What to expect from Euler deconvolution estimates for isolated sources

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Abstract

Euler deconvolution is a technique based anomaly measurements, its derivatives and structural index. The objective is to estimate the base level of the data, horizontal and vertical positions of the source, usually assuming a tentative structural index. Derivatives are calculated using different techniques and different methodologies exist to indicate the correct structural index. Most of the approaches define the correct depth estimates based on structural index estimated. We modeled anomalies magnetized by vertical and inclined directions, generated by an isolated magnetic monopole and calculated its analytical derivatives in order to show what expect from Euler deconvolution estimates for single sources. We then analyzed estimates over the source influence area and at the borders. These examples clearly show an outstanding pattern for depth and base level estimates when the correct structural index is used. The estimates at these cases, and only at these cases, are constant over the anomaly position. For these cases, this pattern can clearly identify the correct structural index, related to the nature of the source.

Introduction

Euler deconvolution was proposed in the nineties (Reid et al., 1990) and became widely accepted as a semi-automatic interpretation method for magnetic and gravimetric data interpretation. The technique is based in Euler homogeneity equation and applies for single point sources related to geological structures (Reid and Thurston, 2014). It relates anomaly measurements, its derivatives and the structural index. The objective of Euler deconvolution is to obtain estimates of the base level of the data, horizontal and vertical positions of the source, assuming a tentative structural index, in most of the cases.

The structural index, that is related to the nature of the source, can only be integer (Reid et al., 2014; Reid and Thurston, 2014), otherwise the anomaly decay with distance would change in a discontinuous way as the distance source to observer changes. This was clearly shown by Ravat (1996) using a dipole and a circular thin disk (arbitrarily shaped source).

The gradients are usually calculated using numerical methods. According to Reid et al. (2014) horizontal gradients can be calculated using splines or finite differences, while vertical gradients normally require Fourier methods. However, Fourier methods are also used to calculate horizontal gradients (Blakely, 1996) and these methods' stability depend on the signal-to-noise level. Pašteka et al. (2009) regularized the derivatives in Fourier domain to keep them stable in cases with low signal-to-noise ratio. Other methodology to calculate gradients in Euler deconvolution is the equivalent layer principle (Leao and Silva, 1989) done by Barbosa et al. (1999), Silva and Barbosa (2003) and Melo et al. (2013).

Thompson (1982) noticed the relationship between the correct structural index and correct depth estimates. He observed that the correct structural index produces the smallest spread of solutions. Thus, he proposed a criterion to identify the correct structural index based in the clustering of depth solutions. Reid et al. (1990) followed this line of tight depth estimates and proposed their criterion. On the other hand, Barbosa et al. (1999) noticed that the correct structural index is the one that yield the minimum correlation, in modulus, between the estimated base level and the observed total-field anomaly for 2D data, Melo et al. (2013) used this tecnique in 3D data. Silva and Barbosa (2003) gave the theoretical basis for selecting solutions assuming a null base level in their formulation. They defined the behavior of horizontal estimates at the borders of the anomaly. Over the source, they showed that horizontal estimates form plateaus of constant values and these estimates are correct regardless of the structural index. While vertical estimates form plateaus of values close to the correct one but they are only correct when the correct structural index is used.

In this paper we generated synthetic anomalies provided by a single isolated monopole (Telford et al., 1990) and do not calculate the derivatives. Instead, we used analytical derivatives of a monopole in Euler deconvolution in order to avoid any error or undesired effect that generated by numerical derivatives. Assuming three tentative structural indices, we then analyze the estimates using wrong and correct structural indices. These estimates are landmarks in the analysis of Euler estimates and can identify the correct structural index. Values of declination and inclination in synthetic tests were based on Chulliat et al. (2014).

