Reduction to the pole in Fourier domain – good and bad filtering of real data in Brazil

Felipe F. Melo* and Valéria C. F. Barbosa, Observatório Nacional

This paper was prepared for presentation during the 16th International Congress of the Brazilian Geophysical Society held in Rio de Janeiro, Brazil, 19-22 August 2019.

Abstract

Reduction to the pole (RTP) is a popular filter that simulates how the magnetic data would be if both the geomagnetic field and the magnetization of the source were vertical. This process locates the horizontal position of the source through the maximum amplitude of the reduced-to-the-pole anomaly; therefore, it assists in the geologic interpretation. To perform properly the RTP is necessary the knowledge of the magnetization directions of the source and the ambient field. However, the magnetization direction of the source is rarely known. Other issue in RTP is its instability at low latitudes; at these locations, the traditional RTP results are useless. We analyze the RTP results of simulated magnetic data at different latitudes, generated by sources with induced and remanent magnetizations. These synthetic tests closely resemble real data from Brazil. Then, we apply the RTP to total-field anomalies in distinct locations from south to north part of Brazil. These locations encompass data from mid to low latitudes: Dom Feliciano Belt, Goiás Alkaline Province and Carajás Mineral Province. The reduced-to-the-pole anomalies of these datasets exemplify good and bad filtering of real data in Brazil.

Introduction

The reduction to the pole (RTP) shifts the maximum of the magnetic anomaly to a position directly over its respective source modifying the shape of anomaly. This process simulates the field as if the anomaly was measured in the magnetic pole and the source had vertical magnetization, and therefore, the interpretative process is simplified. The RTP was introduced by Baranov (1957), formulated in Fourier series by Bhattacharyya (1965) and improved to take advantage of the fast Fourier transform algorithm by Kanasewich and Agarwal (1970). The operation can be viewed as a linear transformation and treated like a filtering operation (Gunn, 1975).

In low latitudes, the RTP is unstable and produces useless results because the filtering operation generates elongated artifacts in the direction of the declination, making the interpretation impracticable. This problem is well known and different solutions exists in both space and Fourier domains. In space domain, Silva (1986) employed the equivalent source technique and treated the operation as an inverse problem and Li et al. (2014) applied a regularization term in the RTP data and used an equivalent layer with positivity as constraint to reduce the undesirable artifacts. In Fourier domain, Hansen and Pawlowski (1989) stabilized the operation with a Wiener filtering technique and Mendonça and Silva (1993) used a truncated Taylor series. Still in Fourier domain, Swain (2000) introduced a pseudo-inclination into the denominator of the filter in order to reduce the amplification of the wavenumbers and Li and Oldenburg (2001) inverted the Fourier transform of the observed magnetic data with explicit regularization. On the other hand, from a practical point of view, MacLeod et al. (2003) used the Total Gradient Analysis (3-D analytical signal) to locate the sources at low magnetic latitudes.

In this work, we simulate total-field magnetic anomalies produced by sources located at high, mid and low latitudes. Notice that, magnetic anomalies simulated in distinct latitudes correlate to magnetic inclinations of the incident field. Then, we apply the RTP to these anomalies and analyze the results for these cases when the sources have induced and remanent magnetizations. These tests show that RTP pass from stable to unstable according to the decreasing of the inclination. Finally, we applied the RTP to three field datasets located from south to north part of Brazil: at the Dom Feliciano Belt (Santa Catarina state), the Goiás Alkaline Province (Goiás state) and Carajás Mineral Province (Pará state). These datasets, which have induced and remanent magnetizations, are located in regions from mid to low latitudes and the results show how real data filtering with standard RTP can produce good and bad results.

Method

In the wavenumber domain and in polar coordinates (Silva, 1986) the RTP filter is defined as

$$RTP(r,\theta) = \frac{1}{[jcos(I)cos(D-\theta)+\sin(I)][jcos(i)cos(d-\theta)+\sin(i)]}, \quad (1)$$

where $j=\sqrt{-1}$, I and D are, respectively, the inclination and declination of the geomagnetic field, i and d are, respectively, the inclination and declination of the source magnetization vector, and $\theta = arctan(k_y/k_x)$ is the wavenumber direction, where k_x and k_y are the wavenumbers in the $x-$ and $y-$directions. On the denominator of equation 1, the first and second terms inside the brackets depend on the magnetization directions of, respectively, the geomagnetic field (I and D) and the magnetized source (i and d). In Fourier domain, we multiply the RTP filter (equation 1) by the magnetic data, and then we transform the filtered data back to space domain. Finally, the interpreter can apply several interpretation methods to the reduced-to-the-pole anomalies or interpret them in a straightforward way.

