Bassrei (CPGG/IGEO/UFBA & INCT-GP)

Abstract

A procedure for well log acoustic velocity prediction is formulated in a probabilistic standpoint based on the regularized least-squares framework. This inversion technique is aimed at solving the relevant problem of estimating the sonic log in wells where this log is absent. The statistical approach combines rock physics relationships between the model and data parameters. The maximum a posteriori estimate of the compressive wave velocity is derived and the results are generated from three different regularization parameters. In addition the posterior covariance model and correlation matrices are calculated and interpreted. The method validity is confirmed by the analysis of well log data from the Jequitinhonha Basin.

Introduction

A drilling program aims to estimate the formation evaluation of a reservoir. Traditionally, it is achieved by the well log petrophysical processing and interpretation. Thereby borehole geophysics plays a key role in the exploration and development of hydrocarbon reservoirs as its data can be used to characterize the geology around the well (Serra and Abbott, 1982). This technique is based on acquiring high resolution data in depth near the underground strata directly through a drilled well. In this way, in situ informations are obtained by various kinds of probes with different vertical resolutions and depths of investigation from the vicinity of the borehole wells (Ellis and Singer, 2007).

The sonic acoustic well logging records the slowness of the compressional seismic wave, also called as P-wave, propagating through the rock layers (Bassiouni, 1994). The P-wave velocity, which is the inverse of the slowness, provides essential information for reservoir exploration and development programs and therefore it is an indispensable tool in geophysical surveys. For example, the sonic log can be related to formation porosity and lithology. It can also be used to support and calibrate seismic data, to estimate mechanical rock properties and detect fractured zones and over-pressure conditions. Therefore, the absence of this log makes necessary the development of a technique for predicting the P-wave sonic log.

In this work, a regularized least-squares method that uses the theory of probability, as it is based on Bayes' theorem, is developed and tested on a real data set. The solution of the inverse problem is represented as the maximum a posteriori probability (MAP) estimate of the acoustic velocity, which is equivalent to the most probable model on the posterior Gaussian distribution (Mosegaard and Tarantola, 2002). Rock physics relationships between the model and data parameters are combined in a probabilistic point of view in order to predict the P-wave velocity log in two wells from Jequitinhonha Basin. The sonic well logs are available. However they are not used in the inversion procedure but for the purpose of validating the methodology. The efficiency of the employed approach is proved and quantified in terms of the relative and root mean squared errors between the inversion outcomes and the well sonic log.

Method

According to Tarantola (2005), the marginal posterior probability density function related to the model $\sigma_m(\mathbf{m})$, which gives the probabilities of all possible models, is the product of a constant and an exponential function:

$$\sigma_m(\mathbf{m}) = \text{const} \cdot \exp\left(-J(\mathbf{m})\right) \tag{1}$$

where $J(\mathbf{m})$ is the objective function, so that the $\mathbf{m}$ that minimizes $J(\mathbf{m})$ is the solution of the inverse problem. Besides:

$$\begin{aligned} 2J(\mathbf{m}) =& (\mathbf{Gm}-\mathbf{d_{obs}})^T \mathbf{C_D^{-1}} (\mathbf{Gm}-\mathbf{d_{obs}}) + \\ & (\mathbf{m}-\mathbf{m_{prior}})^T \mathbf{C_M^{-1}} (\mathbf{m}-\mathbf{m_{prior}}), \end{aligned} \tag{2}$$

where

- $\mathbf{G}$ is a forward mapping linear operator that maps the model $\mathbf{m}$ into the data $\mathbf{d}$ parameters;
- $\mathbf{d_{obs}}$ is the observed data;
- $\mathbf{C_D}$ is the covariance matrix related to the data and the theory. ;
- $\mathbf{m_{prior}}$ is the prior or reference model;
- $\mathbf{C_M}$ is the covariance matrix related to the model.

