Seismic trace analysis using minimum phase and singular value decomposition methods. Application to ground-roll attenuation.

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Abstract

The spiking deconvolution aim is to correct the effect of the wavelet in seismic trace by apply an inverse filter, assumed to be minimum delay, and to get an estimate of reflectivity. In order to compensate propagation and absorption effects one way is to use a time-variant deconvolution where a different inverse filter is computed and applied for each output sample position. We modify this procedure by estimating a minimum-delay wavelet for each time-sample position of the seismic trace. This gives a decomposition of the seismic trace as a sum of minimum-delay wavelets, each multiplied by a reflectivity coefficient.

We done the SVD decomposition through moving windows in the matrix which contains the minimum phase wavelets in each column. The seismic trace can be represented as a sum of the eigenimages of the wavelet matrix multiplied by the reflective function. In this way we can obtain an estimate of the reflectivity function through the inversion of this system.

This seismic signal decomposition method has a range of applications in the data processing and interpretation of the seismic signal. We use this method to ground-roll attenuation present in land seismic data. Improvements in data quality are evident in prestack data panels and velocity analysis.

Introduction

Due to the effects of propagation and absorption on earth the wavelet is not stationary (Yilmaz, 1987). To compensate for this effect, a time-varying deconvolution filter can be applied (Clarke, 1968). Given an autocorrelation function from within each window in seismic trace a new deconvolution filter is calculated and applied for each sample.

The method that we present starts with a time gate in the data. From the autocorrelation function we compute a spiking deconvolution filter which is minimum delay. The inverse of this is an estimate of a local minimum delay wavelet. This procedure is repeated for each time sample position. The seismic trace can be represented by a sum of these wavelets multiplied by the sample values of the reflectivity series. This is a linear equation where the data vector is equal to a lower triangular wavelet matrix times the reflectivity vector. This equation can be solved recursively to obtain the reflectivity series (Porsani et al., 2012).

We perform the SVD decomposition through slide spatial windows (Porsani et al., 2013) in the matrix that contains the minimum phase wavelets and then we represented the seismic traice through a sum of each eigenimage of the wavelets matrix times the reflectivity function. We did an estimate of the reflectivity function by the inversion of this system for each eigenimage of the matrix of the minimum phase wavelets. This seismic trace representation carries information about the frequency content of original seismic trace in each component of the decomposed signal.

We use this methods, minimum phase decomposition and singular value decomposition (MPD+SVD) to ground roll atenuation. Since coherent noise ground-roll is predominant at low frequencies, we can eliminate the first portions of the decomposed signal that contains the unwanted event.

Minimum phase decomposition

We consider a seismic trace $\mathbf{x}_t$ with ns samples. Taking a gate of length lw we have $\mathbf{x}_t^{lw}$, where $lw \leq ns$. The local autocorrelation function $\mathbf{R}_x$ is

$$\mathbf{R}_x = \sum_{k=1}^{lw} \mathbf{x}_t \mathbf{x}_{t+k} \tag{1}$$

After autocorrelation has been calculated from inside the window in a seismic trace, we get the equations normal system (2) which is used to obtain the Wiener-Levinson filter coefficients.

$$\begin{bmatrix} R_0 & R_1 & \dots & R_n \\ R_1 & R_0 & & \vdots \\ \vdots & & \ddots & R_1 \\ R_n & \dots & R_1 & R_0 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} = \begin{bmatrix} E_{g,n} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{2}$$

where $\mathbf{g}$ is the unitary predictive filter or Wiener-Levinson filter.

If we take a normalization condition on the filter and taking into account that an autocorrelation matrix is a Toeplitz matrix, we can calculate the filter coefficient through Levinson's recursion (Porsani, 1986).

