1D inversion of electromagnetic logs in the characterization of carbonate formations from Santos basin –Brazil

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Abstract

Geophysical inversion involves the estimation of the parameters of a hypothetical model from a set of observations. The model responses may be nonlinear functions and least squares techniques prove to be useful for performing the inversion. The linearized Marquardt-Levenberg technique was used and the procedure proved to be robust and efficient. The objective of this work was to evaluate a robust scheme to reverse the resistivity data of Induction Log Deep (ILD) and Induction Log Medium (ILM) tools. The characterization of a geological carbonate system, using the inversion, was performed to interpret the apparent resistivity data obtained in the Santos Basin pre-salt well profile. The direct problem lies in the exact mathematical solution of the electromagnetic response of a stratified medium traversed by a well. To validate the interpretation, the geological description was used in conjunction with information from a set of profiles.

Introduction

The inversion of electrical and electromagnetic profile data for the distribution of resistivity along the well is a non-linear and non-exclusive problem. Practical data are by nature inaccurate, inconsistent, and bandwidth limited, and consequently there are an infinite number of models that can satisfy a given set of data. The purpose of the inversion is to determine some model that adequately explains our observations and also satisfies any constraints imposed by the physics of the problem (MEJU, 1992)

A one-dimensional inversion (1D) methodology was used to interpret Induction Log Deep (ILD) and Induction Log Medium (ILM) resistive profiles. This type of interpretation has been widely used to calculate the response of plane-parallel layer models in geophysical methods such as electro-resistivity, induced polarization and magnetotelluric (WU, 1968, apud Carrasco et al 2004). The inverse algorithm is based on the damped least squares technique, also known as ridge-regression, a method that was proposed by Marquardt (1970).

Many advantages are considered when using this inversion method: convergence from initial models with low precision, stability in the presence of geological noise and rapid obtaining of estimates of statistical parameters (PELTON; RIJO; SWIFT, 1978).

A variety of methods were developed to address the problem (Inman, 1975; Johansen, 1977; Meju, 1988). One of the problems that hinders the iterative refinement of the initial model is that the estimated corrections may be unstable, producing unrealistic results. Levenberg (1944) suggested that these undesirable effects can be avoided during the successive application of the Taylor approximations by culling the absolute values of the corrections applied to the initial model. This idea was later used by Marquardt (1963, 1970) and Hoerl and Kennard (1970) to develop a useful nonlinear algorithm; and the technique is known as ridge regression or the Marquardt-Levenberg method (apud MEJU, 1992)

Each regression requires many evaluations of the direct problem, and to achieve speed at a reasonable cost, it is important to reduce the time, optimizing the calculation of the direct problem, something that was previously obtained by Carrasco and Carrasquilla (2003).

Thus, the objective was to aid in the zoning and resistive characterization of carbonate in the Santos Basin. and also investigate the uniqueness in the geological models employed.

Context Geologic

The pre-salt covers a total area of approximately 227,000 km², which is 800 km long and, in certain regions, 200 km wide. The Santos Basin is in this environment, 350 km from the coast (Figure 1). It is bordered to the north by the Cabo Frio High, which separates it from the Campos Basin and to the south by the Florianopolis High, which separates it from the Pelotas Basin (Bruhn et al., 2003).

The geological petroleum system of the Santos Basin is restricted to the subsalt configuration (pre-salt range); it has, as potential petroleum-generating rocks, black shales rich in organic matter, interspersed with carbonates, that were deposited in a paleolacustrine environment (formations Itapema and Piçarras of the Guaratiba Group) and; As reservoirs, carbonates of the Itapema (coquinas) and Barra Velha (microbialites) formations, both of the Guaratiba Group, and may also occur in siliciclastic rocks (Formation Piçarras) and fractured basalts (Camboriú Formation).The migration of hydrocarbons generated in the rift section occurred through direct contact between the generating rocks and the reservoir rocks of the rift section. Nevertheless, the presence of an overlapping layer of salt (Ariri Formation) was likely responsible for an almost perfect sealant for this oil system (Papaterra, 2010).

The models used in this work are based on real data from a well in the Santos Basin, the modeling section is composed of a sequence of geoelectric layers representing the carbonates.