It is important to state that this approximation is valid when $\mathbf{d_{obs}}$ and $\mathbf{m_{prior}}$ are independent and all functions are built as Gaussian distributions. Besides, a covariance matrix is always symmetric and it measures how much each of the posterior models vary from the mean with respect to each other. The data and theory covariance matrix was considered as a diagonal matrix and its elements are equal to the variation of the caliper log data, considering the drill bit diameter as its mean, as the caliper log offers a quality control of all log data (Rider, 1996).

Considering the weight matrix $\mathbf{W_M}$ such that $\mathbf{C_M^{-1}} = \mathbf{W_M}^T\mathbf{W_M}$, Equation (2) becomes

$$2J(\mathbf{m}) = [\mathbf{W_D}(\mathbf{Gm}-\mathbf{d_{obs}})]^2 + [\mathbf{W_M}(\mathbf{m}-\mathbf{m_{prior}})]^2. \quad (3)$$

The model weight matrix can be represented as a regularized model, such as $\mathbf{W_M} = \mathbf{R}/\sigma_M$. Then σ_M plays the role of the regularization parameters and $\mathbf{R}$ was chosen as the one dimension Laplacian operator. Therefore, the solution of the inverse problem is given by

$$\widetilde{\mathbf{m}} = \left(\mathbf{G^T W_D^T W_D G} + \frac{1}{\sigma_M^2}\mathbf{R^T R}\right)^{-1} (\mathbf{G^T W_D^T W_D d_{obs}} + \frac{1}{\sigma_M^2}\mathbf{R^T R m_{prior}}). \quad (4)$$

Additionally, the posterior model covariance matrix is equal to

$$\widetilde{\mathbf{C}}_\mathbf{M} = \left[\mathbf{G^T W_D^T W_D G} + \frac{1}{\sigma_M^2}\mathbf{R^T R}\right]^{-1}. \quad (5)$$

The posterior model correlation matrix $\widetilde{\mathbf{Cr}}_\mathbf{M}$ is determined from $\widetilde{\mathbf{C}}_\mathbf{M}$ by the relationship:

$$\widetilde{Cr}_M(i,j) = \frac{\widetilde{C}_M(i,j)}{\sqrt{\widetilde{C}_M(i,i)}\sqrt{\widetilde{C}_M(j,j)}} \quad (6)$$

A correlation matrix evaluates the degree of association between two variables, and its elements are always between -1 and 1. Zero correlation means that there is no linear relationship between two models of different depths. A perfect linear relationship happens when the correlation coefficient is equal to 1.

Equation (3) can be written as:

$$2J(\mathbf{m}) = \|\mathbf{r_D}\|^2 + \|\mathbf{r_M}\|^2, \quad (7)$$

where $\|\mathbf{r_D}\|^2$ is the data residual and $\|\mathbf{r_M}\|^2$ correspond to the model residual. They are, respectively, given by

$$\|\mathbf{r_D}\|^2 = [\mathbf{W_D}(\mathbf{Gm}-\mathbf{d_{obs}})]^2, \quad (8)$$

and

$$\|\mathbf{r_M}\|^2 = [\mathbf{W_M}(\mathbf{m}-\mathbf{m_{prior}})]^2. \quad (9)$$

Thus the optimal σ_M, that minimizes $J(\mathbf{m})$ is found when there is a balance between $\|\mathbf{r_D}\|^2$ and $\|\mathbf{r_M}\|^2$. The L-curve can be used to determine σ_M^{opt}.