Equation 1 shows that to perform a correct RTP is necessary the knowledge of the magnetization directions of both the geomagnetic field and the source. Moreover, it shows that, assuming induced magnetization (i.e., $I = i$) and in regions close to the magnetic equator ($I \sim 0°$) the filter is defined as a function of $-cos^{-2}(D-\theta)$. In this case, the filter is unstable and elongated artifacts in the direction of the declination dominate the filter; in fact, these artifacts make the interpretation impracticable.

Synthetic tests

We performed three tests simulating total-field anomalies with null intensity at different latitudes, consequently, with different geomagnetic inclinations. For each test, we simulated three prisms with dimensions of 500 m northing and down, 1000 m easting, top at 500 m and magnetization of 5 A/m (Blakely, 1996; Uieda et al., 2013). The sources are outlined by dashed purple contour before and after filtering. The simulated grid is equally spaced every 100 m, with 120 points northing, 100 points easting and the acquisition flight height is -150 m. In all tests, we performed the RTP assuming the correct magnetization direction of the geomagnetic field. Throughout this work, the Figures are shown using the colour scale by Niccoli (2014).

High-latitude anomalies - In the first test, the geomagnetic field has $I = 60°$ and $D = -50°$, simulating the field at a high latitude. The southernmost source has induced magnetization and the others have remanent magnetization, with $i = -50°$ and $d = 30°$. Figure 1a shows the total-field anomaly and Figure 1b shows the reduced-to-the-pole anomaly by assuming that all sources have induced magnetization. In Figure 1b we notice that the southernmost anomaly is correctly reduced to the pole because the anomaly is predominantly positive and decays softly in all directions. This sound RTP result is because the southernmost anomaly was produced by a source actually having an induced magnetization (i.e., $I = i$ and $D = d$). However, the two other reduced-to-the-pole anomalies in Figure 1b, which were produced by sources with remanent magnetization, exhibit a dipolar shape and elongated artifacts, confirming the failure of the RTP in these anomalies, as expected. In a real scenario, where the magnetization direction of the sources are different to each other, the filter (equation 1) could not be applied on the entire area; hence, it should be applied on piecewise area.

Mid-latitude anomalies - In the second test, the geomagnetic field has $I = -20°$ and $D = -20°$, simulating the field at a mid-latitude. The southernmost source has induced magnetization and the others have remanent magnetization, with $i = 50°$ and $d = 30°$. Figure 1c shows the total-field anomaly and Figure 1d shows the reduced-to-the-pole anomaly by assuming that all sources have induced magnetization. As in the previous test, the southernmost anomaly, which was produced by a source magnetized by induction only, is correctly reduced to the pole because the dipolar characteristic of the observed total-field anomaly (Figure 1c) is completely suppressed (Figure 1d). Like the previous test, the two other reduced-to-the-pole anomalies in Figure 1d, which were produced by sources with remanent magnetization, still exhibit a strong dipolar characteristic, indicating a poor filtering, as expected. The stronger elongated artifacts in Figure 1d are more accentuated than in the previous test (Figure 1b) because the smallest inclination of the geomagnetic field.