Method

Data

We used open well data (A1) (Figure 2) that perforated the same carbonate reservoir from the pre-salt section of the Santos Basin. The data contain the geophysical well profiles, such as caliper, gamma rays, sonic, resistivity, neutron and density. Also, laboratory data, where plugs and measured values of permeability and porosity were analyzed.

The Forward Problem

The development of the 1D direct algorithm is based on the exact solution of the Green problem for the determination of the electromagnetic field inside a stratified medium, crossed by a well and excited by a well tool composed of a transmitter and a receiver.

Inverse Problem

Using an expansion in Taylor series, a nonlinear equation of the type:

$$\Delta \mathrm{G} = \mathrm{A}\Delta\mathrm{P}, \tag{1}$$

can be linearized in the form:

$$\Delta G_{\mathrm{i}} = \mathrm{G}\,(\mathrm{P},\mathrm{X}^{\mathrm{i}}) - \mathrm{G}\,(\mathrm{P}^{0},\mathrm{X}^{\mathrm{i}}),\ \mathrm{i} = 1,\ \mathrm{N}; \tag{2}$$

thus

$$[A]_{\mathrm{ij}} = \frac{\partial \mathrm{G}(\mathrm{P},X)}{\partial P_{\mathrm{j}}}\Bigg|_{\substack{\mathrm{X}=\mathrm{X}^{\mathrm{I}},\\ \mathrm{P}=\mathrm{P}^{0}}} \tag{3}$$

and

$$\Delta P_{\mathrm{j}} = P_{\mathrm{j}} - P_{j}^{0},\ \mathrm{j} = 1,\ \mathrm{M} \tag{4}$$

The term ΔG represents the difference between the apparent resistivity measured in the well profile and the resistivity calculated by the initial P^0 model. The element of matrix system A_{ij} is the derivative of Equation (1) with respect to the j-th parameter of the model evaluated with P0 in the i-th layer spacing. The vector ΔP is the difference between the parameters of the final adjustment and those of the initial model. Equation (3) is the linear estimate of the required correction in the unknown j-parameter and P is the vector of unknown parameters. Eq. (1) represents a system of linear equation N in unknown M, and since this is a non-linear problem, the solution of Eq. (1) provides only a linear estimate of Δ P. The inverse of least squares is

$$\Delta\ \mathrm{P} = (\mathrm{A}^{\mathrm{T}}A)^{-1}\mathrm{A}^{\mathrm{T}}\Delta\mathrm{G} \tag{5}$$

The solution must be iterated, starting each time from the current estimate P^0 of the parameter set.

The method starts by calculating the eigenvalues of the $\mathrm{A}^{\mathrm{T}}A$ matrix. Small eigenvalues indicate a quasi-unique system, which is unstable in the presence of noisy data. The average difference between $\Delta\mathrm{P}$ estimated and true $\Delta\mathrm{P}$ becomes very large. The ridge regression method (Seber and Wild, 1989) seeks to reduce this difference during the damping iteration process of the diagonal terms of $(\mathrm{A}^{\mathrm{T}}A)$ (Hasnaoui et al, 2003). The ridge regression estimates of ΔP_{rr} is:

$$\Delta P_{\mathrm{rr}} = (\mathrm{A}^{\mathrm{T}}A + \alpha I)^{-1}\mathrm{A}^{\mathrm{T}}\Delta\mathrm{G} \tag{5}$$

Thus, the inversion procedure consists of finding a parameter vector that minimizes ΔG, since Equation (1) was established to linearize the system, and the solution is reached after several iterations. This procedure is repeated until the 'G' vector difference is minimized based on the least squares criterion. The solution is dependent on the initial model. Thus, it is important to use, at the beginning of the inversion process, additional reliable geological information to describe the initial model (INMAN et al., 1973).

a) Damped square minimums

Any mathematical function can be linearized, as described in Equation (1), but there will always be an error ξ in this process, in the form:

$$\Delta\mathrm{G} = \mathrm{A}\ \Delta\mathrm{P} + \xi, \tag{6}$$

where A is the sensitivity matrix which relates the variations in the ΔP parameters to the variations in the ΔG data. When the value of x is minimal, we can obtain ΔP in the least squares direction:

$$\Delta\mathrm{P} = (\mathrm{A}^{\mathrm{T}}\mathrm{A})^{-1}\ \mathrm{A}^{\mathrm{T}}\mathrm{G}. \tag{7}$$

