Mathematical modeling of the 2D geoelectric problem using resistor network

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Abstract

We are proposing in this paper a mathematical model formulation of the geoelectric method based on medium discretization through resistive elements. The method employs the Kirchhoff's and Ohm's law to the mathematical problem formulation. The model was validated by comparison between the experimental results and literature results. The comparative results showed that the modeling has the sensitivity to identify layers and isolate bodies with resistivity contrast.

Introduction

The geoelectric method is based on the electric current conduction in subsurface. There are many arrangements of electrodes used in the method: Schlumberger, dipole-dipole, Werner. All of them employ a set of current electrodes, where a potential difference (ΔV) is applied and the current injected into the ground is measured, and power electrodes, where ΔV is measured through a voltmeter, the different arrangements depend on the geometry of the electrode assembly. The apparent resistivity (calculated by measurements of current and voltage on the electrodes, and the distances between them) is related to the resistivity of the subsurface geological structures, so that the correct interpretation of the results can provide important information about the area studied in local scale. The method is applied to shallow geophysical problems, with good results in the location of groundwater, studies of soil contamination, determination of the salt wedge in coastal areas and characterization of shallow geological structures.

In this work we performed the modeling of a vertical electrical survey (VES) using the Schlumberger arrangement (array). The physical medium is represented by a discretization in which the basic elements are resistors associated with a rectangular network in a plane. The electric current is applied to two nodes of the network and the voltage is measured in two other nodes, so the problem is considered as an association of resistors. Subsequently, scaling factors are calculated to fit the network to the field survey dimensions and calibrations with the purpose of converting resistances into resistivities.

Method

The proposed direct model is a discretization of a vertical plane, in subsurface, through a rectangular resistors network, according to the Figure 01:

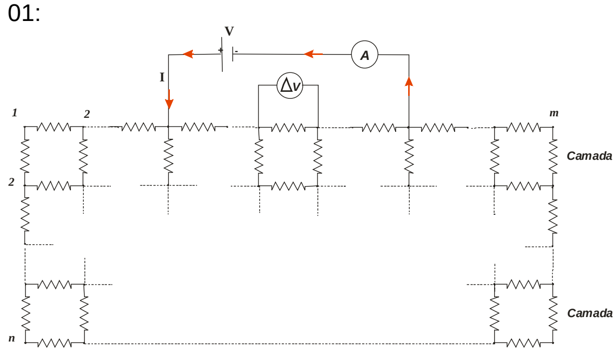

Figure 01 – Medium discretization through a regular resistors network.

The resistor network can be sized according to the problem scale and the desired accuracy.

The direct problem formulation is derived through Kirchhoff's Laws for electrical circuits: law of knots and law of meshes and Ohm's Law (Reitz, J. et al., 1982).

To represent a model of horizontal layers, we will adopt equal resistance for each circuit horizontal layer as showed in the Figure 01.

The values of the resistors and the voltage V applied to the current electrodes (A and B of Figure 2) are given and through the solution of a linear system, the values of all electric currents in the circuit are calculated. For the proposed formulation, with a circuit with *m* horizontal nodes and *n* vertical nodes, we will have:

m x n = number of nodes equations

(m-1)x(n-1)+2 = number of mesh equations

2(n x m) -n-m+1 = total number of unknowns (circuit electric currents)

We then have an overdetermined linear system:

$$\begin{cases} \text{equação dos nós} \\ i_{11} + i_{12} + i_{13} + i_{14} = 0 \\ i_{21} + i_{22} + i_{23} + i_{24} = 0 \\ \vdots \\ i_{w1} + i_{w2} + i_{w3} + i_{w4} = 0 \\ \text{equação das malhas} \\ r_{11} i_{11} + r_{12} i_{12} + r_{13} i_{13} + r_{14} i_{14} = 0 \\ r_{21} i_{21} + r_{22} i_{22} + r_{23} i_{23} + r_{24} i_{24} = 0 \\ \vdots \\ r_{v1} i_{v1} + r_{v2} i_{v2} + r_{v3} i_{v3} + r_{v4} i_{v4} = 0 \\ r_{s1} i_{s1} + r_{s2} i_{s2} + \cdots = V \\ r_{c1} i_{c1} + r_{c2} i_{c2} + \cdots = V \end{cases}$$

Where, $w = m \times n$ e $v = (m-1) \times (n-1)$

After determining all the currents in the circuit we can calculate ΔV, through Ohm's law, which is the voltage at the potential electrodes (M and N).

