Using the Benford's Law to analyze the sonic reflectivity

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Abstract

The caliper log can be sensitivity to borehole enlargement due drilling effect on different rock formations. The density of the lithology is affected by mud infiltration and the consequence is unrealistic values on the well-to-seismic-tie. In this work we use, in the first time, the statistical Benford's Law (BL) to analyze and check on the existence of errors or inconsistencies on the reflectivity log. The Benford's analysis is applied on the reflectivity series of two wells of the Viking Graben field. Effect of sonic and density logs despiking as well as the size of sample window to find the BL distribution were analyzed. The results on the real dataset analyzed shows that it is possible to find a optimum coefficient for the despiking process on the well logs according to the Benford's Law and that the segments of the reflectivity where the caliper log is stable the conformity with the Benford's Law is higher.

Introduction

The earth is composed of rock's layers with different lithology and physical properties. By the seismic point of view, these layers are represented by different densities and velocities in which seismic waves propagates through them. The product of density by velocity is the seismic impedance and it is the impedance contrast between adjacent rock layers that causes the reflections that are recorded along a surface profile (Yilmaz, 2000).

There are an extensive usage of Frank Benford (1938) on natural science. From extensive survey M. Sambridge et al. (210) suggest that BL without artificial constraints or human interference may be used in sciences for data sets verification with sufficient dynamic. Regarding topics related to Geosciences, Mark J. Nigrini and Steven J. Miller (2007) applied the BL in hydrology in order of authenticate and validate check on databases dealing with water bodies. A. Geyer and J. Martí (2012) perceived that anomalies in volcanological data sets may be detected when comparing the data with Benford's law expected frequencies.

The objective of this paper is to test the conformity of the reflectivity of two well logs from the Viking Graben data set and test the potencial utility of using Benford's Law on sonic reflectivity. We use the Frank Benford (1938)'s Law to verify the inconsistence on reflectivity logs due the mud infiltration (on the formation) as well as wellbore landslide effect due effect of drilling. We also perform a analysis of the despiking process on well logs and how it is related with the Benford's Law.

Motivation

Macedo et al. (2017) performed well-tie procedures in two logs (A and B) with seismic section from the Viking Graben field. Among other things, they verified that low correlation between synthetic and real seismic trace was related to borehole enlargement or shrink depicted in the caliper log. Figure 1a show that the segment with the lower correlation coefficient $c_1 = 0.409$ corresponds to the zone where the caliper log is unstable, which directly affects the sonic and density measurements and, consequently, directly affects the quality of the well to seismic tie. Based on that, one of our objective is to test the conformity of the Benford's Law on the reflectivity series on those zones where the caliper log is unstable.

Keys (1998) mentioned on their article about the Viking Graben data set problems during the acquisition of data along the well A, which might be related with this anomalous zone on the caliper log. This first segment is related with the lithology of the formation above the Cretaceous unconformity at 1.97 s. The Paleocene and Cretaceous rocks above the unconformity are deep water clastic sediments and as they are deposited in a slope in the basin, the formation contain turbidites. On the second $c_2 = 0.805$ and third segment $c_3 = 0.839$, it is possible to note that when the caliper log gets more stable, the correlation increases although the shale content also increases due to the Jurassic rocks associated with deepwater shales.

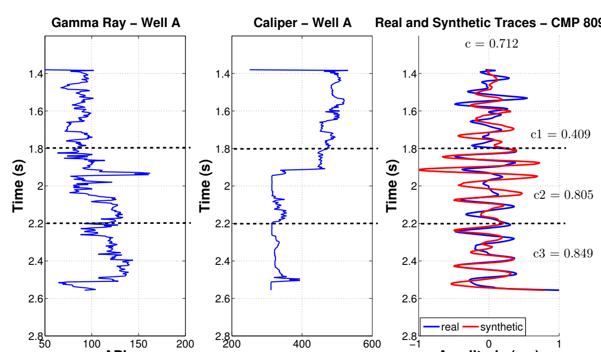


Figure 1: Well tie for the well A with the migrated seismic section and the deterministic estimated wavelet and the gamma ray and caliper logs.