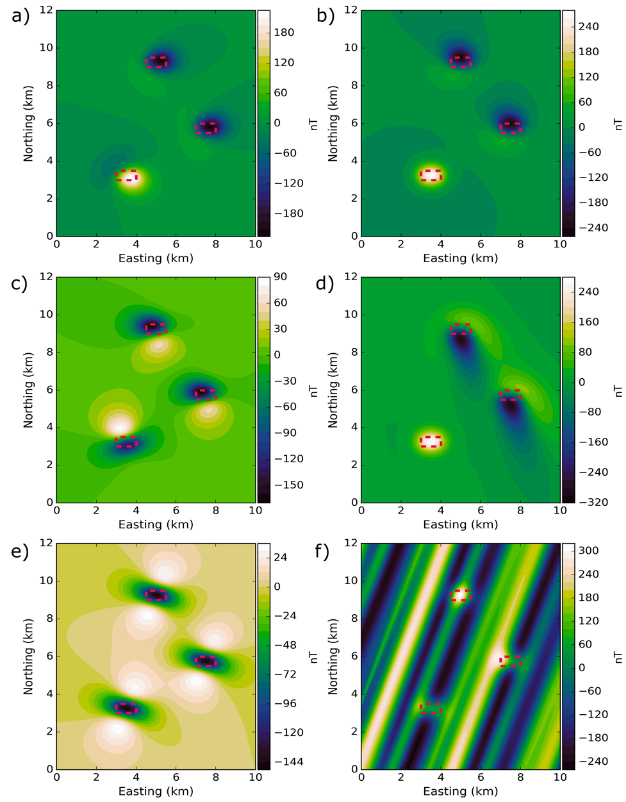

***Figure 1** – Simulated anomalies, where the southernmost source has induced magnetization, at distinct latitudes and the RTP. The dashed purple rectangles outline the sources before and after filtering. a) Total-field anomaly with $I = 60°$, $D = -50°$. b) Reduced-to-the-pole anomaly of (a), the anomaly with induced magnetization was correctly RTP, the others failed and present artifacts. c) Field with $I = -20°$, $D = -20°$. d) RTP of (c), the anomaly with induced magnetization was correctly RTP, the others failed and exhibit straps. e) Field with $I = 1°$, $D = 20°$ and all anomalies with induced magnetization. f) RTP of (e), the elongated artifacts dominate the filtered data in the direction of the field declination.*

Low-latitude anomalies - In the third test, the geomagnetic field has $I = 1°$ and $D = 20°$, simulating the field at a low latitude. In this test, all sources have induced magnetization. Figure 1e shows the total-field anomaly and Figure 1f shows the reduced-to-the-pole anomaly by assuming that all sources have induced magnetization. Regardless of the premise of induced magnetization, the filtering failed because of the low inclination of the simulated geomagnetic field. The most striking feature of Figure 1f is that elongated artifacts in the direction of the declination dominate the filtered data. In this case, the anomalies of the three sources cannot be even correctly located and they are not single peaks. This scenario is worse when RTP is applied to interfering anomalies at low latitudes.

Application to real data set

We applied the RTP in three distinct data sets from Brazil, encompassing areas from mid to low latitudes. Figure 2 shows the three selected airborne surveys, highlighted in yellow, and the areas used in this work, blue dots. These airborne surveys are located from south to north of Brazil: Paraná-Santa Catarina states, Goiás state and Carajás province. The values of magnetic inclination and declination used to perform the RTP were computed from Chulliat et al. (2014), according to the year of acquisition.

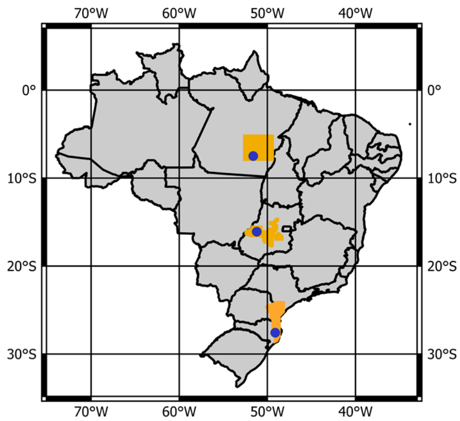

***Figure 2** – Selected airborne surveys in Brazil, highlighted in yellow, and the areas used in this work, blue dots. The airborne surveys are: Paraná-Santa Catarina states (southernmost survey), Goiás state (central survey) and Carajás province (northernmost survey).*

Paraná-Santa Catarina airborne survey

The Dom Feliciano Belt (DFT) is a region in the southern part of Brazil dominated by granitic (the Florianópolis Batholith) and mylonitic rocks (Hartmann et al, 2003; Scheibe et al., 2005). We focus our study in the Anitápolis alkaline-carbonatitic complex, in Santa Catarina state, in the southwest part of the DFT. This circular concentric body, subject to apatite exploitation, intrudes the granitoid rocks of the Santa Catarina basement (Biondi, 2005).