This system can be solved by the least squares method (Helene, 2006):

$$I = (A^T A)^{-1} A^T d$$

Where:

A = matrix of coefficients of the system

I = vector of unknowns (electric currents)

d = vector of independent terms

After choosing the dimensions of the resistor network it is necessary to perform a calibration of the model with the objective of converting the resistors electrical resistance into the electrical resistivity of the medium and correction of the edge effect. This calibration is done through reference data available in the bibliography.

It is important to point out that this direct model can be used to represent more complex geological situations, such as: dikes, spills, faults and batholiths, whenever there is a resistivity contrast with the basement.

Results

1- Experimental bench arrangement

As a first test to validate the direct model, we assemble an electric circuit as in the Figure 2:

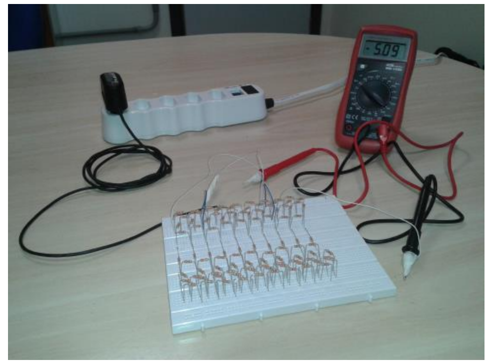

Figure 02 - real electric circuit.

We measured the potential values at each point and compared them with the direct model result. Figure 3 shows that the modeled ΔV values are compatible with the values measured experimentally:

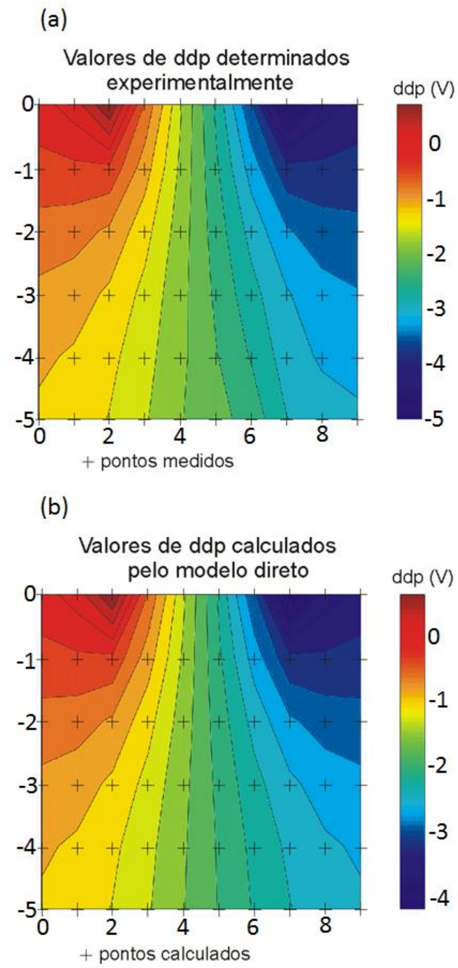

Figure 3 - Comparison between the experimental data and the mathematical model data. a) electric circuit result. b) modeling result.

Then we generate models for different situations where we simulate VESs changing the position of the current injection electrodes thus obtaining a graph of apparent resistivity versus opening of the electrodes and also plots a chart of equipotential, comparing when possible with the results obtained in the literature.

2- Semi homogeneous plane

In this example, the objective is to model the variation of apparent resistivity with the opening of the current electrodes in an ideal situation (Fig.4).

Figure 04 - Example of the homogeneous medium model.

As we can see in Figure 05, the apparent resistivity graph for the homogeneous model is constant as expected.

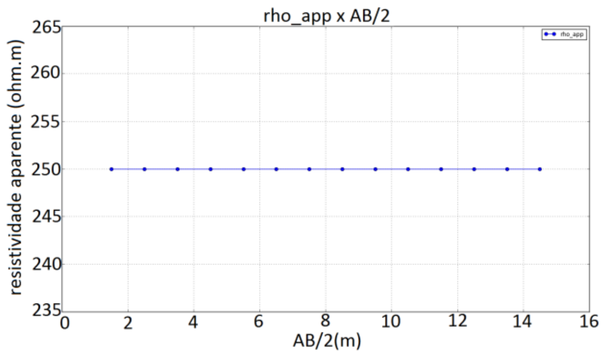

Figure 05 - Apparent resistivity graph for a homogeneous medium model.