In this expression was multiplied, on both sides of Equation (1), the generalized inversion operator for determined and determined systems (PELTON; RIJO; SWIFT, 1978):

$$\mathrm{H} = (\mathrm{A}^{\mathrm{T}}\mathrm{A})^{-1}\ \mathrm{A}^{\mathrm{T}}. \tag{8}$$

For these cases, matrix A can be reversed quickly. For this reason, multiply both sides of Equation (1) by A^{T}, where 'T' represents the matrix carries. Thus:

$$\mathrm{A}^{\mathrm{T}}\Delta\mathrm{G} = \mathrm{A}^{\mathrm{T}}\mathrm{A}\Delta\mathrm{P}. \tag{9}$$

If the problem were "well-put" and the initial model very close to the final answer, we could simply reverse $\mathrm{A}^{\mathrm{T}}\mathrm{A}$ to get ΔP. However, it is difficult to propose an accurate initial model, and often include parameters that are not accurate. As a result, $\mathrm{A}^{\mathrm{T}}\mathrm{A}$ is approximately singular and, by Equation (3), one can produce a change in the parameters and increment the residuals of the $\Delta\mathrm{G}^{\mathrm{T}}\Delta\mathrm{G}$ square numbers (INMAN, 1975).

Once the initial model is selected, the purpose of the inversion is to minimize the error between the calculated and observed data:

$$\xi = \Delta\mathrm{G}^{\mathrm{T}}\Delta\mathrm{G}. \tag{10}$$

If the problem were exactly linear, only a determination would be necessary. However, most geophysical problems are non-linear and usually require many iterations to obtain a satisfactory solution from an initial model (PETRICK; PELTON; WARD, 1977).

The Marquardt-Levenberg technique, or chain regression technique, was applied by Inman (1975) and Rijo et al. (1977) in the inversion of electric survey data using the traditional method of least squares. The main modification in our approach concerns the minimization method, which also leads to changes in the application of the regression chain technique (Marinho & Lima, 1995; Marinho, 1997).

To stabilize $A^T A$ and avoid divergence, Levenberg, Foster and Marquardt (Inman, 1975) proposed adding a small positive constant 'k' to the elements of the main diagonal before the inversion process, so that Equation (3) becomes:

$$\Delta P^* = (A^T A + k\, I)^{-1} A^T \Delta G \tag{11}$$

where 'I' is the identity matrix. In a linear problem, an optimal value of "k" is the one that provides the best fit in the least squares sense. If the value of "k" is too high, Equation (7) approaches the inverse gradient method, which although stable, converges slowly. If 'k' has a value close to zero, Equation (7) approaches the Newton-Raphson method, which, unlike the previous case, is fast, but diverges. In practice it is advisable to test multiple values of 'k' on a logarithmic scale to minimize the residue of the least squares damped. This technique, which changes the value of 'k' through iterative methods, is known as a 'ridge-regression' (PELTON; PELTON; WARD, 1978, apud Carrasco et al 2004).

The function of the regularization parameter 'k' is to cushion small eigenvalues of $A^T A$', causing instability and, while at the same time producing the smallest effect on the larger eigenvalues associated with well-defined parameters. Thus, any small eigenvalue will be incremented by the constant "k" and the matrix inversion ($A^T A$+ k I) will be more stable. Good results have been obtained using the Marquardt (1970) technique with a high k value (approximately 1) when the initial value is far from the solution, which is the gradient method. A small value of k (values equal to or less than 0.01) is equivalent to include low eigenvalues in the estimator and is used when the estimator is close to the minimum, which is the Newton-Raphson technique. Increasing the value of 'k' is similar to discarding the small eigenvalues and eigenvectors associated with it. Each eigenvector is a linear combination of the original parameters of the model (resistivities and thicknesses). (INMAN et al., 1973, apud Carrasco et al 2004).