In Figure 2, it is shown the relations between the gamma ray log, the caliper log and the well to seismic tie for the well B. As the Paleocene and Cretaceous rocks are deposited in a slope in the basin, the Cretaceous unconformity that on well A is at approximately 1.97 s, on well B it is approximately at 2.4 s, so the beginning of the synthetic trace for the well B is just below the unconformity. Therefore, it covers the Jurassic rocks. For this well, although the caliper log is relatively stable along the hole trace, the first segment of the trace shows a lower correlation coefficient. This might be due to the faults associated with the Cretaceous unconformity that constitute the oil and gas traps and the deep water shales associated with it, as mentioned by Madiba and McMechan (2003) and as can be seen by the gamma ray log. As the shale content tends to decrease and the caliper log gets stable, the quality of the well tie increases and the excellent result of a correlation of $c_2 = 0.988$ for the second part of the trace was obtained.

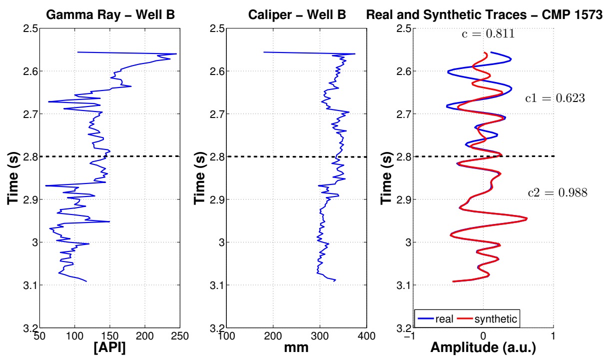


Figure 2: Well tie for the well B with the migrated seismic section and the deterministic estimated wavelet and the gamma ray and caliper logs.

Theoretical background

In this section, we present the theoretical framework used to establish the analysis of reflectivity logging by the Frank Benford (1938).

Sonic reflectivity

All procedures to create a synthetic trace and/or to estimate the wavelet to proceed in the seismic-well tie on the work of (Macedo et al., 2017) is based on the classic convolutional model of the seismogram and its assumptions. The recorded seismogram $s(t)$ can be modeled as the convolution of the Earth's reflectivity $r(t)$ with the seismic wavelet $w(t)$ plus recorded noise $n(t)$:

$$s(t) = w(t) * r(t) + n(t). \tag{1}$$

For the equation 1 be valid, it is considered that the earth is made up of horizontally deposited layers of constant velocity and that an impulsive seismic source generates a compressional pressure wave (P-wave) that interacts on layer boundaries with normal incidence. Therefore, no shear waves (S-waves) are generated. The reflection coefficient R_c associated with the boundary between layers is defined as:

$$R_c(i) = \frac{\rho_{i+1}v_{i+1} - \rho_i v_i}{\rho_{i+1}v_{i+1} + \rho_i v_i}, \tag{2}$$

where ρv is the acoustic impedance. As the pressure wave travels through the earth, its amplitude decays because of wavefront divergence and frequency attenuation due absorption effects of rocks. This change of the wavelet with the time and depth is not incorporated on the convolutional model of the recorded seismogram, in other words, the convolutional model assumes that the wavelet is stationary. Other assumption, which is important in this work relies on fact that earth reflectivity is aleatory. This is an important premise for the usage of Benford's Law.

Benford Law

The Benford Law is also known as first digit law and is a observation of the frequency distribution of leading digits in many sets of numerical data. Frank Benford (1938) analyzed the digit patterns of 20 data sets with a total of 20.229 observations. The results showed that 30.6 percent of the numbers had a 1 as the first digit, 18.5 percent of the numbers had a 2 as the first digit, with 9 being the first digit only 4.7 percent of the time. The first digit of a number is the leftmost non-zero digit, the minus sign or decimal points are ignored. For example, suppose your grocery bill gave \$1,500.00 and the chance of your win at the lottery is 0.00000145%. In both cases, the first non-zero digits are D = 1. Benford noticed the logarithmic pattern in the distribution of digits and derived the formulas for the expected frequencies of the digits:

$$P_D = log_{10}(1 + \frac{1}{D}) \tag{3}$$

where P_D is the probability of occurrence of first digit (non-zero) and D=1, 2, 3, . . . ,9.