The real aeromagnetic data were acquired between 2009 and 2011 (CPRM, 2011) over São Paulo, Paraná and Santa Catarina states. The flight lines in the north–south direction were acquired every 500 m, the tie lines were acquired every 10 km and the flight height was approximately constant at -100 m. The data set is gridded with the same size in the $x-$ and $y-$directions, 125 m.

Figure 3a shows the total-field anomaly of the intrusion and the inset shows the location of the DFT. Notice that the shape of the anomaly resembles a dipolar anomaly. Figure 3b shows the reduced-to-the-pole anomaly assuming induced magnetization with $I = i = -37°$ and $D = d = -18.2°$. The anomaly was perfectly reduced to the pole, because it is predominantly positive exhibiting a smooth decay to zero. We stress that the dipolar characteristic of the observed total-field anomaly (Figure 3a) is eliminated in the reduced-to-the-pole anomaly (Figure 3b) whose shape resembles to be produced by an isolated vertical intrusion.

Goiás state airborne survey

The Goiás Alkaline Province (GAP) is a region in the central part of Brazil subject to mafic-alkaline magmatism (Marangoni and Mantovani, 2013). The study area is in the northern part of the GAP (Melo et al., 2013; Melo and Barbosa, 2018), this region is characterized by mafic-ultramafic alkaline complexes; these plutonic intrusions present remanent magnetization (Dutra et al., 2014; Marangoni et al., 2016; Zhang et al., 2018).

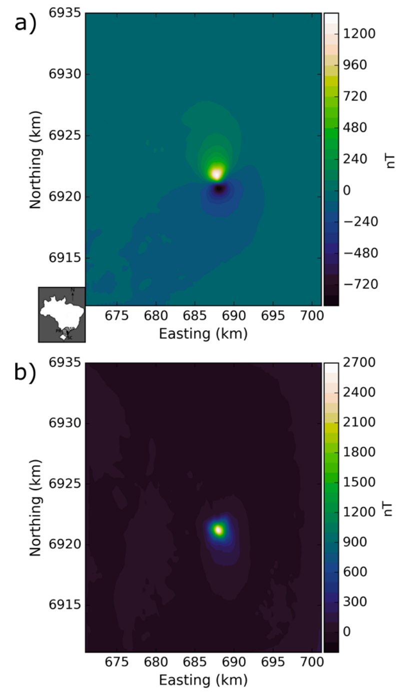

***Figure 3** – a) Anitápolis alkaline-carbonatitic complex at the Dom Feliciano Belt, the inset shows the location. b) Reduced-to-the-pole anomaly of (a) assuming induced magnetization.*

The real aeromagnetic data were acquired in 2004 (CPRM, 2004) over Goiás state. The flight lines in the north–south direction were acquired every 500 m, the tie lines were acquired every 5 km and the flight height was approximately constant at -100 m. The data set is gridded with the same size in the $x-$ and $y-$directions, 125 m.

Figure 4a shows the total-field anomaly of the northern part of GAP and the inset shows the location of the GAP. The numbers indicate the main alkaline intrusions in this region: (1) Montes Claros de Goiás complex, (2) Diorama, (3) Córrego dos Bois complex, (4) Fazenda Buriti complex, and (5) Arenópolis. Diorama intrusion is pointed out by a dashed rectangle because we will further analyze this anomaly separately.

Figure 4b shows the reduced-to-the-pole anomaly of the total-field anomaly shown in Figure 4a assuming induced magnetization with $I = i = -19.5°$ and $D = d = -18.5°$. The RTP failed because the sources have remanent magnetization. Notice that the dipolar characteristic of the observed total-field anomaly (Figure 4a) is not eliminated in the reduced-to-the-pole anomaly (Figure 4b) and elongated artifacts appeared. Then, we used the magnetization direction of the robust estimates from Oliveira et al. (2014) i.e.: $i = -71.4°$ and $d = -23.4°$. Figure 4c shows the reduced-to-the-pole anomaly of the total-field anomaly shown in Figure 4a assuming the remanent magnetization and the results are acceptable because the anomalies are predominantly positives and the dipolar characteristic of the total-field anomaly was almost entirely suppressed. Zhang et al. (2018) performed the RTP in the some anomalies at the same area, Montes Claros de Goiás and Córrego dos Bois complex (labeled 1 and 3 in Figure 4). The reduced-to-the-pole anomaly obtained by Zhang et al. (2018) is comparable to the one shown in Figure 4c, although Zhang et al. (2018) estimated different values of inclination and declination from Oliveira et al. (2014). These results are similar with different magnetization directions due to the ambiguity in potential fields.