After filter coefficients calculation **g** inside the window lw, you can get the minimum phase wavelets **w** by inversion of the following equation system (Robinson, 1967)

$$\mathbf{g}_n^{lw} * \mathbf{w}_n^{lw} = \delta_{mn}, \tag{3}$$

where δ is the Kronecker's delta, and $\delta = \begin{cases} 1, & \text{if} \quad t = 0 \\ 0, & \text{if} \quad t \neq 0 \end{cases}$

We can represent the seismic trace by a matrix-vector notation

$$\begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ \vdots \\ x_{ns} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ w_0(1) & 1 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & 0 & 0 \\ w_0(lw) & \ddots & \ddots & 1 & 0 \\ 0 & \ddots & \ddots & w_{ns-1}(1) & 1 \end{pmatrix} \begin{pmatrix} r_0 \\ r_1 \\ \vdots \\ \vdots \\ r_{ns} \end{pmatrix} \tag{4}$$

The equation (4) is

$$\mathbf{x} = \mathbf{W}\mathbf{r} \tag{5}$$

Singular value decomposition of the matrix W

The singular value decomposition (SVD) is an importante algebra theorem that we used to decompose **W** of $ns \times ns$ into a weighted sum of unit rank matrices.

The SVD decomposition of the matrix **W** in singular values, can be written in reduced form (Golub and Van Loan, 1996) as follows

$$\mathbf{W} = \mathbf{U}\Sigma\mathbf{V}^T \tag{6}$$

where,

$\Sigma = diag\{\sigma_1, \ldots, \sigma_N\}$ is the matrix of the singular values $\sigma_1 \geq \ldots \geq \sigma_N \geq 0$

$\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_N]$ it is the array of the eigenvectors of the covariance matrix $\mathbf{W}\mathbf{W}^T$ associated with the time dimension.

$\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_N]$ it is the array of the eigenvectors of the covariance matrix $\mathbf{W}^T\mathbf{W}$ associated with space dimension.

The matrices **U** and **V** are unitary and orthogonal. Another way to write the equation (6) is

$$\mathbf{W} = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \mathbf{v}_k^t = \sum_{k=1}^{r} \widetilde{\mathbf{W}}_k, \tag{7}$$

r is the rank of the matrix **W**.

According to equation (7), the matrix **W** can be seen as a sum of unit rank matrices, weighted by their respective singular values (Freire, 1986). Each scalar product $\mathbf{u}_k\mathbf{v}_k^t$ defines a unit rank matrix, called by Andrews and Hunt (1977) as a eigenimage of **W**. Then $\widetilde{\mathbf{W}}_k = \sigma_k \mathbf{u}_k \mathbf{v}_k^t$ represents the *k-th* eigenimage of the matrix **W**.

In order to avoid unnecessary operations on **W** array that contains a large number of zeros, we find it useful to organize the wavelets into a new array now represented by $\mathbf{W}min$ of order $lwav \times ns$, where $lwav$ is the length of the wavelets and ns the number of samples of the seismic trace.

The SVD decomposition is done adaptively as proposed by Porsani et al. (2013), where for each column of the matrix $\mathbf{W}min$, we collect a subset of N neighboring columns, taking as reference the central column from which we want to obtain the SVD decomposition, in this way we will obtain a window that will go through the entire matrix obtaining the SVD decomposition in the central column of the slide window. This procedure preserves the relative amplitude and highlights the continuity and coherence of reflection events (Silva, 2015).

After obtaining the SVD decomposition of the matrix $\mathbf{W}min$ we can represent it as follows

$$\mathbf{W}min = \widetilde{\mathbf{W}}min_1 + \widetilde{\mathbf{W}}min_2 + \ldots + \widetilde{\mathbf{W}}min_L = \sum_{i=1}^{L} \widetilde{\mathbf{W}}min_i \tag{8}$$

$\mathbf{W}min_i$ is the *i-th* eigenimage of the $\mathbf{W}min$ array after SVD decomposition.

After we calculate the minimum phase wavelets, we organize these matrices that have the same structure as the matrix **W** of the equation (4) filled with 1 in its main diagonal.