The Wyllie's equation (Wyllie et al., 1956) is represented by:

$$\frac{1}{v_p} = \frac{\phi}{v_p^f} + \frac{1-\phi}{v_p^m}. \quad (10)$$

where ϕ is the rock porosity, v_p is the P-wave velocity and v_p^f and v_p^m are, respectively, the P-wave velocity in the fluid that fills the pore space and the P-wave velocity in the rock matrix. Considering that the slowness s_p is the inverse of v_p, it can be written that:

$$\phi + \frac{s_p^m}{s_p^f - s_p^m} = \frac{1}{s_p^f - s_p^m}\cdot s_p. \quad (11)$$

In this way, the model $\mathbf{m}$ is equal to the slowness of the P-wave. Additionally, by using the neutron porosity log and the fluid and rock matrix slowness, $\mathbf{d_{obs}}$ is built as follows:

$$d_{obs,i} = \phi_i + \frac{s_{p,i}^m}{s_{p,i}^f - s_{p,i}^m}. \quad (12)$$

Besides, according to Equation (11), $\mathbf{G}$ is a diagonal matrix so that, for each depth of investigation i, the element $G(i,i)$ is calculated as:

$$G(i,i) = \frac{1}{s_{p,i}^f - s_{p,i}^m}. \quad (13)$$

The prior model is constructed using Gardner's equation (Gardner et al., 1974), which estimates sonic logs, from the density log:

$$v_p = 360\rho^4. \quad (14)$$

For a depth of investigation i:

$$m_{prior,i} = s_{p,i} = \frac{1}{v_{p,i}} = \frac{1}{360\rho_i^4}. \quad (15)$$

Results

The data used in this work are from two wells (1-BAS-68 and 1-BAS-80) located at the Jequitinhonha Basin and they were provided by PETROBRAS. This basin is situated in the state of Bahia. In this basin, there are thirty one exploratory drilled wells, all located on land or in the proximal zone.

Wells 1-BAS-68 and 1-BAS-80 have 3586 and 2586 depths of investigations, respectively. For each of these depths, the rock matrix slowness was defined by using lithological data, that are related to ditch samples.

Well 1-BAS-68

According to the lithology data set of well 1-BAS-68, only calcarenite and calcilutite are present in this well, then limestone was considered as the rock matrix for both lithologies. Besides water with 15% of NaCl was chosen as the fluid present in the rock pores. Therefore, for this well, $v_p^m = 6.40$ km/s and $v_p^f = 1.50$ km/s (Carmichael, 1988).

The procedure outcome for three different values of σ_M (0.0001, 1 and 1000) are presented along with the sonic well log v_p^{well} and the caliper log on Figure 1. From Figure 1a it is possible to see that the predicted velocity log is very smooth as the regularization parameter is very small. A small σ_M makes the result not dependent on the forward operator and standard deviation of data and theory, that is perceptible when Equation (4) is analyzed. In Figures 1b and 1c, it is visible that the estimated sonic log and the sonic well log have the same trend and they are quite similar. The caliper log indicates a serious caving about 3950 m. As it compromises the quality of the data used in the procedure (the neutron porosity and density logs), the inversion efficiency decreases and it explains the larger differences between v_p^{well} and $v_p^{\sigma 2}$ and $v_p^{\sigma 3}$ around that depth. The relative errors between the well and the estimated sonic logs for σ_M^1, σ_M^2 and σ_M^3 were, respectively, equal to 9.76%, 3.61% and 3.65%. Additionally, the root mean squared errors were, respectively, equal to 0.539 km/s, 0.199 km/s and 0.201 km/s.

The posterior model covariance matrices for σ_M^1, σ_M^2 and σ_M^3 are shown in Figures 2. The larger covariance values are on the matrix main diagonal. Besides, for these regularization parameters, the values outside the main diagonal are close to zero. Therefore, the models of different depths are almost independent of each other. Besides, the smaller σ_M is, the bigger the components of $\widetilde{\mathbf{C}}_{\mathbf{M}}$ are. For Figures 2b and 2c, the values outside the main diagonal are quite small, which creates a misinterpretation that they are diagonal matrices.

Figure 3 shows the posterior model correlation matrices for σ_M^1, σ_M^2 and σ_M^3. For the three presented matrices, the correlation is equal to 1 only where there is a autocorrelation, i.e., the correlation of an element to itself, what represents the components of the matrices main diagonal. A small σ_M increases the correlation values.