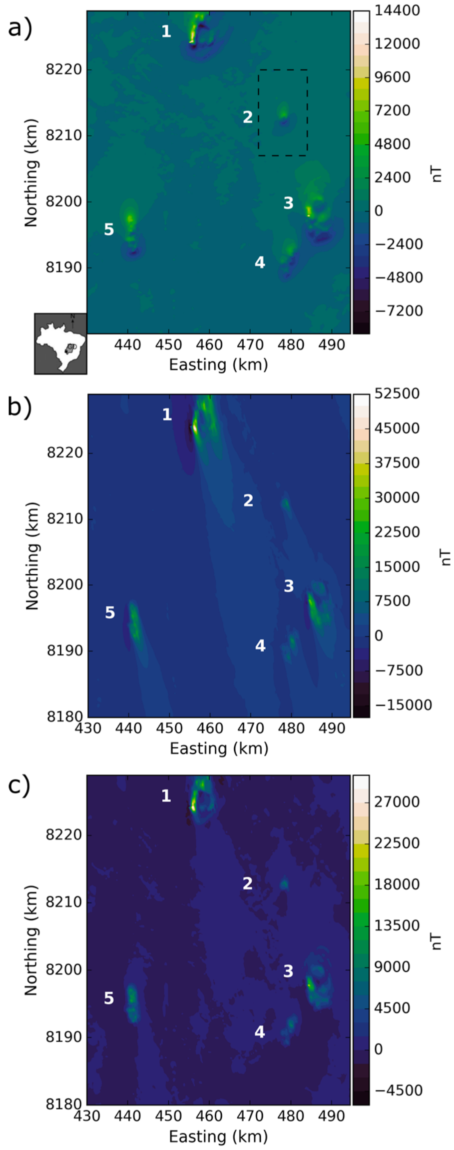

Figure 4 *– a) Total-field anomaly in the northern portion of GAP. The numbers indicate the main alkaline intrusions in this study area: 1, Montes Claros de Goiás complex; 2, Diorama; 3, Córrego dos Bois complex; 4, Fazenda Buriti complex; and 5, Arenópolis. The inset shows the location of GAP b) Reduced-to-the-pole anomaly of (a) assuming induced magnetization. c) Reduced-to-the-pole anomaly of (a) assuming remanent magnetization.*

We outline in our study the Diorama anomaly, labeled 2 in Figure 4, this anomaly is located in Goiás state, in the northern part of GAP. In this area, subvolcanic intrusions, dikes, plugs, and sills of picrite are common (Dutra et al., 2012). Figure 5a shows the total-field anomaly of Diorama anomaly, this area is outlined by the dashed rectangle in Figure 4a. The shape of the total-field anomaly of Diorama (Figure 5a) resembles a strong dipole and a weak dipole in the north part. In fact, according to Melo and Barbosa (2018), the anomaly can be generated by more than one vertical intrusion (plugs). Figure 5b shows the RTP data assuming the remanent magnetization previously used and the results are acceptable because the anomaly is predominantly positive. Actually, the RTP was able to distinguish, and to confirm, at least two sources, a main anomaly as a vertical intrusion and a weaker anomaly in the northern part, as pointed before.