Results

In Figure 2 we present the basic set of profiles for well A1. The profiled section is characterized by a thick layer of carbonate ranging from 250m to 275m deep, with a resistivity ranging from 20 to 200 ohm-m.

In Figures 3 and 4 we present data from the ILD and ILM induction profiles respectively for well A1 along with the initial model. The zoning of the initial model, also presented in Figure 4 and 5, was elaborated with the aid of the profiles of gamma rays, porosity and induction, which allowed to define better the thicknesses of the layers. The magnitudes of the resistivities of the layers of this model were chosen considering the average value of each zone. The green lines represent the direct ILD and ILM models, the rectangular lines in blue are the model used and in red the actual data used as a reference for direct modelling.

In Figures 5 and 6 we have the result of the inversion of the modelling of the ILD and ILM induction profiles respectively for well A1, together with the direct model. The green lines represent the direct models ILD and ILM, the lines in cyan are the answers obtained from the inversion and in red the real data. Although carbonate is a very complex rock, good adjustments were obtained in both inversions.

Conclusions

The variety of resistivity records offers opportunities for reservoir characterization. At the same time, they also make consistent interpretation a much more complex task. Without modeling and inversion, the large amount of information inserted within this large number of geophysical records may even be confusing. This large number of profiles is a challenge for professionals in geoelectrical interpretation. In this work, the profile profiles ILD (Induction Log Deep) and ILM (Induction Log Medium) profiles were performed satisfactorily. The answers show that it is possible to reverse with a good adjustment of the values close to the real ones.

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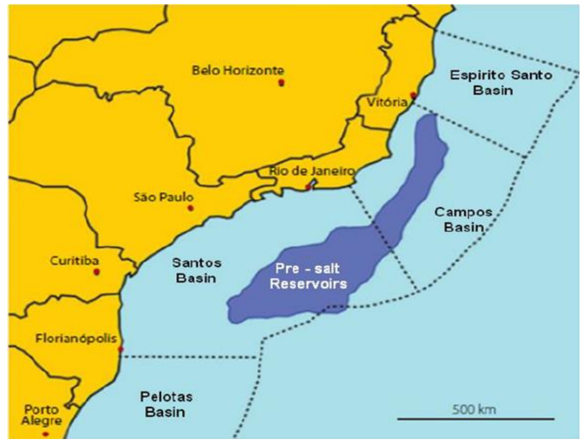

Figura 1 Pre-salt location map in the Santos Basin (Adepted from Riccomini et al., 2012).