We can now represent the seismic trace as follows

$$\mathbf{x} = \widetilde{\mathbf{x}}_1 + \widetilde{\mathbf{x}}_2 + \ldots + \widetilde{\mathbf{x}}_L = \sum_{i=1}^{L} \widetilde{\mathbf{x}}_i \tag{9}$$

and,

$$\sum_{i=1}^{L} \widetilde{\mathbf{x}}_i = \widetilde{\mathbf{W}}_1 * r + \widetilde{\mathbf{W}}_2 * r + \ldots + \widetilde{\mathbf{W}}_L * r \tag{10}$$

This is results in the following system of equations

$$\widetilde{\mathbf{x}} = \widetilde{\mathbf{W}}\mathbf{r} \tag{11}$$

Reflectivity function estimation

The system of equations (11) allows us to obtain an estimate of the reflectivity function for each $\widetilde{\mathbf{x}}_i$ solving the following equation

$$\widetilde{\mathbf{r}} = \widetilde{\mathbf{W}}^{-1}\widetilde{\mathbf{x}} \qquad (12)$$

The $\widetilde{\mathbf{W}}_i$ arrays that have 1 in their main diagonal and are triangular lower arrays allows us to solve the system quickly through backsubstitution.

The reflectivity function **r** will be represented as follows

$$\mathbf{r} = \widetilde{\mathbf{r}}_1 + \widetilde{\mathbf{r}}_2 + \ldots + \widetilde{\mathbf{r}}_L = \sum_{i=1}^{L} \widetilde{\mathbf{r}}_i \qquad (13)$$

Results

We insert the proposed method in the filtering step in 2D seismic land data processing. We done the processing of the seismic line of the Tacutu basin located in Brazil. The distance between geophones are 50 meters and the interval between shots are 50 meters. The data are computed until $t = 4$ seconds with a sampling interval $\Delta t = 0.004$ seconds.

We have in Figure1 select a seismic trace form the data to apply $MPD + SVD$ decomposition and we represent the seismic trace as the equation (9). We can observe that the components of the decomposed data have their high frequency content removed as the number of decompositions increases. We can observe the effect of decomposition on the amplitude spectrum. We note that the first component of the decomposed signal is very similar to the original signal when the number of decomposition is small (Figura 2). We notice that the sum of all the decomposed traces restores the original seismic trace.

After we have selected an common-shot gather and we have obtained an estimate of the reflectivity function after 20 decompositions according to equation (13). Then we eliminated a first and a second decompositions where the noise of high amplitudes and low frequencies was present. At the end we add the remaining components.

In figure 3(a) we have a common-shot gather that is highly contaminated by the coherent noise ground-roll, the noise is presented in the section in the form of a cone of high amplitudes and low frequencies, these characteriristics masks the reflections in the seismogram. In figure 3(b) we have a common-shot gather after $MPD + SVD$ filtering with a rejection of the first and second eigenimage of the matrix that containing the reflectivity estimate. In Figure 3(c) is a filtered common-shot gather with an Automatic Gain Control (AGC) application with a 2 seconds window, we noticed that with a noise attenuation the AGC performance was much more efficient when compared to a section that has a low signal-to-noise ratio.

After filter application we sorting the data into Common Midpoint gathers and we done supergathers. In Figure 5 we have a supergather and the semblance panels of this supergather. The coherent noise makes it difficult to pick velocities mainly in the shallow parts of the basin. In Figure 6 shows that the ground-roll attenuation using $MPD + SVD$ decomposition filter for this basin helped to pick velocities mainly in the shallow parts of the basin.

Conclusion

The $MPD + SVD$ filter is an important tool to seismic signal decomposition and to give an estimate of the reflectivity function. The method that we presented was very efficient to ground-roll attenuation. Since each component of the decomposed data carries information of the frequency content of the original data, we can also to attenuate noises in other frequencies bands and to get some information about the signal attenuation on the earth.