To measure the fit between the BL prediction with the values of the real distribution, we use the mean absolute deviation (MAD). A alternative to the MAD is the use of the goodness fit equation of M. Sambridge et al. (210)

$$\phi = \left[1 - \left(\sum_{D=1}^{9} \frac{(n_D - nP_D)^2}{nP_D}\right)^{\frac{1}{2}}\right] \tag{4}$$

where n_D, P_D and n are the number of observed data with first digit D and the proportion of data expected with first digit D from (1) and n is the total of sample in the data, respectively. Nigrini (2012) established the boundary values for the mean absolute deviation (MAD) that verify whether the data set is in conformity with the Benford Law or not:

•MAD < 0.006 : close conformity
•MAD < 0.012 : acceptable conformity
•MAD < 0.015 : marginal conformity
•MAD > 0.015 : non conformity

We used those boundaries to verify the conformity of the reflectivity of our data set.

Real Data Example

We perform the following tests on our real data example:
1) As the Benford's Law deals with aleatory sets of numbers, we need to verify if the reflectivity, that according to the classical assumptions of the convolutional model is a aleatory process, is in conformity with the Benford Law.

2) In general, it is necessary large sets of data to apply the Benford's Law. However, this number varies according to the nature of the data. The second test is to verify the influence of the number of samples of the reflectivity on the Benford's Law.

3) From the information of previous papers (Macedo et al. (2017)) it is possible that the anomalies during the drilling produced anomalies on the caliper log and it affected the values of the density and sonic logs and, as a consequence, the reflectivity. The third test was to verify if in the portions where the caliper log is unstable the conformity with the Benford Law decreases

4) As the well logs come with noisy spikes and it is a common procedure to perform the despiking. The procedure to perform the despike is to set a limit value for the spikes on the sonic and density log. If the measured point does not exceeds that limit, the value of the real log is used. If the measured point exceeds that limit, the value of the smoothed log is used. We verified the influence of the despiking process on the Benford's Law and if it is possible to find a optimum limit coefficient for the despiking process.

The well log information is from two wells designed Well A and Well B located in Viking Graben field on a seismic line. We used the segment of the logs where there were no zero-values and as on well B the information about density log was insufficient, the log was completed using the Gardner's relation (Gardner, 1974) that relates compressional velocity with the bulk density of the lithology where the wave travels. The results of our first test shows that the reflectivity for both wells are in close conformity with the Benford's Law, as depicted on figure 4.

The number of samples of reflectivity that makes the Benford Law consistent is different from the both well logs. While on well A from around 5500 samples the reflectivity is in close conformity with BL, for the well B the close conformity appears with around 2000 samples, as shown on figure 5.

When comparing the caliper log from both wells on figure 3, it is noticed that the caliper on well B is stable while the caliper on well A is unstable until the depth of 1949m. Therefore, on well A, we verify the conformity with the BL of this portion where the caliper is unstable and compare with the portion where the caliper is unstable. The mean absolute deviation (MAD) is higher (MAD=0.0040) where the caliper is unstable, comparing to where it is stable (MAD=0.0031).

Well log data are essential input for petrophysical analysis, such as well to seismic tie. The first step in any project that has well log data is well log audit and edit. In all cases the log data will require some editing, normalization, and interpretation before they can be used in a reservoir study and the despiking process to remove noisy spikes is one of the most common procedures. To perform the despike is this work, we set a limit value for the spikes on the sonic and density log. If the measured point does not exceeds that limit, the value of the real log is used. If the measured point exceeds that limit, the value of the smoothed log is used. We already shown that the reflectivity (in its pure