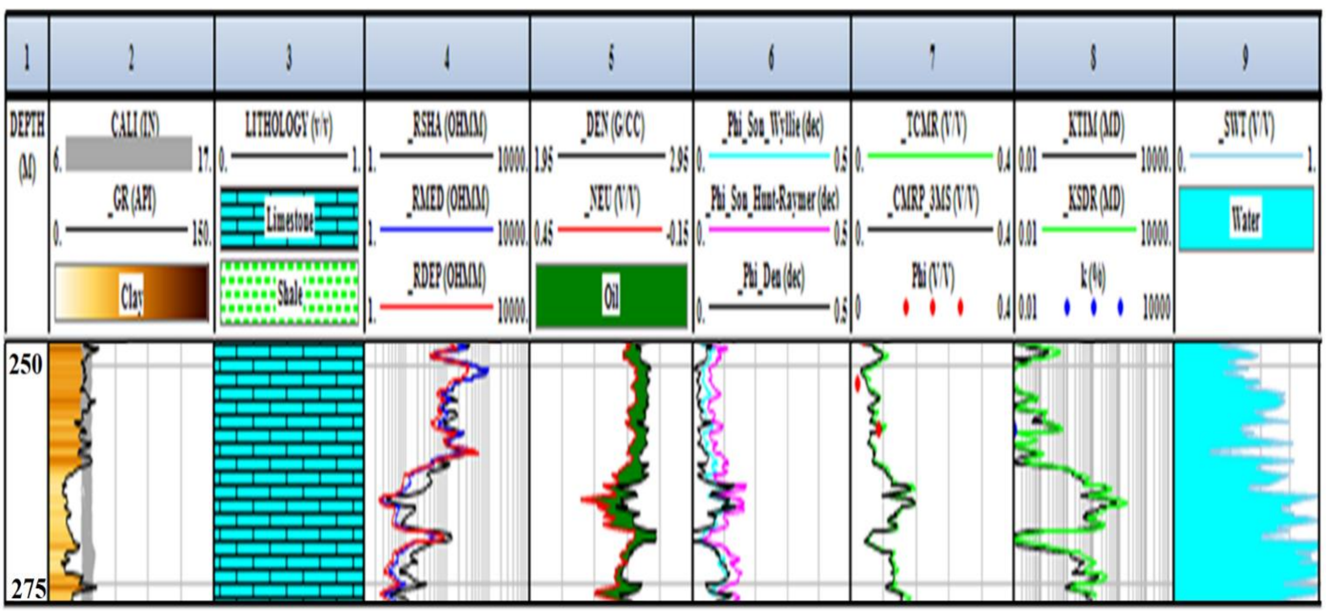

Figura2 Dataset used in the study, consisting of well logs (curves) and porosity and permeability laboratory measurements (red dots).

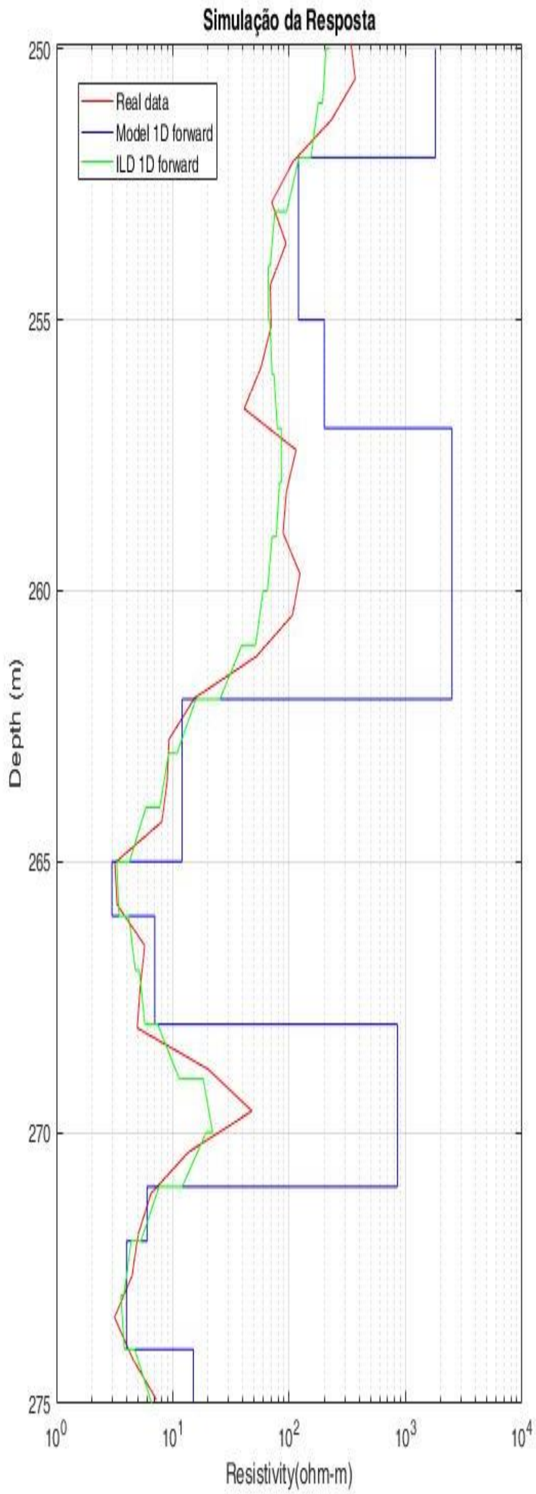

Figura 3 Synthetic logs of ILD (green)