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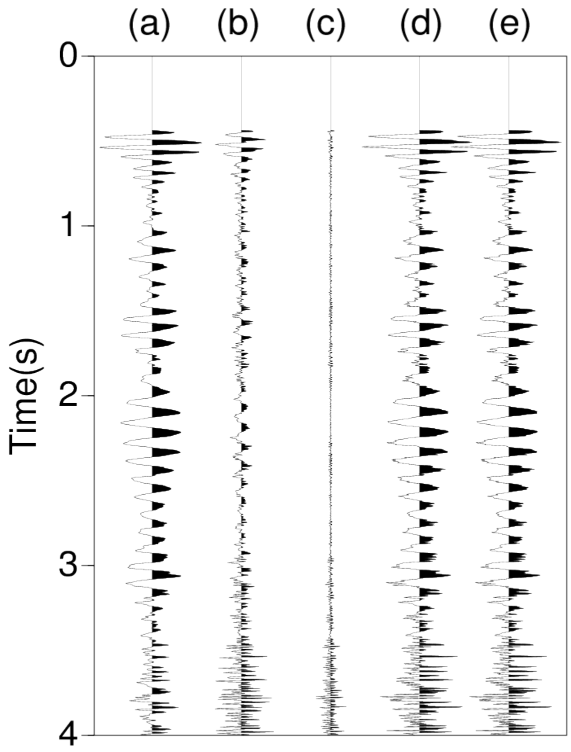

Figure 1: Seismic trace decomposition: Decomposition 1 (a), Decomposition 2 (b), Decomposition 3 (c), Sum of all decompositions (d) and seismic trace original (e) .

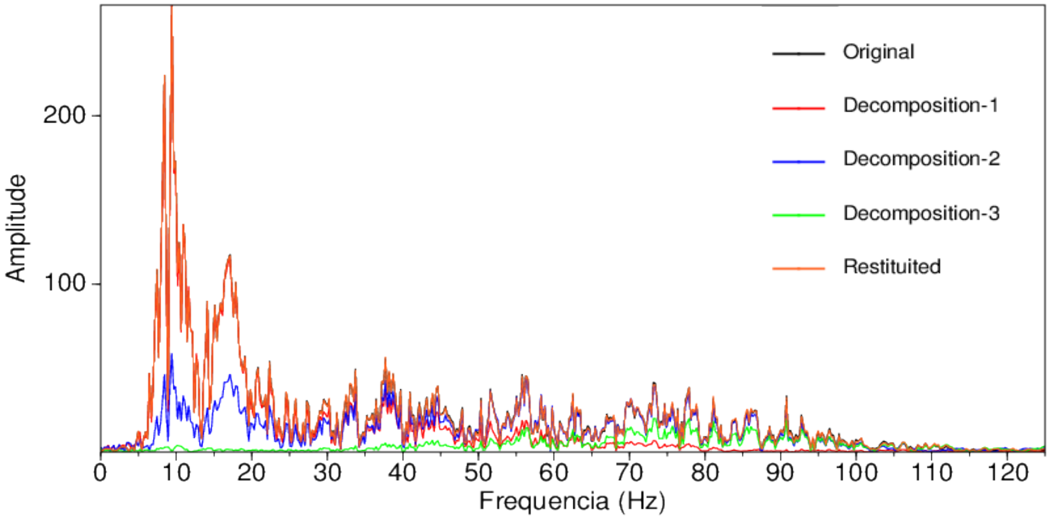

Figure 2: Amplitude spectrum of the original trace and amplitude spectrum of its decompositions.

