An elastic full-waveform inversion approach for anisotropic media

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Abstract

Full waveform inversion (FWI) updates the velocity model by forward modeling synthetic shots that are then compared to the real data, and the residuals are used to define the direction in which the model must be updated. Differences between the synthetic and real data due to incomplete physics will be incorrectly assigned to the velocity, which can result in a degradation of the velocity model. For example, a lack of anisotropy in the modeling can often lead to the overestimation of the velocity. We have therefore developed a 3D anisotropic elastic FWI code, which we test using a modified version of the SEAM model, using both an anisotropic elastic update and an isotropic acoustic update. The anisotropic elastic update recovers accurately the correct model, giving us confidence in our anisotropic elastic FWI code. On the other hand, the isotropic acoustic update seriously degrades the velocities, illustrating the detrimental effects that an incomplete parametrization can have on an FWI update.

Introduction

In the exploration geophysics industry, full waveform inversion (FWI) is a method to update an existing velocity model. FWI can add considerable detail to the velocity model and this can translate to better migrated images, particularly when using reverse time migration (RTM), which makes full use of the details in the velocity model without smoothing it.

FWI works by attempting to match the observed wavefield. Using the initial velocity model, a round of forward modeling is performed. The forward modeled shots are compared to the real data shots, and the differences (known as the residuals) are back-propagated and cross-correlated with the forward field. This yields the gradient, which shows the direction the velocity model must be updated to minimize the difference between the synthetic and real data (Tarantola, 1986).

However, the update is dependent on the accuracy with which the forward modeled shots can match the real data. Differences due to an incomplete parametrization of the physics will be incorrectly assigned to the velocity model and will appear as systematic errors. Hence, it is critical to parametrize the physics in a way that is realistic enough to explain the data, by including anisotropy and elastic effects.

We have therefore developed an anisotropic, elastic FWI 3D code to model the subsurface as accurately as possible. We present the case for anisotropic elastic FWI, and test our code using a synthetic 3D model. As well as the anisotropic, elastic inversion, we also carry out an isotropic acoustic inversion to illustrate the different result obtained.

Anisotropy

Anisotropy refers to the fact that the velocity of p and s-waves depends on the direction, rather than being the same in all directions (known as isotropy). The horizontal velocity is always larger than the vertical velocity [$V_p(90) \approx V_p(0)(1+\varepsilon)$, Thomsen, 1986], although the velocity at intermediate angles (≈45°) can be smaller if δ is negative.

An isotropic FWI code will assume the horizontal and vertical velocities are the same. When given data that has experienced predominantly the horizontal velocity, such as refractions and wide-angle reflections, FWI will use the vertical velocity, which will be too small to explain the data. To better match the data, FWI will increase the velocity.

However, this represents the incorrect migration velocity, so that in the image migrated using the updated model, the geological layers will be systematically too deep, and the well-markers will no longer tie. This is a simple illustration of the importance of including anisotropy in FWI.

Elastic wave equation

The acoustic wave equation,

$$\frac{1}{V_P^2\rho}\frac{\partial^2 u}{\partial t^2} - \nabla.\left(\frac{1}{\rho}\nabla \mathrm{u}\right) = f$$

is often used to propagate the wavefields, in RTM and FWI. It is scalar by nature, and therefore isotropic, although approximations can be made to solve anisotropic acoustic wave equations (Alkhalifah, 2000). This equation cannot model any phenomenon related to s-waves.

The elastic wave equation,

$$\partial_t^2 u_i = \sigma_{ij,j} + f_i.$$

on the other hand, provides a complete description of dynamics of the wave: both the phase and the amplitudes are accurately represented. Mode conversion occurs at the interfaces between different layers, so that part of the energy originally propagated in p-waves is transferred to s-waves. This leads to amplitude variations with offset being correctly modeled.

Hence, an FWI code using the elastic wave equation will be able to match the amplitudes of the data better than using the acoustic approximation. In addition, the elastic wave equation is vectorial and can naturally include anisotropy.

Method

We have therefore developed an anisotropic elastic 3D FWI code to update the p-wave velocity model for seismic exploration.

A quasi-Newton scheme is used, which requires the first derivative of the cost function with the model parameters (the gradient), as well as an approximation of the second derivative (the hessian) and a step size.

To propagate the wavefields and calculate the gradient, we use an elastic, tilted transverse isotropy (TTI) anisotropy finite difference scheme to propagate waves through the parameter model (p-wave and s-wave velocities, anisotropy parameters and density).