Figure 6a shows the modeled result, and can be seen from Figure 6b that the features of the equipotential lines are very similar in the area delimited by the rectangle:

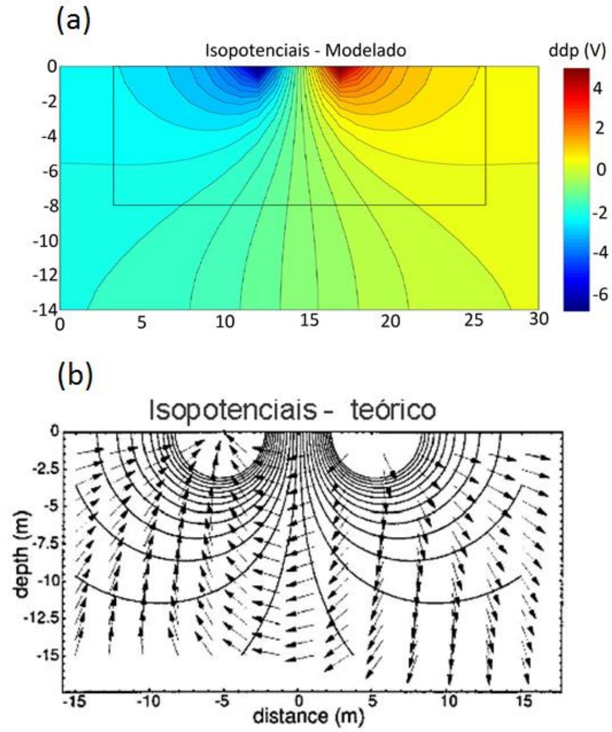

Figure 06 - Comparison between modeling and theoretical equipotential graphs. a) modeling result. b) theoretical reference result Taken from Herman, R. (2001).

The distortions found outside the rectangle are related to an edge effect, this is because the resistor network has a finite dimension and Figure 6b shows the isovalues for an infinite half-plane. This edge effect has been corrected by a numerical calibration.

3- Model of horizontal layers.

In examples, 3.1 and 3.2 we show the results for a two and three-layer model, respectively, Figures 7 and 10. In Figures 08 and 11 we can see that the proposed model satisfactorily represents the theoretical graph of apparent resistivity obtained in the literature. In Figures 9 and 12 we can see that the model also represents the distortion effect of the potential lines at the boundary between two media with different resistivities (geological contact), according to Telford (1990) pg. 527.

3.1 Two Layer Model

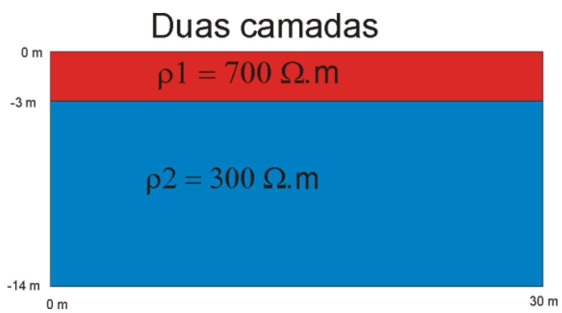

Figure 07 - Example of the two-layer model.

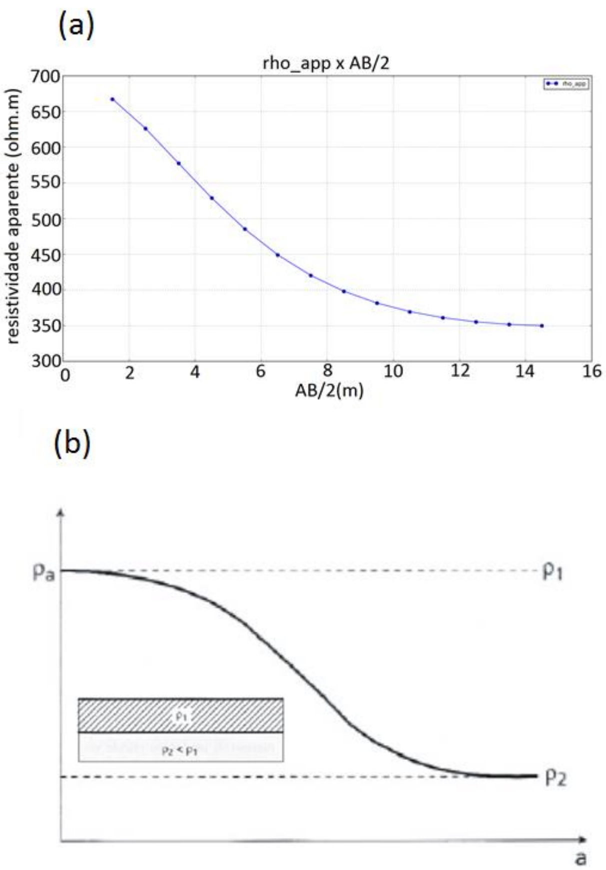

Figure 08 - Comparison between the modeling and theoretical apparent resistivity graphs for two layers. a) modeling result. b) theoretical reference result Taken from Kearey, P., Brooks M., Hill, I. (2009).