The L-curve was calculated and 10000 regularization parameters were tested between 0.01 and 10^6 and the optimal σ_M was equal to 40072. The results generated by this parameter will not be shown as the predicted sonic log using σ_M^{opt} and also the posterior model covariance and correlation matrices were not so different from the outcomes computed using σ_M^2 and σ_M^3.

<u>Well 1-BAS-80</u>
The lithology data of well 1-BAS-80 shows that this well log contains calcilutite, marl and shale. Therefore, for each depth of investigation, if the presented lithology is calcilutite, shale or marl, the adopted values for v_p^m, will be, respectively, equal to 6.4 km/s, $v_p^m = 4.8$ km/s and $v_p^m = 6.0$ km/s (Carmichael, 1988). As in 1-BAS-68, it was considered that the fluid in the rock pores is water with 15% of NaCl.

Figure 4 shows the inversion result for three different values of σ_M (0.0001, 1 and 1000) along with the sonic well log v_p^{well} and the caliper log. Once again, the outcome for a very small regularization is very smooth, as it is visible in Figure 2a. Figures 2b and 2c show that the predicted sonic log and the sonic well log are quite similar, even though the caliper log present very inconstant values, which influence the data and theory covariance matrix. The velocity peaks present on the sonic well log is related to the lithology calcilutite and they are also found in the estimated log. The relative errors between the well and the estimated sonic logs for σ_M^1, σ_M^2 and σ_M^3 were, respectively, equal to 12.71%, 4.43% and 4.80%. Additionally, the root mean squared errors were, respectively, equal to 0.287 km/s, 0.102 km/s and 0.108 km/s.

The posterior model covariance matrices for σ_M^1, σ_M^2 and σ_M^3 are shown in Figures 5. The models of different depths are almost independent of each other as in well 1-BAS-68 and the values outside the main diagonal are close to zero. Figure 6 shows the posterior model correlation matrices for σ_M^1, σ_M^2 and σ_M^3.The correlation is equal to 1 only for the components of the matrices main diagonal and the others are almost close to zero .

The L-curve was also calculated for this well, but the results generated by this parameter will not be presented in this work for the same reason why they were not shown for well 1-BAS-68.

Conclusions

From statistical and theoretical relationships among well logs as neutron porosity, density and caliper, a regularization technique based on the Bayesian philosophy that allows the prediction of the sonic log was proposed and tested on a real data set. The procedure is simple and suitable for application. Three different regularization parameters were used in order to estimate the P-wave velocity log. The choice of an appropriate parameter plays a role in the inversion process quality. The posterior model covariance and correlation matrices were also computed and discussed, showing that a model for any depth is almost independent from the other models. The comparison between the outcomes and the sonic well log shows that the method is operative and efficient.

Acknowledgements

The authors would like to thank CNPq and PETROBRAS for the project National Institute of Science and Technology in Petroleum Geophysics (INCT-GP) and FINEP for sponsoring the CTPETRO Network in Exploration Geophysics (Rede-01). O. C. A. de Aragão would like to thank CAPES for a M.Sc. scholarship. A. Bassrei would like to thank CNPq for a research fellowship (308.690/2013-3).

References

Bassiouni, Z. [1994] Theory, measurement, and interpretation of well logs. Society of Petroleum Engineers Textbook Series, Richardson.

Carmichael, R. S. [1988] Practical Handbook of Physical Properties of Rocks e Minerals. CRC Press Inc, Florida.

Ellis, D. V. and Singer, J. M. [2007] Well logging for earth scientists. Springer, Dordrecht.

Gardner, G. H. F., Gardner, L. W. and Gregory, A. R. [1974] Formation velocity and density - the diagnostic basics for stratigraphic traps. *Geophysics*, **39**, 770-780.

Mosegaard, K. and Tarantola, A. [2002] Probabilistic approach to inverse problems. In Lee, W. H. K., Jennings, P., Kisslinger, C. and Kanamori, H. (Eds.) *International Handbook of Earthquake and Engineering Seismology*. Academic Press, 237-265.

