Inversion of P- and S-wave VSP traveltimes in a homogeneous moderately anisotropic medium

Ivan Pšenčík and Bohuslav Růžek, Czech Academy of Sciences, Czech Republic

Abstract

Success of traveltime inversion in anisotropic media depends strongly on the selection of the parameterization of the medium and on an efficient forward modelling scheme. We react to these requirements by using the so-called weak-anisotropy (WA) parameters instead of stiffness-tensor elements or elastic parameters in Voigt notation, and by using the forward modelling formulae based on the weak-anisotropy approximation. In the performed tests, we use P- and S-wave traveltimes generated in a synthetic VSP (vertical seismic profiling) experiment to recover complete anisotropy of a homogeneous model. The performed tests represent a preparation step for applications of traveltime inversion in inhomogeneous anisotropic media.

Introduction

There are two possible approaches to traveltime inversion in anisotropic media. In the first, we assume that we know the type of the anisotropy of the medium, and we seek parameters specifying this type of anisotropy and angles specifying the orientation of the anisotropy in the space. In the other approach, we seek a complete set of 21 elastic parameters, and from them we estimate the type of the anisotropy, its orientation and its parameters. Here we concentrate on the latter approach.

We specify anisotropic medium by 21 WA parameters. WA parameters represent a generalization of Thomsen (1986) parameters, which were designed for the parameterization of VTI anisotropy. WA parameters can be, however, used for anisotropy of arbitrary symmetry. The WA parameters represent an alternative to elastic parameters in Voigt notation. Despite their name, the WA parameters can be used for the parameterization of anisotropic media of arbitrary anisotropy, strength and orientation. Their use in formulae describing wave propagation in weak-anisotropy approximation has many advantages. As shown below, it leads, for example to a complete separation of formulae describing P-wave propagation from the corresponding S-wave formulae, without need to use the so-called acoustic approximation (Alkhalifah, 2000). For the definition and more details on WA parameters, see, for example, Farra et al., (2016).

For the forward modelling, we use approximate formulae for P- and S-wave ray velocities based on the weak-anisotropy approximation. Weak-anisotropy approximation formulae represent leading terms of the expansion of exact formulae with respect to WA parameters. The performed tests indicate that the formulae provide results of high accuracy even for anisotropy stronger than 20%, as in the example shown below. Original is the treatment of S waves. Instead of dealing with two separate S waves, we consider the so-called common S wave, whose traveltimes correspond to an average of traveltimes of separate S waves. The advantage of this formulation is the simplicity of the S-wave forward modelling formula, and no need for the identification of recorded traveltimes with faster or slower S waves (whose order may differ in different directions). In inhomogeneous media this will lead to a ray-tracing procedure, which will not collapse in vicinities of shear-wave singularities. The above-described inversion scheme allows us to arrange the equations to be inverted into the system of linear equations for the sought WA parameters, with no need for the two-point ray tracing for the determination of traveltime between a source and a receiver.

Method

For the inversion of P-wave traveltimes, we use an approximate formula, which we derived by ignoring the differences between the directions of phase and ray velocities and differences between the squares of both velocities. Růžek and Pšenčík (2016) show that even such a rough approximation is an appropriate tool for the inversion of noisy data. The approximate traveltime formula for P waves (Farra and Pšenčík, 2003) has the following form:

$$r^2/t_P^2 = \widetilde{v_P^2}(\varepsilon_i, \mathbf{N}_P) = \widetilde{c_P^2}(\varepsilon_i, \mathbf{n}_P)$$

$$= \alpha^2 \Big(1 + 2(\varepsilon_x N_1^4 + \varepsilon_y N_2^4 + \varepsilon_z N_3^4 + \delta_x N_2^2 N_3^2 + \delta_y N_1^2 N_3^2 + \delta_z N_1^2 N_2^2)$$

$$+4[(\varepsilon_{15}N_3 + \varepsilon_{16}N_2)N_1^3 + (\varepsilon_{24}N_3 + \varepsilon_{26}N_1)N_2^3 + (\varepsilon_{34}N_2 + \varepsilon_{35}N_1)N_3^3$$

$$+(\chi_x N_1 + \chi_y N_2 + \chi_z N_3)N_1 N_2 N_3]\Big) . \qquad (1)$$

In a similar way, we obtain the approximate traveltime formula for common S waves (Farra and Pšenčík, 2008). It has the following form:

$$r^2/t_S^2 = \widetilde{v_S^2}(\varepsilon_i, \mathbf{N}_S) = \widetilde{c_S^2}(\varepsilon_i, \mathbf{n}_S)$$

$$= \beta^2 \Big(1 + \gamma_x(N_2^2 + N_3^2) + \gamma_y(N_1^2 + N_3^2) + \gamma_z(N_1^2 + N_2^2)$$

$$+\varepsilon_{45}N_1N_2 + \varepsilon_{46}N_1N_3 + \varepsilon_{56}N_2N_3\Big)$$

$$+\alpha^2\Big(\varepsilon_x N_1^2(N_2^2+N_3^2)+\varepsilon_y N_2^2(N_1^2+N_3^2)+\varepsilon_z N_3^2(N_1^2+N_2^2)$$
$$-\delta_x N_2^2N_3^2-\delta_y N_1^2N_3^2-\delta_z N_1^2N_2^2$$
$$+\varepsilon_{15}N_1N_3(1-2N_1^2)+\varepsilon_{16}N_1N_2(1-2N_1^2)+\varepsilon_{24}N_2N_3(1-2N_2^2)$$
$$+\varepsilon_{26}N_1N_2(1-2N_2^2)+\varepsilon_{34}N_2N_3(1-2N_3^2)+\varepsilon_{35}N_1N_3(1-2N_3^2)$$
$$-2\chi_x N_1^2N_2N_3-2\chi_y N_1N_2^2N_3-2\chi_z N_1N_2N_3^2\Big)\ . \qquad (2)$$

Here, r denotes the source-receiver distance, t_P and t_S are the corresponding P- and common S-wave traveltimes, $\widetilde{v_P}$ and $\widetilde{v_S}$ are approximate P- and common S-wave ray velocities, and $\widetilde{c_P}$ and $\widetilde{c_S}$ are approximate P- and common S-wave phase velocities. The velocities depend on the ray (source-receiver) direction, specified by a unit vector $\mathbf{N}$. We assume that the vector $\mathbf{N}$ is parallel to the unit normal $\mathbf{n}$ to the wavefront (vector parallel to the slowness vector). Further, the velocities depend on the parameters of the medium, ε_i, which are the sought quantities in the inversion. For the specification of an arbitrary anisotropic medium, we use 21 WA parameters related to the elastic parameters in the Voigt notation, $A_{\alpha\beta}$, in the following way:

$$\varepsilon_x=\frac{A_{11}-\alpha^2}{2\alpha^2},\quad \delta_x=\frac{A_{23}+2A_{44}-\alpha^2}{\alpha^2},$$
$$\varepsilon_y=\frac{A_{22}-\alpha^2}{2\alpha^2},\quad \delta_y=\frac{A_{13}+2A_{55}-\alpha^2}{\alpha^2},$$
$$\varepsilon_z=\frac{A_{33}-\alpha^2}{2\alpha^2},\quad \delta_z=\frac{A_{12}+2A_{66}-\alpha^2}{\alpha^2},$$
$$\varepsilon_{15}=\frac{A_{15}}{\alpha^2},\quad \varepsilon_{16}=\frac{A_{16}}{\alpha^2},\quad \varepsilon_{24}=\frac{A_{24}}{\alpha^2},$$
$$\varepsilon_{26}=\frac{A_{26}}{\alpha^2},\quad \varepsilon_{34}=\frac{A_{34}}{\alpha^2},\quad \varepsilon_{35}=\frac{A_{35}}{\alpha^2},$$
$$\chi_x=\frac{A_{14}+2A_{56}}{\alpha^2},\quad \chi_y=\frac{A_{25}+2A_{46}}{\alpha^2},\quad \chi_z=\frac{A_{36}+2A_{45}}{\alpha^2}.$$
$$\gamma_x=\frac{A_{44}-\beta^2}{2\beta^2},\quad \gamma_y=\frac{A_{55}-\beta^2}{2\beta^2},\quad \gamma_z=\frac{A_{66}-\beta^2}{2\beta^2},$$
$$\varepsilon_{46}=\frac{A_{46}}{\alpha^2},\quad \varepsilon_{56}=\frac{A_{56}}{\alpha^2},\quad \varepsilon_{45}=\frac{A_{45}}{\beta^2}. \qquad (3)$$

The symbols α and β in equations (1)-(3) denote the P- and S-wave velocities of a reference isotropic medium. They are used for the definition of WA parameters in equation (3). Equations (1) and (2) are, however, *independent* of α and β.