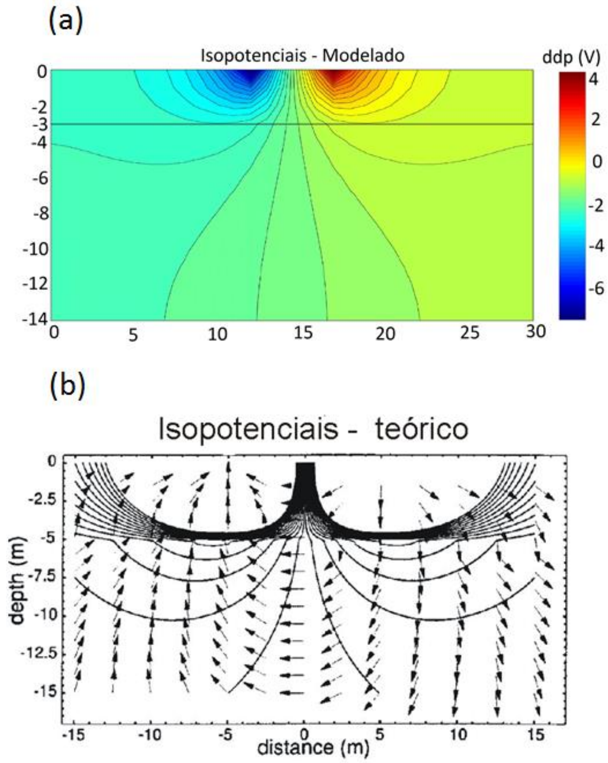

Figure 09 - Comparison between modeling and theoretical equipotential graphs for two layers. a) modeling result. b) theoretical reference results taken from Herman, R. (2001).

3.2 three-layer model

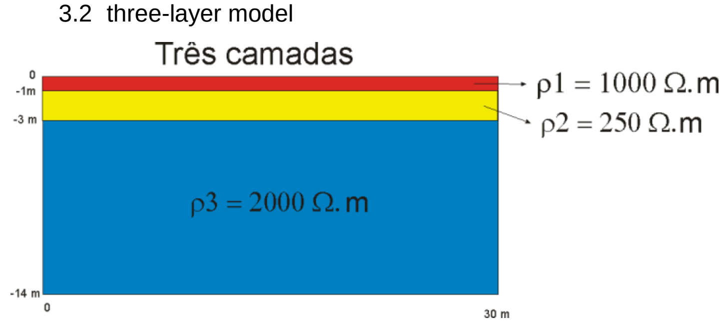

Figure 10 - Example of the three-layer model.