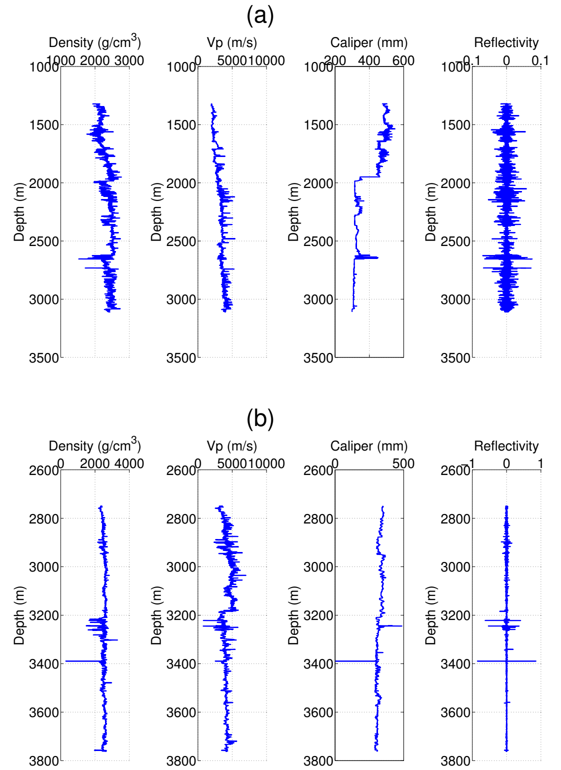


Figure 3: Density log, sonic log, caliper log and the reflectivity without the despiking process for a) the well A and b) well B.

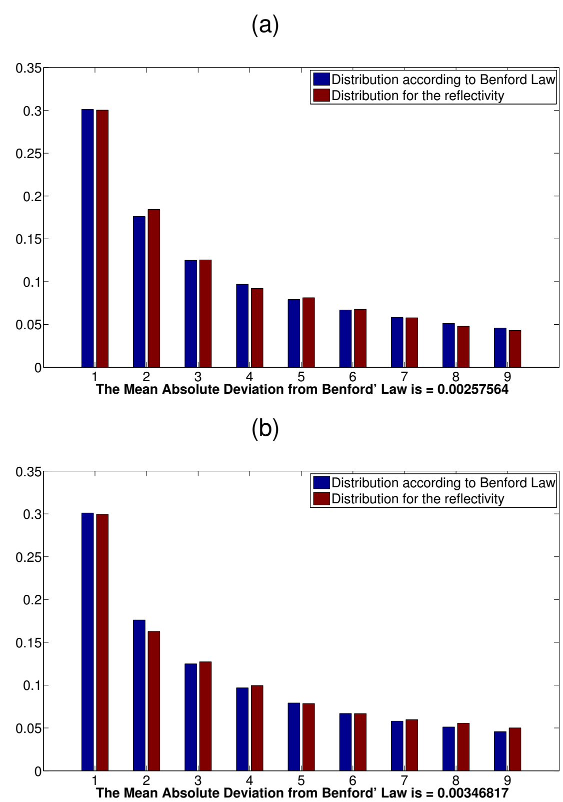
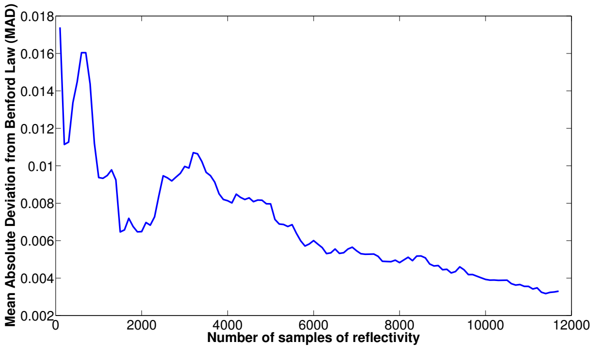


Figure 4: Benford Law distribution. a) Well 4. MAD = 0.0025 , close conformity. b) Well 5. MAD = 0.034, close conformity.