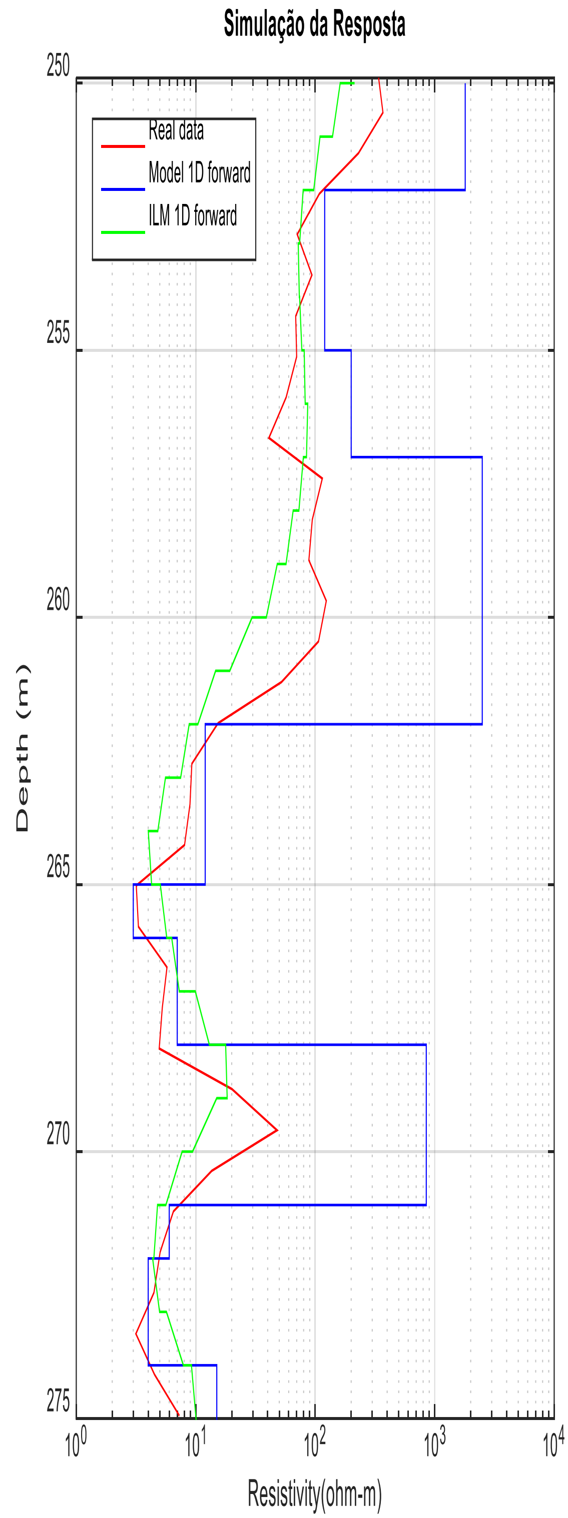

Figura 4 Synthetic logs of ILM (green)