From equation (1), we can see that in the weak-anisotropy approximation, P-wave traveltimes depend on only 15 WA parameters while, as one can see in equation (2), common S-wave traveltimes depend on the complete set of WA parameters. We can also see that each WA parameter in equations (1) and (2) has a different coefficient. This indicates that for a sufficient angular illumination of the medium, all WA parameters can be determined. Because source-receiver distances r, source-receiver directions $\mathbf{N}$ and the velocities α and β are known, equations (1) and (2) represent a set of linear equations for the determination of 15 (when only P-wave traveltimes are available) or 21 (when both P- and common S-wave traveltimes are available) WA parameters. The simplest way how asses sensitivity of WA parameters to the noise in the data is to transform the covariance matrices of right-hand sides of equations (1) and/or (3) into the model covariance matrix. Errors of individual WA parameters are then evaluated as square roots of diagonal elements of the model covariance matrix. In order to solve a system of equations (1) and/or (2), we can thus use a least-square procedure (Press et al., 2007) for their solution.

Examples

We consider a VSP configuration with 4 receivers situated in a borehole at depths of 0.1, 0.4, 0.7 and 1.0 km. Further, we consider 50 sources distributed along 5 profiles on the surface, 10 sources along each profile. The profiles are distributed regularly, with angular step of 72°. The sources are separated by 0.05 km, the closest to the borehole is at the distance of 0.05 km.

For this configuration, we generate exact ray traveltimes using the ANRAY package (Gajewski and Pšenčík, 1990), and impose random Gaussian noise on them. In our studies, we test effects of considering only P-wave traveltimes alone or P- and S-wave traveltimes together, effects of varying noise, etc. We also test chances to recover the type and the orientation of anisotropy symmetry from the estimated WA parameters.

In the following, we present results of one of the above-described experiments. Specifically, we consider only P-wave traveltime data generated in a model of HTI (transversely isotropic with the horizontal axis of symmetry) medium, with axis of symmetry deviated by 45° from the profile 0°. We impose random Gaussian noise of 0.005 s on the exact traveltimes. From the system of equations of the form (1), we then try to estimate 15 P-wave WA parameters. The exact values of the WA parameters used in the generation of exact traveltimes are shown by open circles in Figure 1. The P-wave reference velocity α is chosen equal to 4.22 km/s, to make the WA parameter ε_z zero. The exact WA parameters are also used in equation (1) for the construction of the P-wave phase-velocity surface shown on the left-hand side of Figure2.

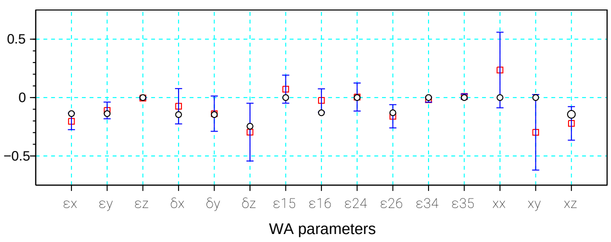

Figure 1: Results of the inversion of traveltimes with 0.005 s noise generated by 50 sources distributed along 5 profiles on the surface of the HTI model, and recorded by 4 receivers in the borehole. Open circles: exact values of WA parameters, red squares: estimated values of WA parameters, error bars are determined as square roots of diagonal elements of the model covariance matrix.

Red squares in Figure 1 denote estimated WA parameters. We can see that all estimated values are situate with error bars (blue). Except WA parameters χ_x and χ_y, the estimated values do not differ much from the exact ones (it is necessary to emphasize that the specific values of estimated parameters depend on the specific realization of noise). The largest errors are connected with WA parameters δ_x, δ_y, δ_z, χ_x, χ_y and χ_z. These parameters depend on elastic parameters in the Voigt notation, see equation (3), which control S-wave propagation. The error bars are expected to reduce when S-wave traveltimes are also considered. It is interesting to note that the estimated values of WA parameters would be closer to exact ones and error bars would be substantially reduced if the sources, were randomly distributed on the surface of the model instead being distributed along 5 profiles, see Růžek and Pšenčík (2016).

In Figure 2, we can compare P-wave phase-velocity surfaces determined from equation (1) by using exact values of WA parameters (left) and estimated WA parameters (right). Although some of the WA parameters are estimated with a significant error, the phase-velocity surfaces in Figure 2 display similar features. The most important are a similar character of axial symmetry in both plots, and a similar orientation of the axis of symmetry.