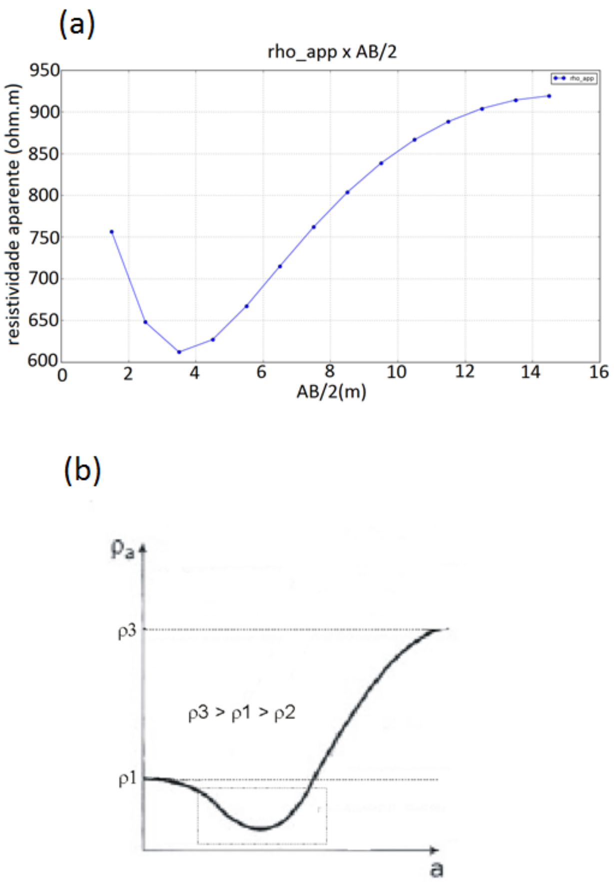

Figure 11 - Comparison between modeling and theoretical apparent resistivity graphs for three layers. a) modeling result. b) theoretical reference results taken from Kearey, P., Brooks M., Hill, I. (2009).

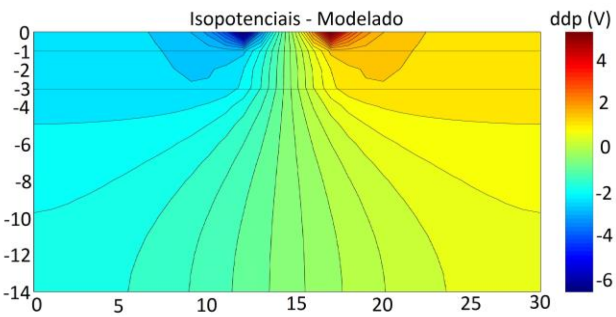

Figure 12 - Graph of equipotential for three layers model.

4- Vertical Dike

In this example, the objective is to show the potentiality of this model to a more complex situation, which is an intrusion with a vertical discontinuity.

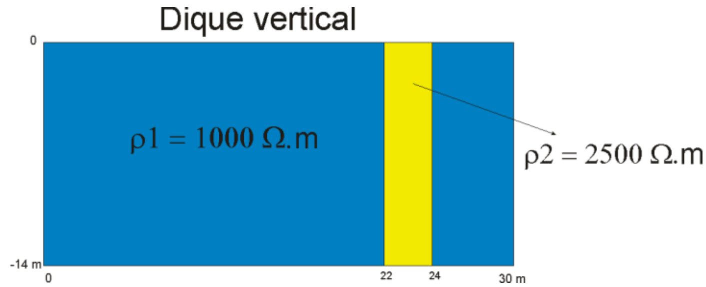

Figure 13 - Example of a vertical dike model.

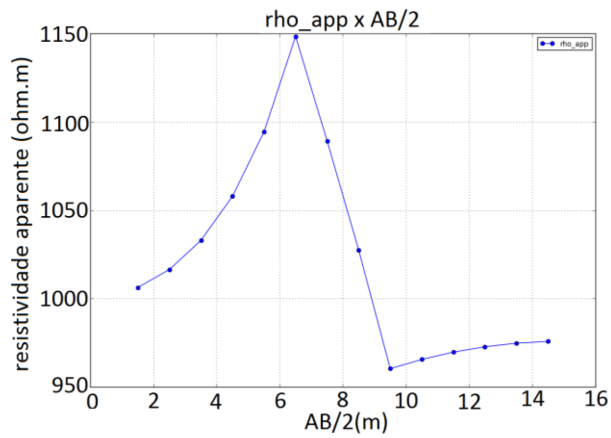

Figure 14 - Graph of apparent resistivity for a SEV on a terrain with a vertical dike.

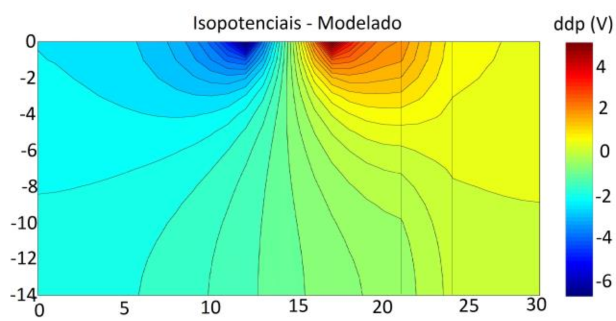

Figure 15 - Graph of equipotential on a terrain with a dike.

In the case of the vertical dike, we can clearly see the distortions caused in both the apparent resistivity graph and the potential graph.

Conclusions

According to the presented results, the modeling proved to be efficient to represent the studied processes, having sensitivity to identify layers and bodies isolated with resistivity contrast. For future research, we intend to implement an inversion method using this model as a direct model input.

Acknowledgments

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