Euler deconvolution

Considering a discrete set of N observations of total-field anomaly, Euler deconvolution (Reid et al., 1990) can be written as a linear system of equations given by:

$$\hat{x}_o \frac{\partial h_i}{\partial x} + \hat{y}_o \frac{\partial h_i}{\partial y} + \hat{z}_o \frac{\partial h_i}{\partial z} + n\hat{b} = x_i \frac{\partial h_i}{\partial x} + y_i \frac{\partial h_i}{\partial y} + z_i \frac{\partial h_i}{\partial z} + nh_i , \qquad (1)$$

where $h_i = h(x_i, y_i, z_i)$ is the i*th* observation of the total-field anomaly at the coordinates (x_i, y_i, z_i), η is the structural index related to the nature or geometry of the source. The parameters estimated are $\hat{x}_o, \hat{y}_o, \hat{z}_o$, related to the horizontal and vertical coordinates of the source, and $\hat{b}$, a base level (i.e., a background value). Therefore, solving this linear system of equations provides four parameters, assuming a tentative structural index. Euler deconvolution works thru the whole dataset using a moving data window scheme. At each window, Euler deconvolution solves the linear system for the four unknowns. In the following, will be present two tests with single anomaly with derivatives calculated analytically, one simulating a vertical incidence and the other one an inclined incidence of the field.

Vertical incidence

We generated a synthetic monopole (structural index = 2) located at x_o = 14 km, y_o = 25 km and z_o = 1 km, intensity of 1 A/m and magnetized by induction. The magnetic anomaly was calculated in a grid of 240 points in *x*-coordinate (north direction) and 200 *y*-coordinate (east direction). The grid starts in x = -10 km and y = 5 km and it is equally spaced at each 0.2 km. Observation level at this test is simulated at z = 1 km and base level is equal to zero. The anomaly was corrupted with pseudorandom Gaussian noise with standard deviation of 0.01 nT and seed 69. The anomaly is in Figure 1a and the derivatives in Figures 1b-1d.

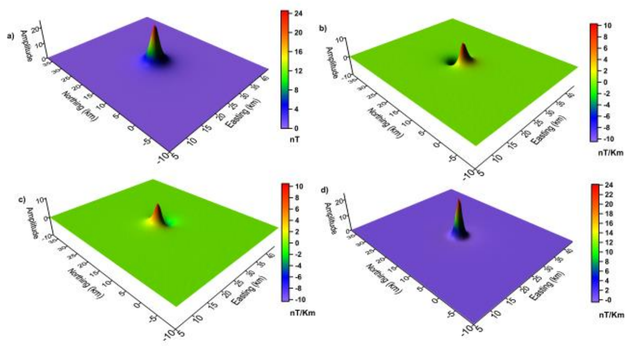

Figure 1 – *Magnetic anomaly generated by a monopole vertically magnetized with base level equal to zero and its analytical derivatives. A) Magnetic anomaly. B) X derivative. C) Y derivative. D) Z derivative.*

We apply Euler deconvolution using an 11 × 11 moving data window and assuming structural indices 1, 2 and 3. We plotted all estimates using the procedure adopted by Silva and Barbosa (2003), the estimates are plotted against the *x*- and *y*-coordinates of the center of the moving data window used in Euler deconvolution.

Figure 2 (a) – (c) displays *x*-estimates and Figure 2 (d) – (f) displays *y*-estimates from indices 1, 2 and 3, respectively. Estimates from all indices exhibit plateaus over the anomaly location. These plateaus mean that on that area all estimated parameters have the same value. However, they are completely flat only in Figures 2 b) and e), these are estimates using SI=2, the correct one for this source. Using other SI is also possible to see the plateaus in Figures 2 a), c), d) and f); however, they have a small amount of undesired values in the center. These values differ a bit from the correct one but nothing that interfere in the final evaluation of the positions. At the borders of the anomaly, estimates are weighted by the coordinates of the moving data window, as pointed by Silva and Barbosa (2003). Regardless of the structural index used, estimates outside the source (at the borders of the anomaly) are the same. An average of horizontal estimates can be done in these plateaus to achieve the correct horizontal estimated value (Li, 2003). This procedure is valid also for depth and base level estimates.