Rider, M. H. [1996] The geological interpretation of well logs. Gulf Publishing Company, Sutherland.

Serra, O. and Abbott, H. T. [1982] The contribution of logging data to sedimentology and stratigraphy. *Society of Petroleum Engineers Journal*, **22**, 117-131.

Tarantola, A. [2005] Inverse problem theory and methods for model parameter estimation. Society for Industrial and Applied Mathematics, Philadelphia.

Wyllie, M. R. J., Gregory, A. R. and Gardner, L. W. [1956] Elastic wave velocity in heterogeneous and porous media. *Geophysics*, **21**, 4170.

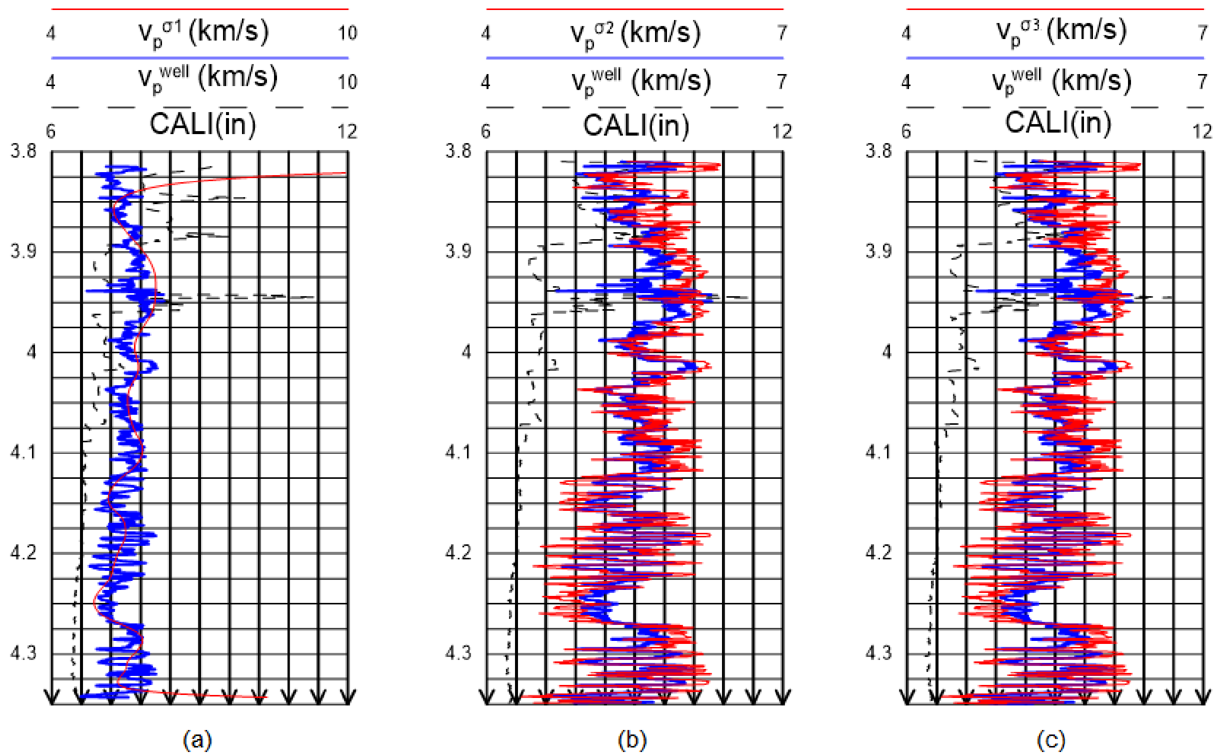

Figure 1: The caliper and the sonic well logs along with the predicted sonic logs related to well 1-BAS-68 for (a) $\sigma_M^1 = 0.0001$, (b) $\sigma_M^2 = 1$ and (c) $\sigma_M^3 = 10000$.