(a)



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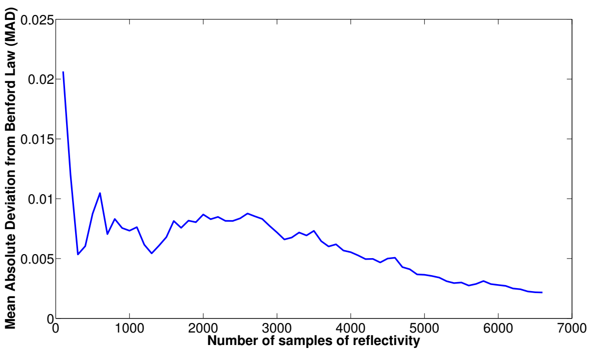
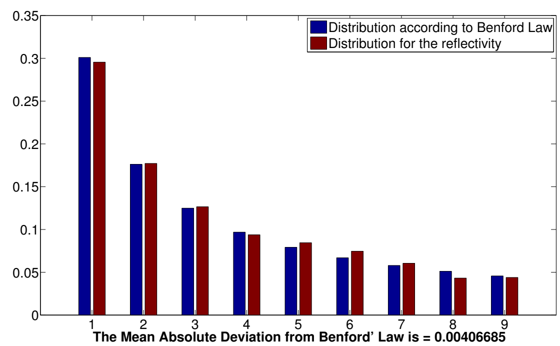


Figure 5: Number of samples X The mean absolute deviation from Benford Law (MAD) for a) Well A and b) Well B.

(a)



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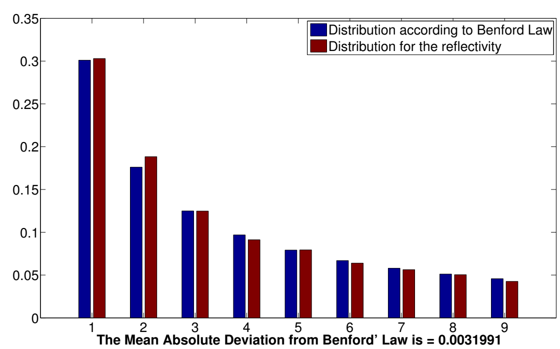


Figure 6: Benford Law distribution for the reflectivity of well A in different portions. a) portion where the caliper log is unstable. MAD = 0.0040 b) portion where the caliper log is stable. MAD = 0.003.

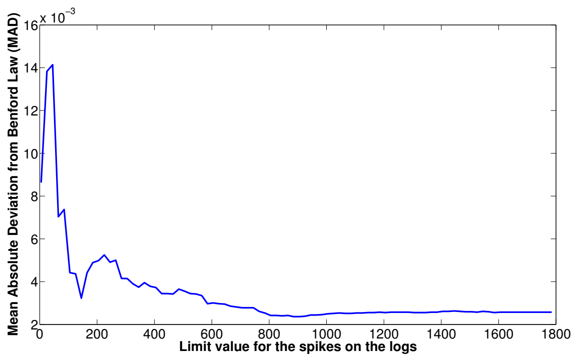
form, without despiking) obeys the Benford Law. However, depending on the limit value adopted for the spikes to perform the despiking process, we verify that the reflectivity looses conformity, as shown on figure 7. Moreover, we verify that on the curve limit of the spikes X MAD, for both wells, there is a local minimum value that might correspond to a optimum limit for the spikes, once those limit values produce logs that are are geologically consistent.

Figure 8 and 9 shows the density and sonic logs with the despiking made with the optimum limit value obtained from the Benford Law analysis.

Conclusions

In this paper we show that the Benford's Law has a potential utility on the analysis of the seismic reflectivity. The reflectivity from both real logs data sets were in close conformity with BL. As the usage of the Benford's Law require a large number of data, it was possible to verify that for the case of the reflectivity it requires an amount of samples of the order of thousands to have a consistent result. We also show that the conformity with BL is higher on the segments of the reflectivity where the caliper log is stable. As for the editing of logs, the Benford's Law show that the despiking process can deviate the conformity of the reflectivity and through a analysis of the limit of the spikes with the mean absolute deviation, it might me possible to find a optimum limit value for the spikes, when proceeding with the despiking. For future works, the authors suggest a analysis of the optimum limit of the spikes for the density log and the sonic logs individually and the effect of the

(a)