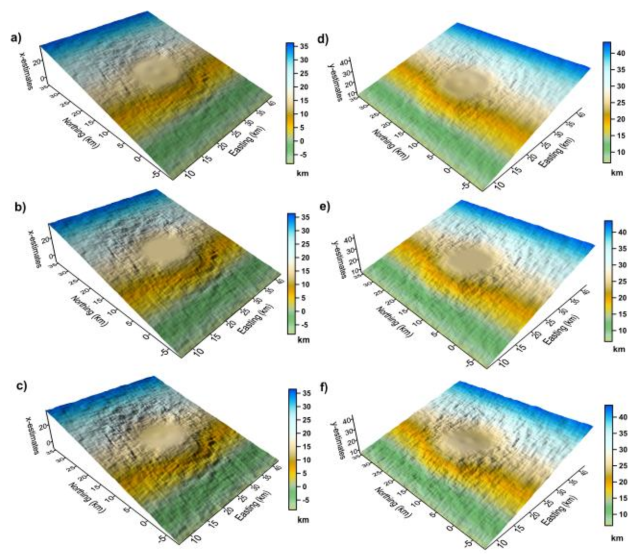

Figure 2 – *X-estimates (a) – (c) and y-estimates (d) – (f) from Euler deconvolution using an 11 x 11 moving data window assuming structural indices 1, 2 and 3, respectively for both estimates. Estimates using the correct structural index are (b) and (e).*

Figure 3 (a) – (c) displays depth estimates assuming indices 1, 2 and 3, respectively. These estimates are the ones used by Thompson (1982), Reid et al. (1990) and others to select the correct structural index based on the clustering of solutions. We can identify in Figure 3 b) that estimates over the source have a constant value and thus form a plateau. Figure 3 b) exhibits the tight clustering of solutions that those authors seek to identify when depth estimates relates to the correct structural index. Only these estimates, using the correct structural index, have correct depth estimates. Using wrong structural indices estimates do not form plateaus. In Figure 3 a) we can see a cavity over the anomaly; this means that those depth estimates are lower than the correct ones. While in Figure 3 c) we can see a prominence over the source; this means that depth estimates at this case are overestimated. Regardless of the structural index used, at the borders of the anomaly the estimates are close to the simulated acquisition level, as pointed by Silva and Barbosa (2003). Depth estimates form ramps at the limit between the borders of the anomaly and the region under the influence of the anomaly. Thus, the minimum variation of depth estimates over the influence zone of the source, bounded by those ramps, can define the correct structural index, for the case of single isolated source.