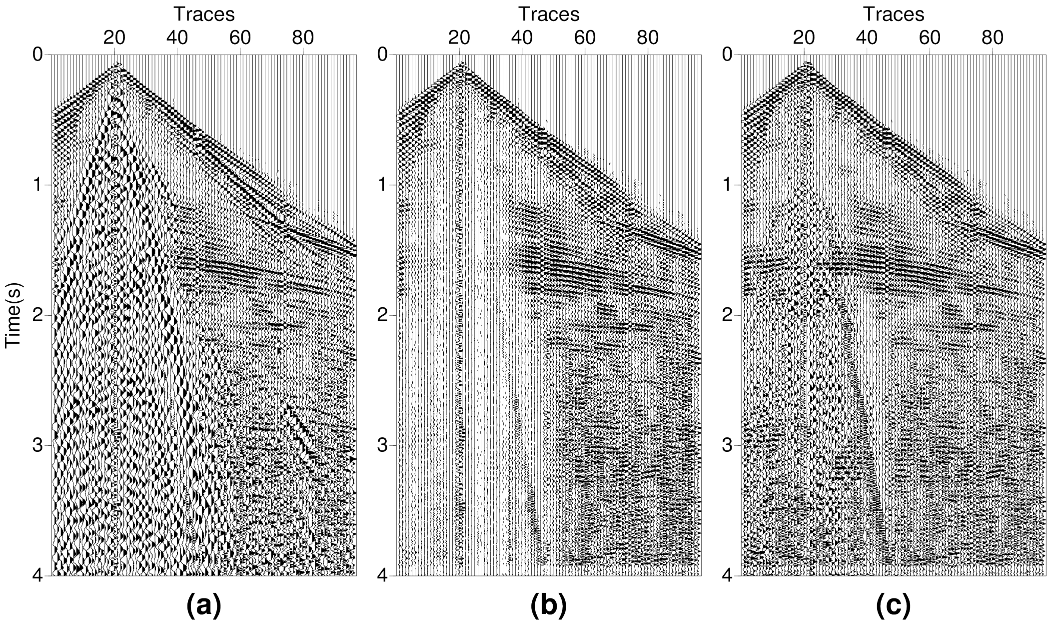

Figure 3: Common-shot gather without filter apply (a), after MPD+SVD filter (b) and MPD+SVD+AGC.

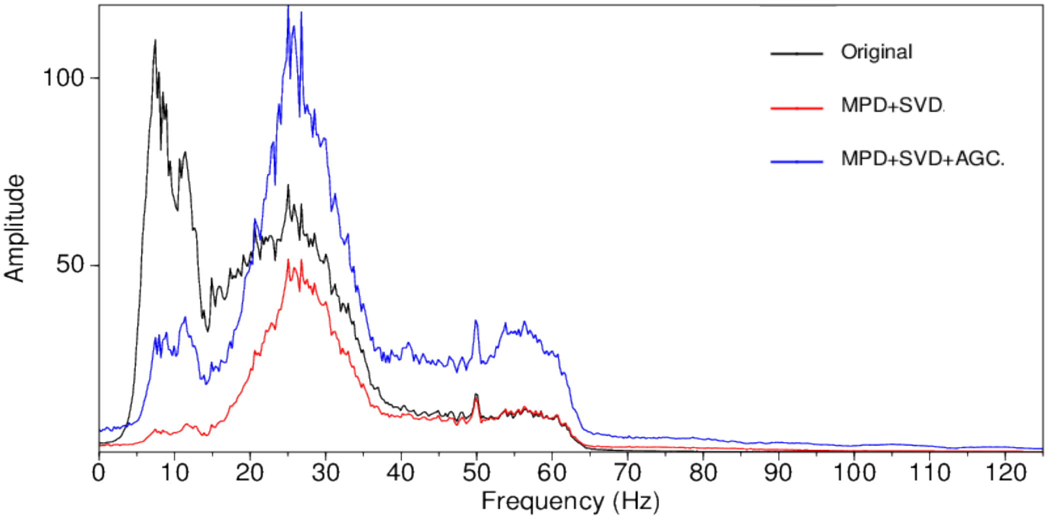

Figure 4: Amplitude spectrum showing the effect of filtering.