Following Liu & Nocedal (1989), we make use of the l-BFGS method to approximate the inverse of the hessian, with the line search method as described by Moré & Thuente (1994).

Synthetic model

In order to test our FWI code, a TTI elastic synthetic dataset was prepared. A subsection of the SEAM (Fehler & Larner, 2008) acoustic isotropic velocity model was cut and used for the p-wave velocity. The section was chosen to represent a sedimentary basin, flanked by salt on one of the edges. An 800 m-deep layer of water with constant velocity of 1500 m/s was added on top, to simulate a moderately deep water environment (see Figure 1). As input p-wave velocity for the inversion, we smoothed the correct velocity model, but kept the correct water velocity. This input velocity model is intended to reflect a good starting model, yet lacking in details.

For the water layer, the s-wave velocity was set to zero. In the sediments and salt the s-wave velocity was set everywhere to $V_s = V_p/(2\sqrt{2})$. The water and salt were assumed to be isotropic, so all anisotropy parameters were set to zero in both regions. Throughout the sediment layers the anisotropic parameters were set to the following constant values: $\delta = 0.02$ $\varepsilon = 0.04$ $\gamma = 0.01$, the azimuth was set to $\varphi = 30°$ and the structural dip to $\theta = 10°$. The density was set to 1 g/cm^3 at the water layer, a constant density of 2 g/cm^3 was used for all the sediments, and 2.5 g/cm^3 was used for the salt. Figure 1 summarizes the parameter model.

Our FWI code was used in forward modeling mode to create the “real” data by using the correct parameter model. These shots were created parallel to the crossline direction, mimicking a marine acquisition geometry with the boat going both forwards and backwards. To cover as much of the velocity model with full-fold shots and receivers, the maximum offset was limited to 3 km.

With such short offsets and an 800m-deep water layer, the update will be driven by reflections, rather than refractions as is most common with FWI using real data. The initial velocity model was good enough to allow this.