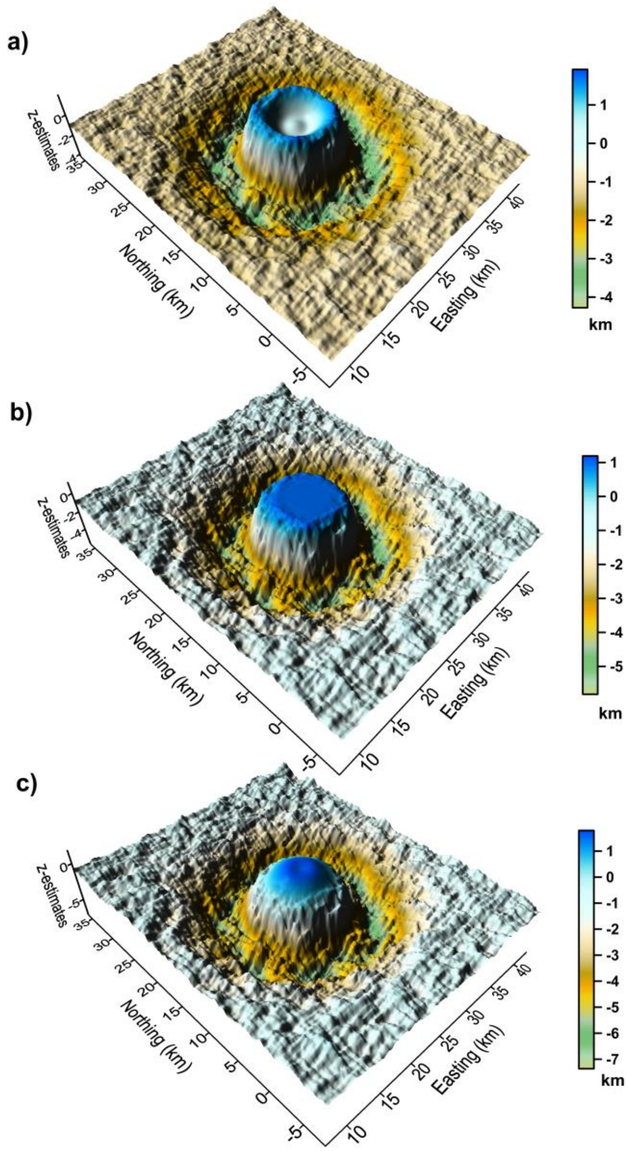

Figure 3 – *Depth estimates of Euler deconvolution using an 11 x 11 moving data window assuming structural indices (a) 1, (b) 2 and (c) 3. Depth estimates using the correct structural index are constant over the source and have the smallest variation.*

Figure 4 (a) – (c) displays base level estimates from Euler deconvolution assuming structural indices 1, 2 and 3, respectively. From these estimates is easy to find an outstanding pattern in the region over the anomaly at Figure 4 b). The estimates showed in Figure 4 b) are from to structural index 2, the correct one for this source. The values of the estimates at this plateau are the correct ones. Let us recall that the base level present in our synthetic data is zero. Moreover, when we compare the range of the estimates, at the color bar for example, is easy to see that the correct estimates have a lowest variation compared to other estimates. Base level estimates of Figures 4 a) and c) are protuberant and show an angular pattern, positively or negatively, depending if the structural index used is bigger or lower than the correct one. At the region outside the source, at the borders of the anomaly, values of base level estimates are the correct one, regardless of the structural index used, in this case it is zero. Thus, the minimum variation of base level estimates over the source identify the correct structural index, for the case of single isolated source.