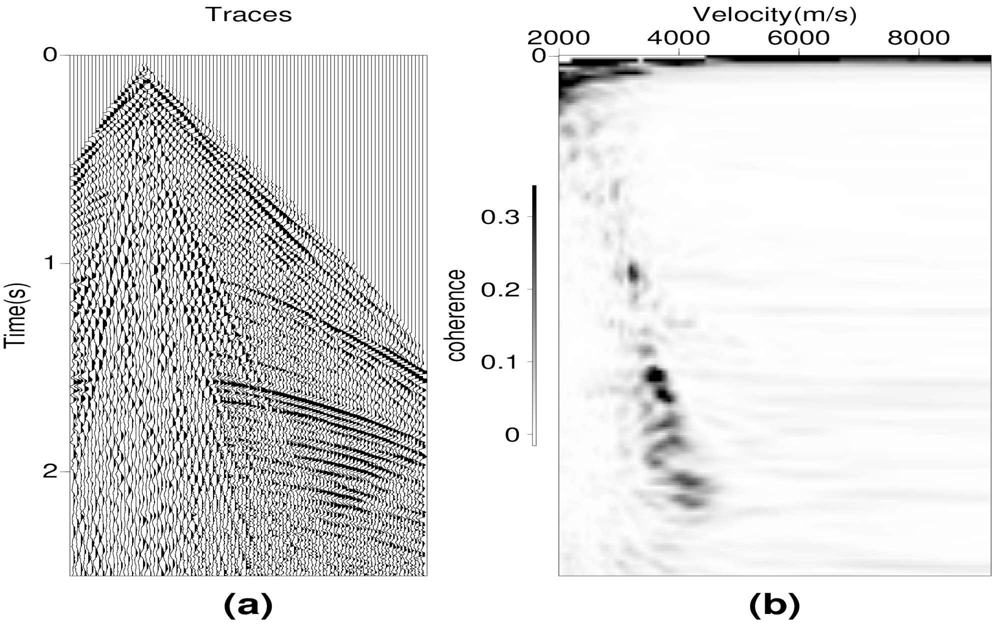

Figure 5: Supergather without application of filters and its panel of semblence.

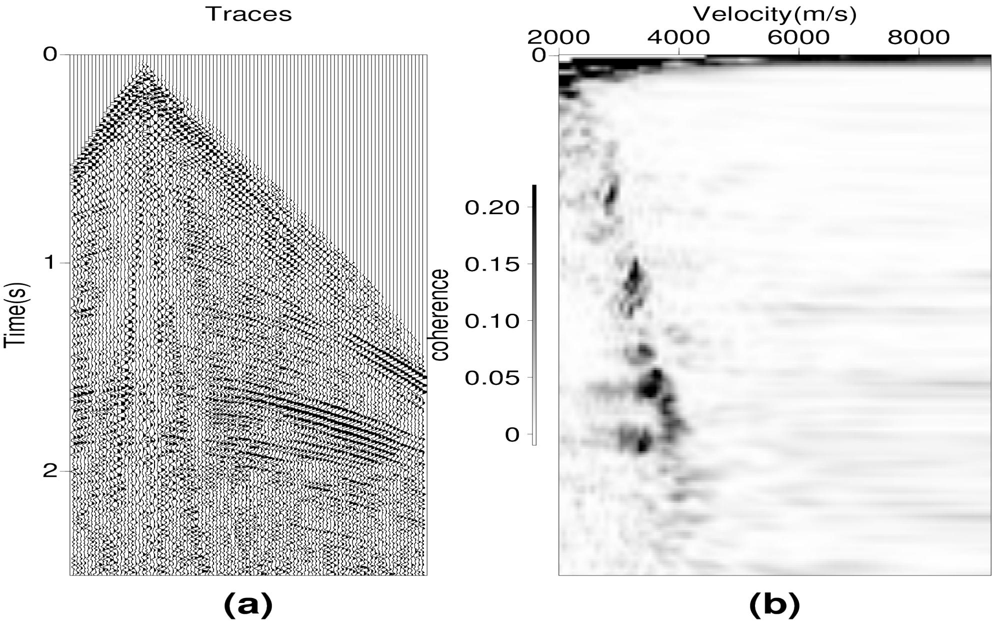

Figure 6: Supergather $MPD+SVD$ filter application and its panel of semblence.