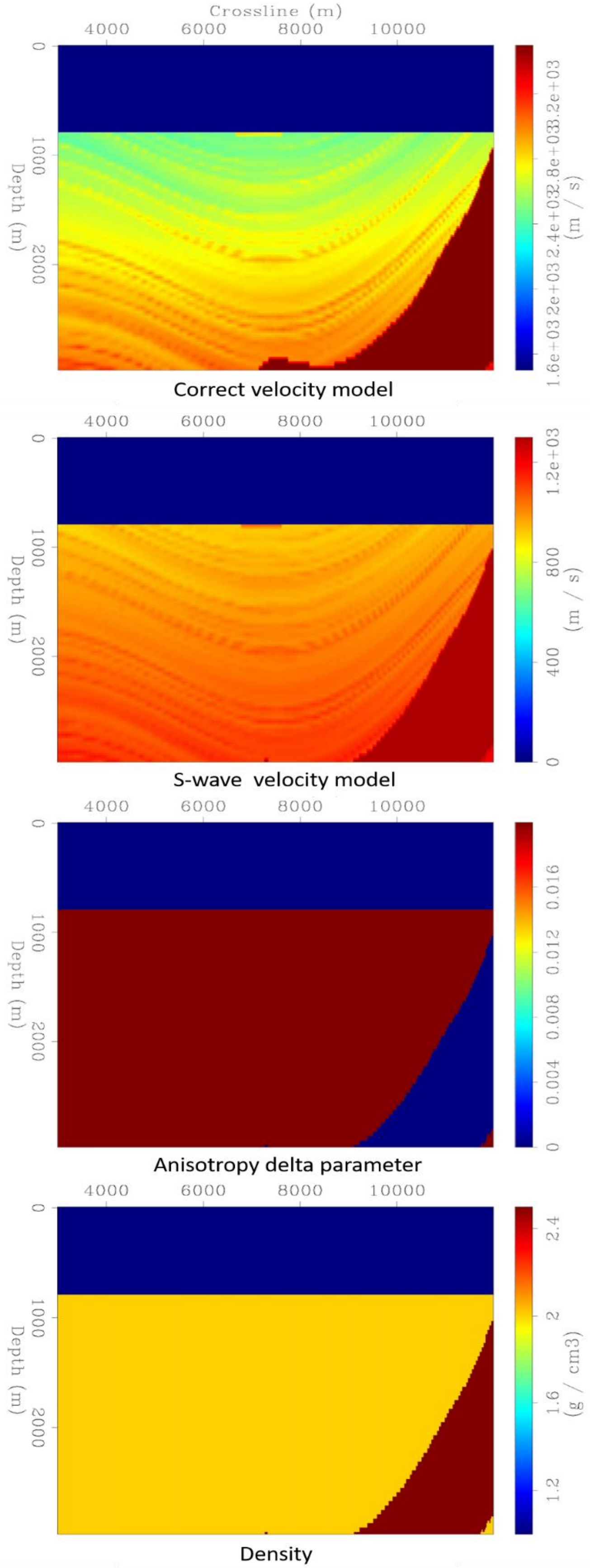

Figure 1*- Slice through the parameter model. From top to bottom: correct p-wave velocity model; s-wave velocity; delta, representative of all anisotropy parameters; density.*

Two different inversions of the p-wave velocity were carried out. One made use of the s-wave velocity and anisotropy parameters (hereafter "anisotropic elastic update"). For the other update, we used the same finite-difference elastic code, but with the s-wave velocity and anisotropy parameters all set to zero ("isotropic acoustic update"). Both inversions made use of the correct density model, and of a multi-scale strategy sequentially using data band-passed to 2-4 Hz, 2-5 Hz, 2-6 Hz and 2-8 Hz.

Results

Figure 2 shows the four p-wave velocity models: correct model, initial model, and the two FWI updates. It also shows the corresponding images from our anisotropic elastic RTM code. Figure 3 shows a velocity profile for all four models, and Figure 4 shows the profile of the migrated images.

Since the initial model was created by smoothing the correct model, it has broadly the correct velocities. However, it lacks details such as the different sediment layers and the velocity inversions. Compared to the correct image, the image migrated with the initial model is very similar, except that the amplitudes of the events around 1200-1600 m depth are slightly weaker, and below 2400 the reflectors are slightly deeper than for the correct model. This is best seen in Figure 4.

The left panel of Figure 2 shows that the anisotropic elastic update recovers the velocity of the sediment layers with great accuracy, except in the vicinity of the salt. The high velocity anomaly just below the water bottom, around crossline 7000, was totally absent from the initial model, yet the anisotropic elastic FWI update recovers it accurately. It is even able to recover velocity inversions, with low velocity layers below higher-velocity ones.