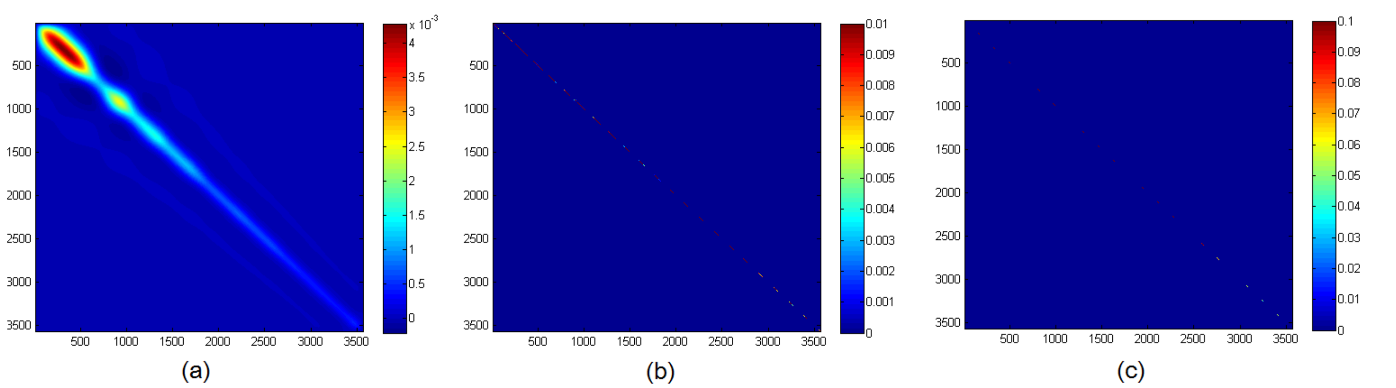

Figure 2: Posterior model covariance matrices related to well 1-BAS-68 for (a) $\sigma_M^1 = 0.0001$, (b) $\sigma_M^2 = 1$ and (c) $\sigma_M^3 = 10000$.

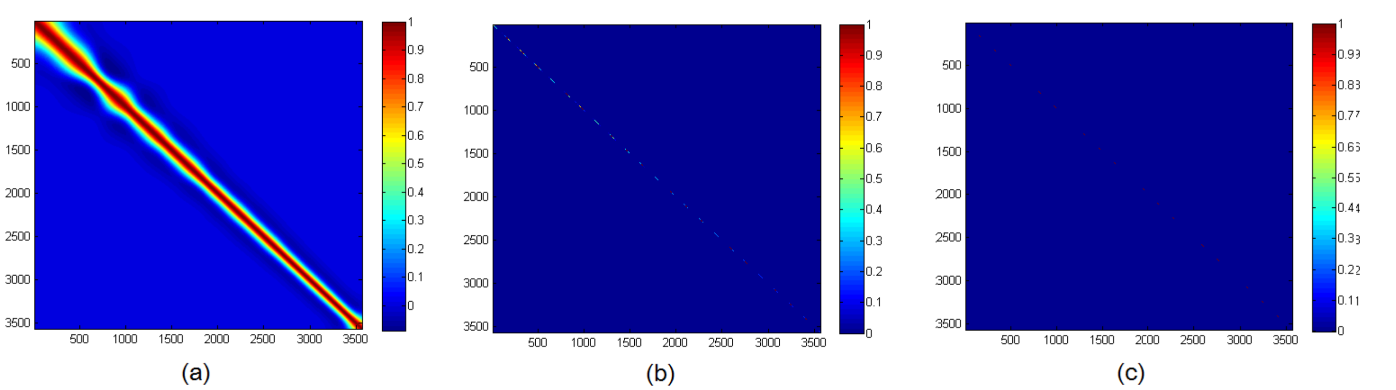

Figure 3: Posterior model correlation matrices related to well 1-BAS-68 for (a) $\sigma_M^1 = 0.0001$, (b) $\sigma_M^2 = 1$ and (c) $\sigma_M^3 = 10000$.