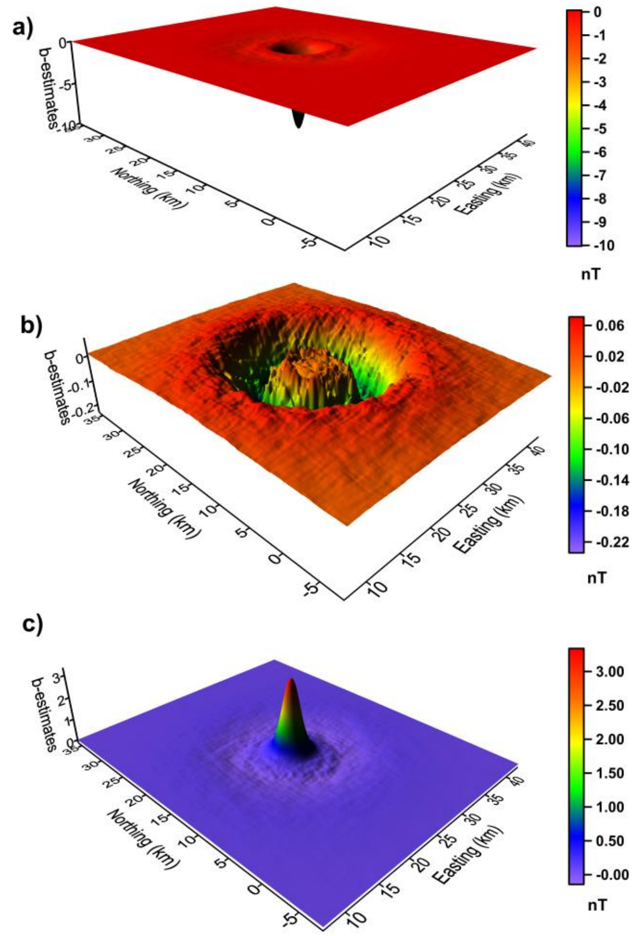

Figure 4 – *Base level estimates of Euler deconvolution using an 11 x 11 moving data window assuming structural indices (a) 1, (b) 2 and (b) 3. Base level estimates using the correct structural index are constant over the source and have the smallest variation.*

Inclined magnetization

At this test, we kept the same geometric parameters of the source and grid. Now, acquisition is at ground level and geomagnetic inclination and declination were changed. The simulated geomagnetic field has inclination of -39º and declination of -22º, values close to the field at the city of Rio de Janeiro. The magnetic anomaly is in Figure 5 (a) and the derivatives in Figures 5 (b) – (d).

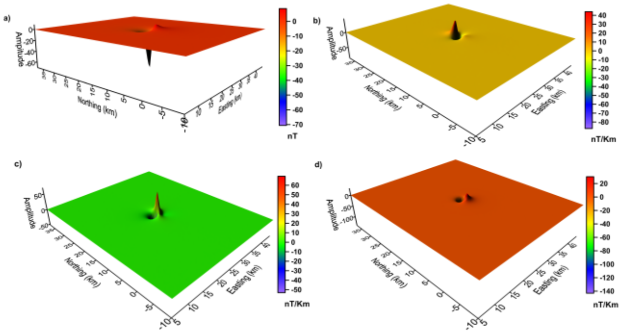

Figure 5 – *Magnetic anomaly generated by a monopole, simulated geomagnetic field has inclination of -39º and declination of -22º, with base level equal to zero and its analytical derivatives. A) Magnetic anomaly. B) X derivative. C) Y derivative. D) Z derivative.*

We apply Euler deconvolution using the same data moving window size, 11 × 11 points, and assuming structural indices 1, 2 and 3.

Figure 6 (a) – (c) displays *x*-estimates and Figure 6 (d) – (f) displays *y*-estimates from indices 1, 2 and 3, respectively. As for the vertical incidence case, estimates from all indices exhibit plateaus over the source. At these estimates, plateaus are bigger than for vertical incidence case; this is due to the inclination and declination of the simulated geomagnetic field. The same pattern of small disturbances appears with estimates using wrong structural indices. At the borders of the anomaly, the same analysis is valid and estimates are weighted by moving data window.

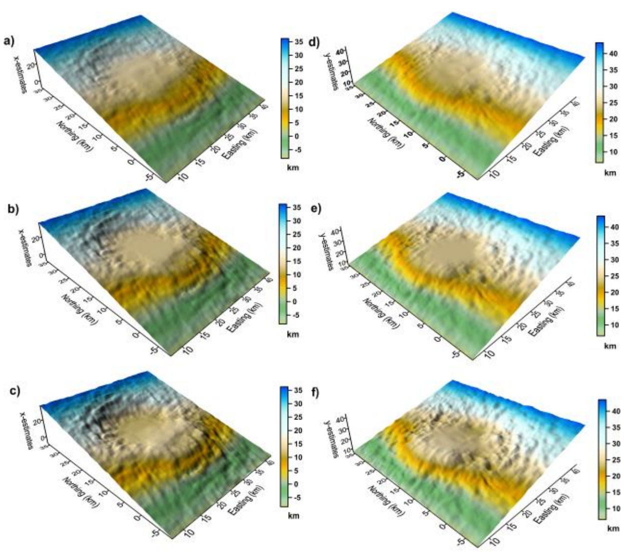

Figure 6 – *X-estimates (a) – (c) and y-estimates (d) – (f) from Euler deconvolution using an 11 x 11 moving data window assuming structural indices 1, 2 and 3, respectively for both estimates. Estimates using the correct structural index are (b) and (e).*