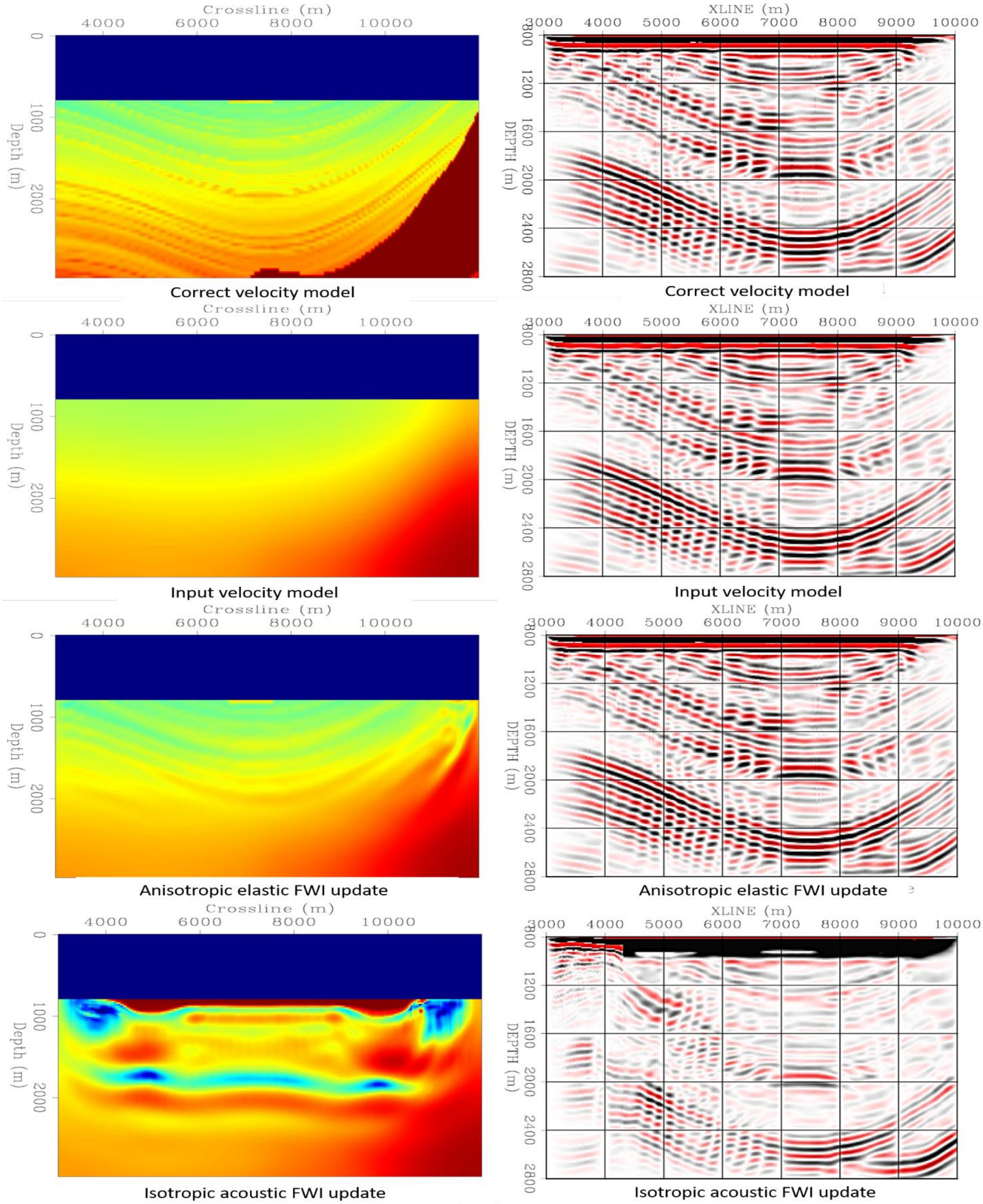

Figure 2 – *Left-hand side, p-wave velocity models. From top to bottom: correct model, input model, anisotropic elastic FWI update, isotropic acoustic FWI update. Right-hand side, images migrated using the corresponding model. The migrations were carried out using data in the range 2-25 Hz, and are only shown from the water bottom downwards.*

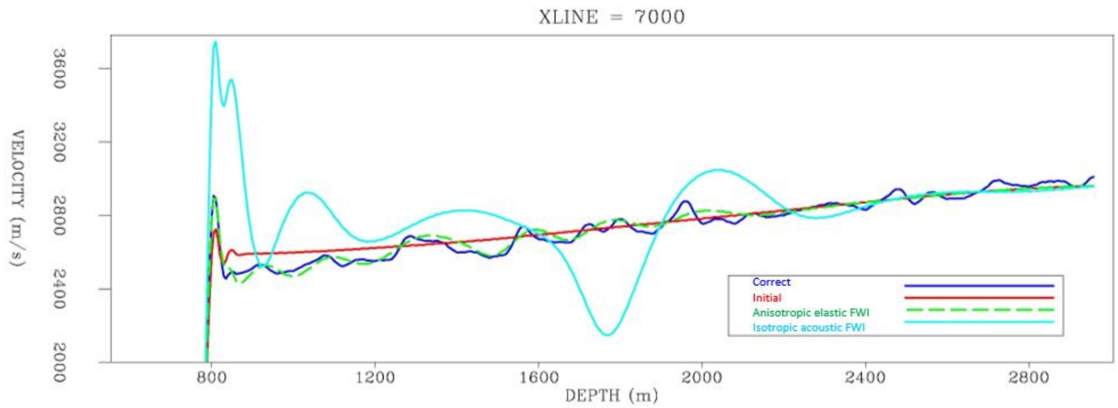

Figure 3- *Velocity profile at XL 7000. Dark blue is the correct model, red is the initial model, green dashed line is the anisotropic elastic update while light blue is the isotropic acoustic update.*

In Figures 2 and 3 we can see that the layers created by the anisotropic elastic update correspond accurately to those of the correct velocity model down to around 2000 m depth. Below this depth the velocity model has not been updated. Near the edge of the salt, we can see signs of cycle-skipping. This is due to the initial model not being accurate enough in this region, which reminds us that particular care must be taken when creating the initial velocity model close to strong velocity contrasts, such as those created by salt or carbonate layers.

Migration using the anisotropic elastic updated model produces images that are virtually indistinguishable from those migrated using the correct model. Between 2000 and 2400 m depth, a very slight shift begins to be noticed, but the events are still essentially at the same depths. Only below 2400 m does the shift become significant, however it remains very small.

Hence, for imaging purposes, the anisotropic elastic update has essentially recovered the correct model.