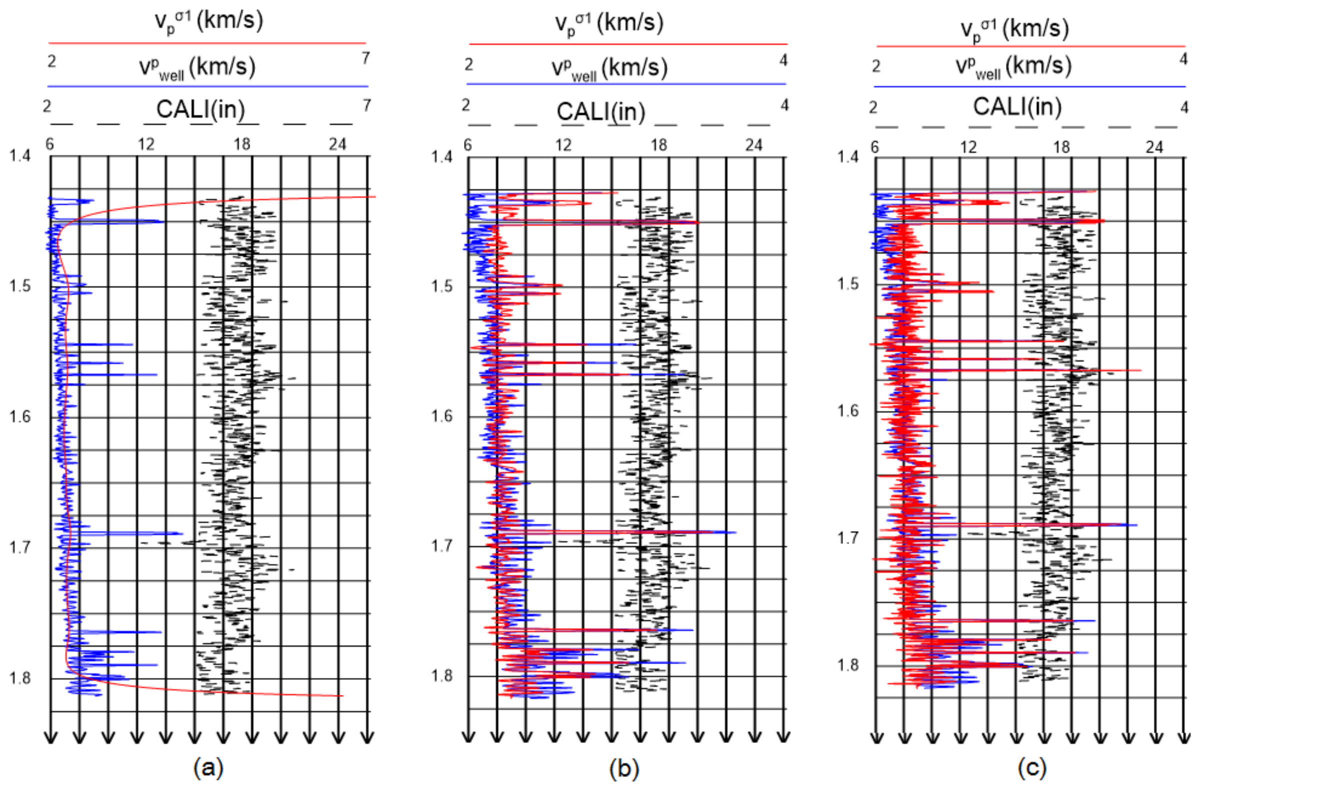

Figure 4: The caliper and the sonic well logs along with the predicted sonic logs related to well 1-BAS-80 for (a) $\sigma_M^1 = 0.0001$, (b) $\sigma_M^2 = 1$ and (c) $\sigma_M^3 = 10000$.