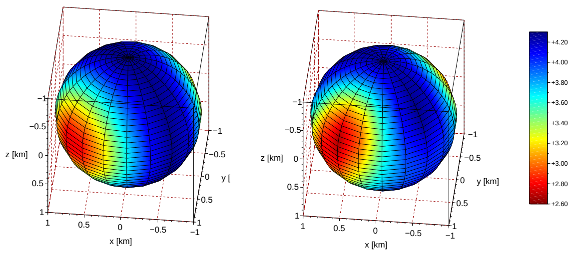

Figure 2: Phase velocity surfaces determined from equation (1) with the use of exact values of P-wave WA parameters (left), and with the use of WA parameters estimated from traveltimes with 0.005 s random Gaussian noise (right).

Conclusions

Although the choice of a homogeneous model in testing the described traveltime inversion scheme is an obvious oversimplification, it brought a series of interesting and important observations.

So far performed tests indicate important advantages of the use of parameterization of the model with WA parameters in comparison with, for example, parameterization with elastic parameters in Voigt notation. All WA parameters are non-dimensional, of a comparable size and retrievable (in inversion formulae, they appear with different coefficients) from the used formulae. In combination with weak-anisotropy approximation, they allow separate treatment of P waves from S waves without need to use the so-called acoustic approximation. As the presented results indicate, even the use of P-wave traveltime data alone offers a possibility to reconstruct the P-wave phase-velocity surface with a reasonable accuracy. An important property of WA parameters, not mentioned above, is their easy transformation from one coordinate system to another, which will play an important role in the traveltime inversion in inhomogeneous media.

The innovative treatment of the two S waves propagating in anisotropic media as a single common S wave also brings advantages. In the described tests in homogeneous media it removes the problem of identification of individual S waves. In inhomogeneous media, its use will be even more important. Tracing common S waves in inhomogeneous anisotropic media is similar to the tracing of P waves. The use of the common S wave concept removes the problems of failures of ray tracing in vicinities of shear-wave singularities.

As mentioned above, the performed tests are, due to the use of a homogeneous medium, oversimplified, and they are thus only useful for analysis of the proposed scheme. The presented procedure can be, however, immediately used for inverting laboratory traveltime (or velocity) measurements on rock samples supposed to be homogeneous. In such an application, the proposed scheme offers an additional advantage. It is the use of the assumption under which equations (1) and (2) were derived, equality of the squares of phase and ray velocities and of their directions. In this way, there is no need to investigate if ray or phase velocity is measured (Dellinger and Vernik, 1994).

Acknowledgments

We are grateful to the project ”Seismic waves in complex 3-D structures” (SW3D) and Research Project 16-05237S of the Grant Agency of the Czech Republic for support.

References

Alkhalifah, T., 2000. An acoustic wave equation for anisotropic media: *Geophysics*, **65**, 1239–1250.

Dellinger, J and L. Vernik, 1994. Do traveltimes in pulse-transmission experiments yield anisotropic group or phase velocities? *Geophysics*, **59**, 1774–1779.

Farra, V. and Pšenčík, I., 2003. Properties of the zero-, first- and higher-order approximations of attributes of elastic waves in weakly anisotropic media. *J.Acoust.Soc.Am.*, **114**, 1366–1378.

Farra, V. and Pšenčík, I., 2008. First-order ray computations of coupled S waves in inhomogeneous weakly anisotropic media. *Geophys.J.Int.*, **173**, 979-989.

Farra, V., Pšenčík, I. and Jílek, P., 2016. Weak-anisotropy moveout approximations for P waves in homogeneous layers of monoclinic or higher anisotropy symmetries: *Geophysics*, **81**, C39–C59.

Gajewski, D. and Pšenčík, I., 1990. Vertical seismic profile synthetics by dynamic ray tracing in laterally varying layered anisotropic structures: *Journal of Geophysical Research*, **95**, 11301–11315.

Press, W.H., Flannery, B.P., Teukolsky, S.A., and Vetterling, W.T., 2007. Numerical recipes. Cambridge University Press, Cambridge.

Růžek, B. and Pšenčík, I., 2016. P-wave VSP traveltime inversion in weakly and moderately anisotropic media: *Seismic Waves in Complex 3-D Structures*, **26**, 17–59, online at “http://sw3d.cz”.

Thomsen, L., 1986. Weak elastic anisotropy: *Geophysics*, **51**, 1954–1966.