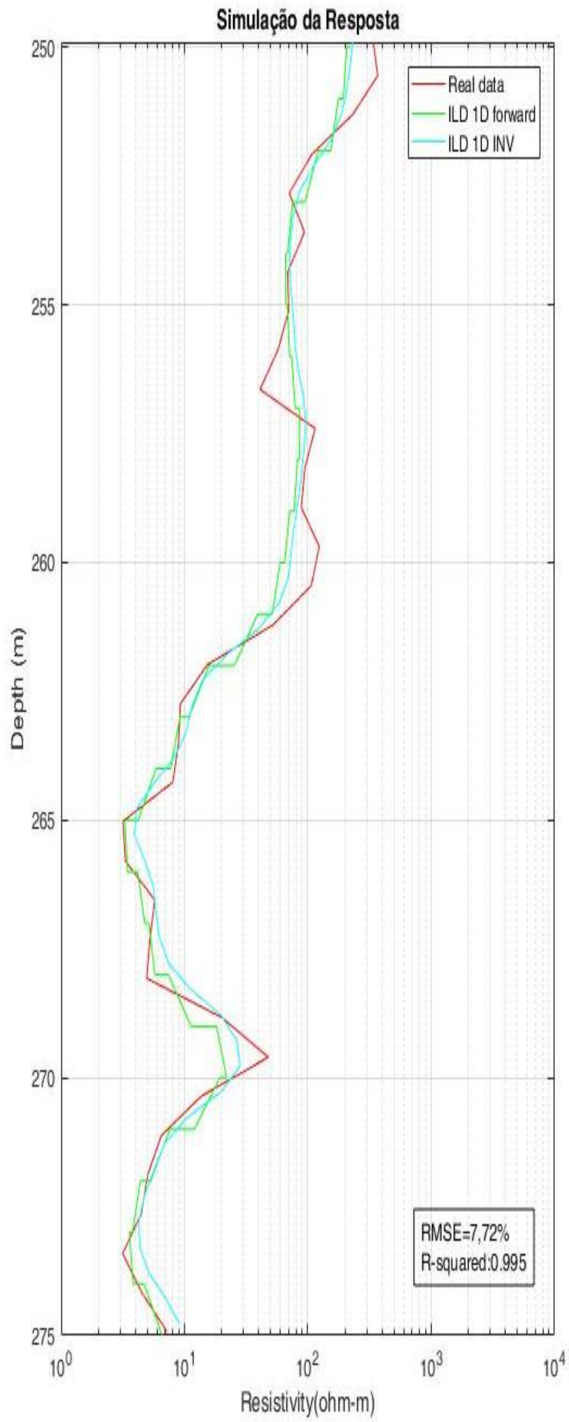

Figura 5 Responses from inversion to log ILD (cyan)

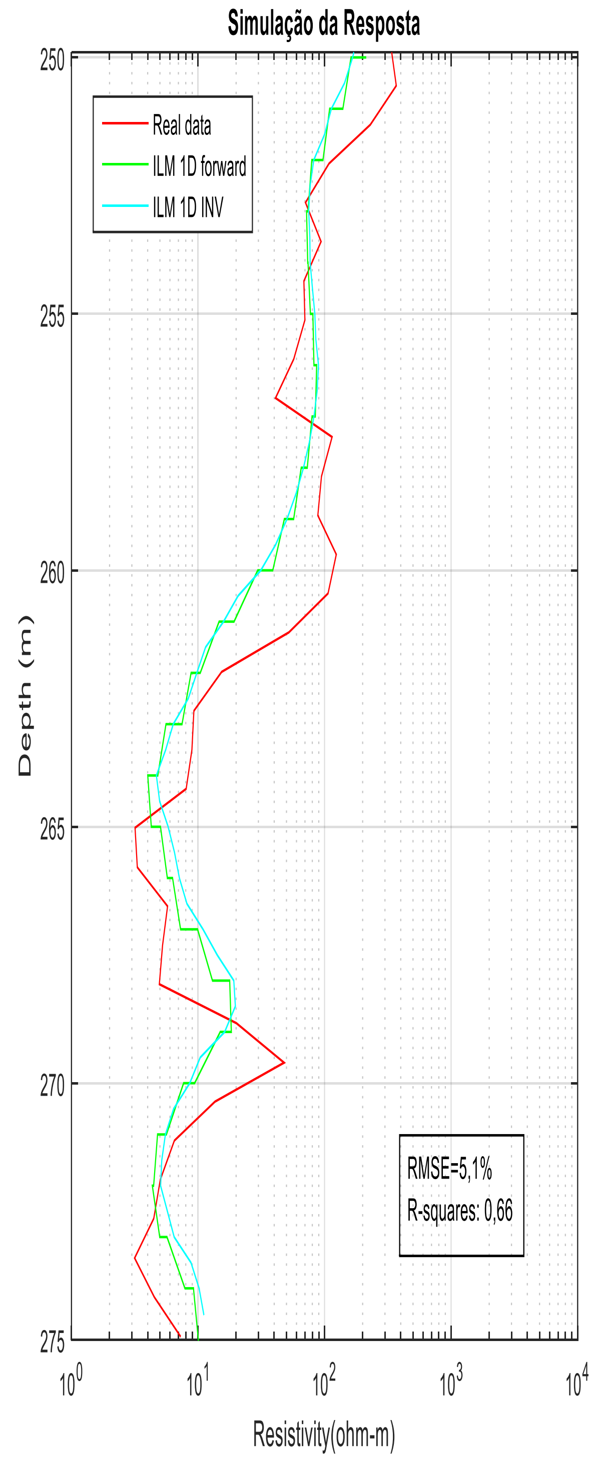

Figura 6 Responses from inversion to log ILD (cyan)