Figure 7 (a) – (c) displays depth estimates from Euler deconvolution assuming indices 1, 2 and 3, respectively. These estimates do not form those ramps (Figure 3), at the interface of the source influence zone and the borders of the anomaly because of the inclination and declination simulated. However, is easy to distinguish between the use of correct and wrong structural index due to the plateau associated to the use of correct structural index, Figure 7 (b). At the borders of the anomaly, the estimated values are close to zero because simulated acquisition is at ground level. Again, the minimum variation of depth estimates over that plateau is still valid to define the correct structural index.

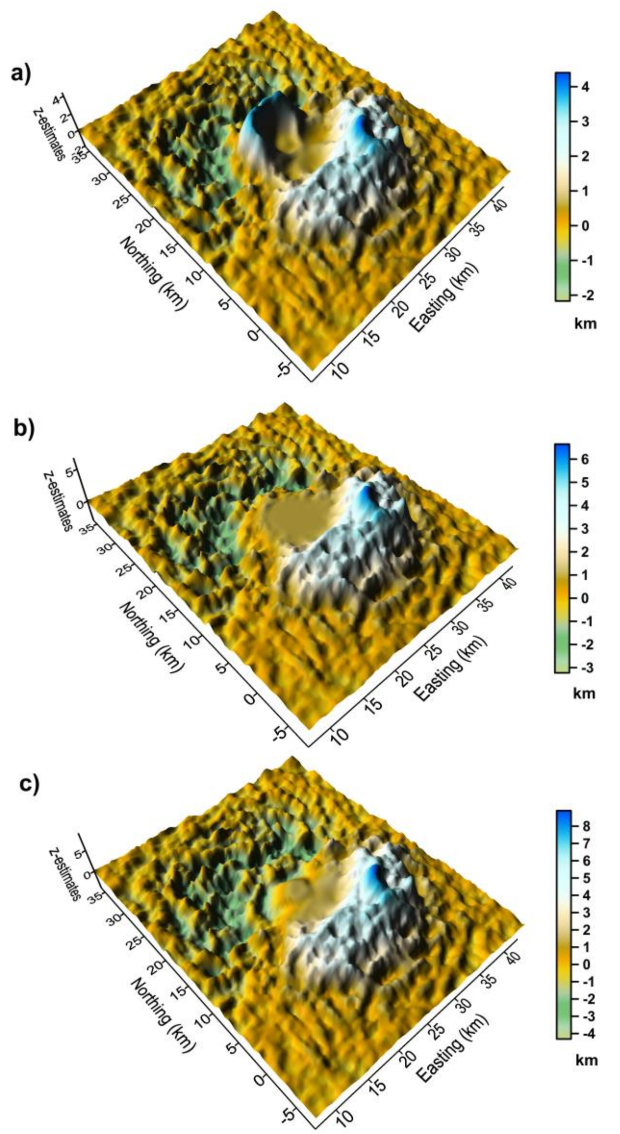

Figure 7 – *Depth estimates of Euler deconvolution using an 11 x 11 moving data window assuming structural indices (a) 1, (b) 2 and (c) 3. Depth estimates using the correct structural index are constant over the source and have the smallest variation.*

Finally, Figure 8 (a) – (c) displays base level estimates from Euler deconvolution are assuming structural indices 1, 2 and 3, respectively. These estimates follow the same pattern of the vertical incidence case, only using the correct structural index a plateau of correct estimates are present. At the borders of the anomaly influence area, base level estimates are correct regardless of the used structural index. Besides the plateau at the correct estimate in Figure 8 (b), comparing the amplitudes range via the color bar is easy to identify that these estimates have a small variation. Thus, like for the vertical incidence test, the minimum variation of base level estimates can define the correct structural index.