Looking at the isotropic acoustic update in Figure 2, we can see that the inversion has degraded the starting model significantly. At the longest offsets of 3 km the angle to the sediments just below the water bottom (800 m) can be as large as 75°. The TTI tilt of 10° makes this 65° or 85° relative to the tilted axis, depending on the direction. In any case, this is means the velocity that the inversion is sensitive to at such angles is close to the horizontal velocity. Looking at the synthetic shots for the isotropic acoustic model (not shown here), the reflections from just under the water bottom are approximately half a cycle off at the longest offsets. Putting together these two factors we can understand why the isotropic acoustic update has found a high-velocity cycle-skipped solution just below the water bottom.

From then onwards, the whole inversion is compromised. The alternating high- and low-velocity layers are a common feature often seen in inversions that are cycle skipped. Once the wrong velocity is placed at a certain layer, the FWI update attempts to compensate this immediately below, which leads to alternating high- and low-velocity layers, with no resemblance to the geology.

We know the initial model was a good one, since the anisotropic elastic update converged. Hence, the difference in the result must be due to the different parametrization, namely the lack of anisotropy.

The image migrated using the isotropic acoustic update has very weak amplitudes and the events are generally too deep (Figures 2 and 4). The high velocity just below the water bottom has also severely degraded the image at this location.

Conclusions

We have developed an anisotropic elastic full waveform inversion code and have successfully tested it on a synthetic 3D dataset based on the SEAM model. The anisotropic elastic inversion recovers correctly the different layers of the velocity model. Migration using this updated

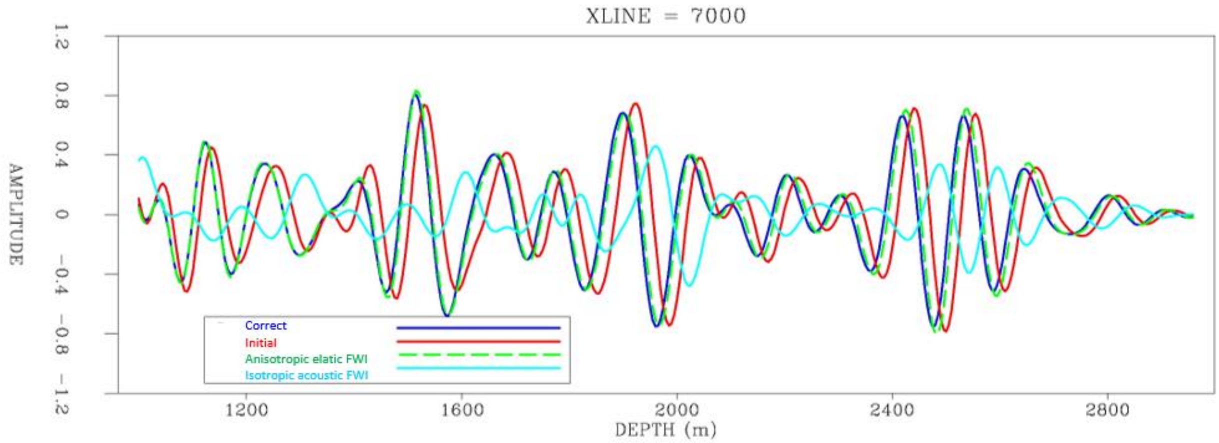

Figure 4 *Profile of the migrated images at XL 7000. Dark blue is the image migrated from the correct model, red from the initial model, dashed green from the anisotropic elastic update, and light blue from the isotropic acoustic update.*

model creates an image almost indistinguishable from the image migrated using the correct model, and only fails near the salt flanks. FWI is known to struggle near strong velocity contrasts, and a more accurate initial model would have been required near the salt flank.

We have also run an isotropic acoustic update, to illustrate the kind of problems that an incomplete parametrization can lead to. Despite using the same initial p-wave velocity model, the isotropic acoustic update has immediately placed a cycle-skipped high-velocity layer below the water bottom, and deeper down the inversion has severely degraded the velocity model. The resulting migrated image is poorly focused and has the wrong depths.

In this particular example, the isotropic acoustic update has turned out to give an extremely poor result, most likely due to the fact that the anisotropy is coherent throughout the entire sediment region. For a more realistic model, where the anisotropy parameters and tilt are not consistent throughout, an acoustic inversion might not necessarily give such a poor result. After all, exploration geophysics has used isotropic modeling for a long time, with successful results. However, our example serves as an illustration of the kind of problems that an isotropic update can suffer from.

Acknowledgments

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