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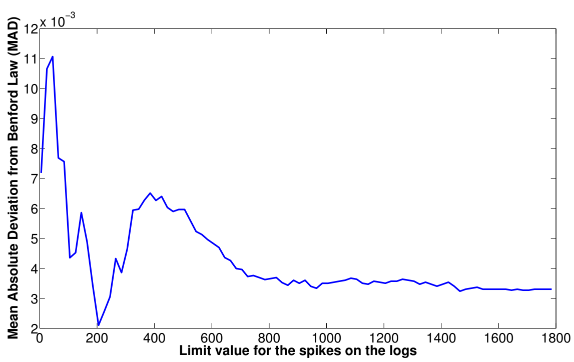
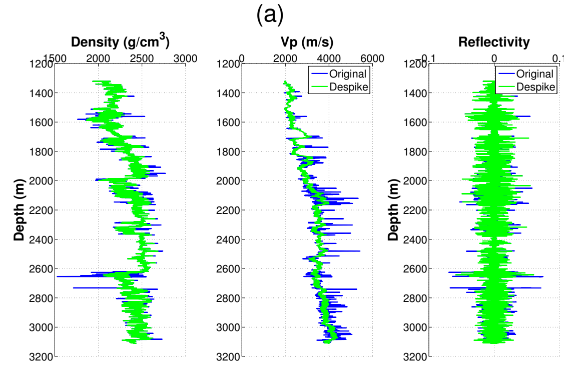


Figure 7: Limit value for the spikes on the logs X Mean absolute deviation from Benford Law. a) For the well A, the limit value for the spike that produce the best conformity with Benford Law and is geologically consistent is 145. b) For the well B, this limit value is 205.

(a)



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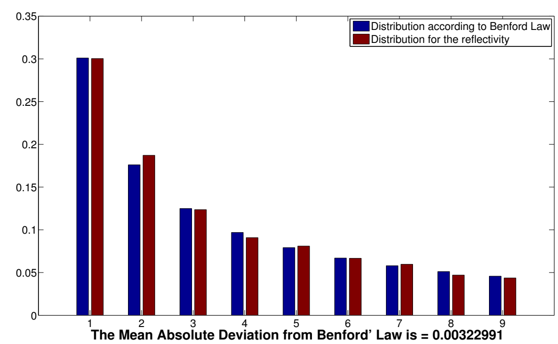
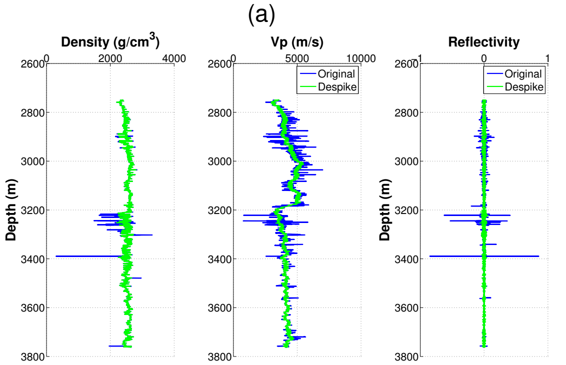


Figure 8: a) Density and sonic logs with the despiking made with the value obtain from the Benford Law analysis for the well A. Limit value for the spikes: 145. b) Benford's distribution of the reflectivity produced with the despiking logs. MAD=0.0032.

(a)



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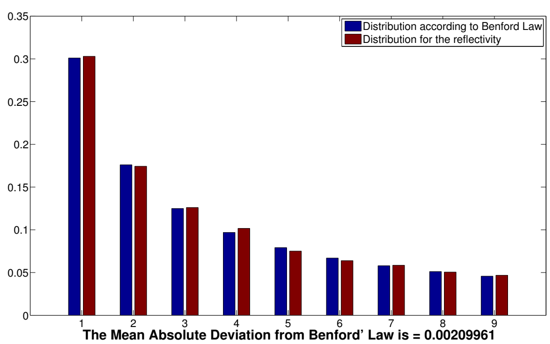

Figure 9: a) Density and sonic logs with the despiking made with the value obtain from the Benford Law analysis. Limit value for the spikes: 205. b) Benford's distribution of the reflectivity produced with the despiking logs. MAD=0.0020.

reflectivity corrected with the optimum limit values on the well to seismic tie.

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