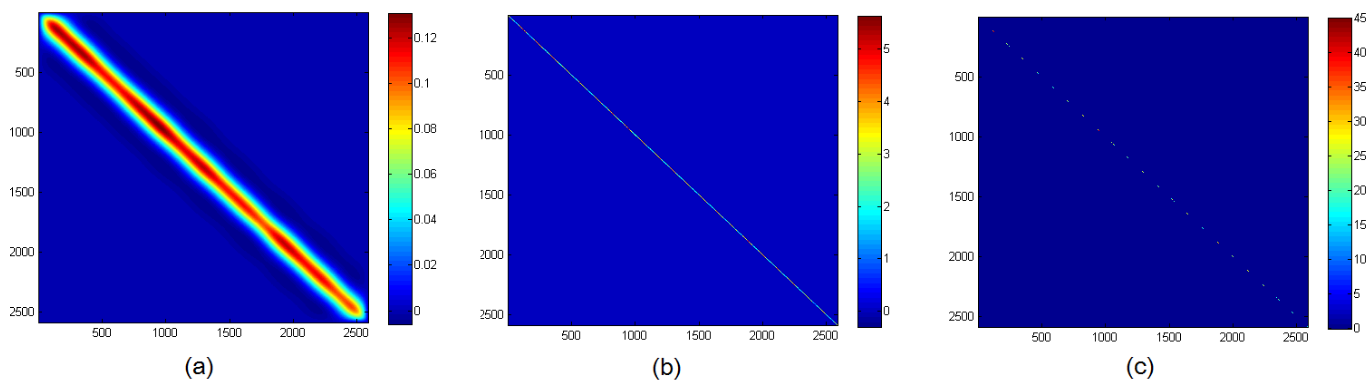

Figure 5: Posterior model covariance matrices related to well 1-BAS-80 for (a) $\sigma_M^1 = 0.0001$, (b) $\sigma_M^2 = 1$ and (c) $\sigma_M^3 = 10000$.

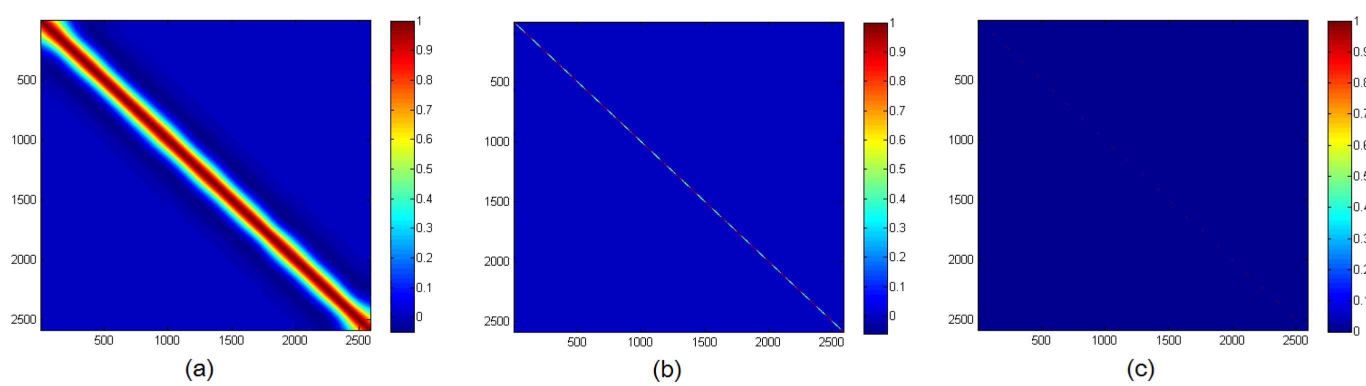

Figure 6: Posterior model correlation matrices related to well 1-BAS-80 for (a) $\sigma_M^1 = 0.0001$, (b) $\sigma_M^2 = 1$ and (c) $\sigma_M^3 = 10000$.