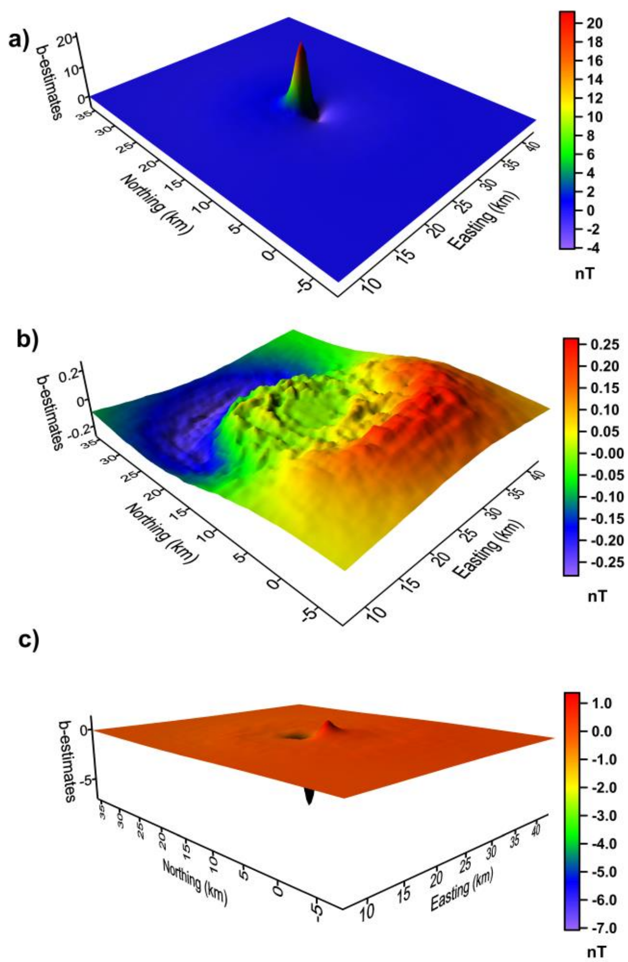

Figure 8 – *Base level estimates of Euler deconvolution using an 11 x 11 moving data window assuming structural indices (a) 1, (b) 2 and (b) 3. Base level estimates using the correct structural index are constant over the source and have the smallest variation.*

Conclusions

We have shown what to expect from Euler deconvolution estimates isolated sources. With this analysis was possible to define that the minimum variation of either depth or base level estimates at the region of the source can define the correct structural index. No further calculation is necessary to define the correct structural index. The average of the estimates over the plateau associated with the correct structural index gives the correct estimated value for any estimates. We performed these analyses in anomalies with different simulated geomagnetic field influence and we extend the results for all cases. Horizontal estimates have almost the same value using or not the correct structural index. Over the source, there are small differences on these estimates that make no difference in the result. At the borders of the anomaly, moving data window coordinates weights the estimates and they are correct, regardless of the structural index used. Over the source, depth and base level estimates show an outstanding pattern forming plateaus only when the correct structural index is used. This pattern allows the identification of the correct structural index. More than that, depth and base level estimates are only true over the source if the correct structural index is used. Variation of base level and depth estimates using the correct structural index is smaller than estimates using the wrong structural index, over the source. At the borders of the anomaly, depth estimates are weighted by acquisition height while base level estimates are the correct base level, regardless of the structural index used, for both cases. These analyses were possible because we have run Euler deconvolution on an anomaly generated by a single isolated synthetic source with high signal-to-noise level. Its derivatives are analytic so no problems related to numerical derivatives influence the results. We can extend these analyses to any source that is valid for Euler deconvolution, any values of base level, acquisition height and any inclination or declination of the source or geomagnetic field, as long as there are no interfering sources.

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