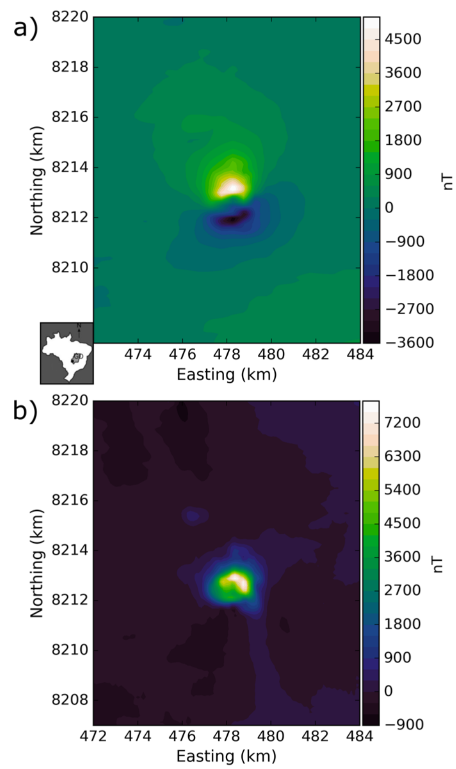

Figure 5 *– a) Diorama anomaly (dashed area in Figure 4a) at the Goiás Alkaline Province, the inset shows the location. b) R*educed-to-the-pole anomaly *of (a) assuming remanent magnetization.*

Carajás airborne survey

The Carajás Mineral Province (CMP) is a highly mineralized metallogenic province in the northern region of Brazil, known for its deposits of gold, copper, iron and manganese, among others (Grainger et al., 2008). We focus our study in the region around the Cristalino copper-gold deposit, which is located in the southeastern part of the CMP, in Pará state; the ores in this deposit are formed by hydrothermal alteration of a volcano-sedimentary sequence (Melo et al., 2017).

The real aeromagnetic data were acquired between 2013 and 2014 (CPRM, 2015) over the Pará state. The flight lines in the north–south direction were acquired every 3000 m, the tie lines were acquired every 12 km and the flight height was approximately constant at 900 m. The data set is gridded with the same size in the $x-$ and $y-$directions, 750 m.

Figure 6a shows the total-field anomaly of the southeastern part of the CMP, the black star shows an approximate location of the Cristalino deposit and the inset shows the location of CMP. Some anomalies with dipolar shape are remarkable in the area. Figure 6b shows the reduced-to-the-pole anomaly assuming induced magnetization with $I = i = -8°$ and $D = d = -20°$. Although Melo et al. (2017) showed the existence of the remanence in the region, we assumed induced magnetization for the RTP. Similarly to the synthetic test shown in Figure 1f, the reduced-to-the-pole anomaly from the southeastern part of the CMP (Figure 6b) failed because of the low inclination of the geomagnetic field. Likewise, the elongated artifacts in the direction of the declination dominate the filtered data (Figure 6b) and, thus, the horizontal locations of the sources are not even correctly inferred by a visual inspection. This happens regardless of the correct values of magnetization direction.

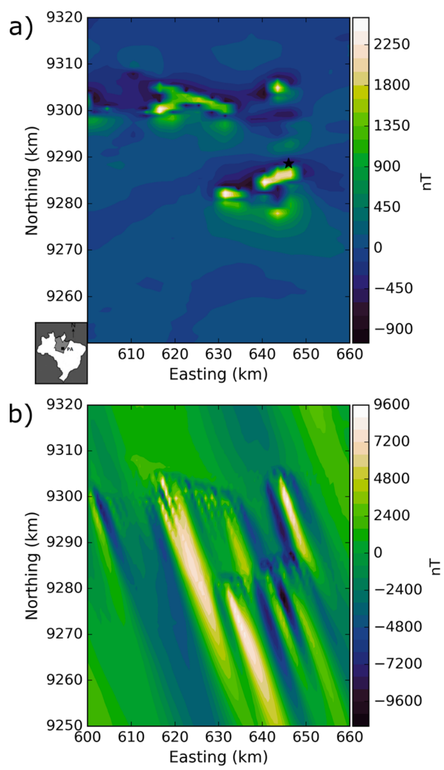

***Figure 6** – a) Total-field anomaly of the southeastern part of the Carajás Mineral Province (CMP). The black star locates the Cristalino deposit and the inset shows the location of the CMP. b) R*educed-to-the-pole anomaly *of (a) assuming induced magnetization. The elongated artifacts dominate the filtered data and RTP failed because of the low inclination of the geomagnetic field.*

Conclusions

The RTP aids qualitative interpretation because it moves the maximum of the magnetic anomaly to a position directly over the center of the causative source modifying the shape of anomaly. The knowledge of the correct magnetization direction of the sources is mandatory to a correct RTP operation. The geomagnetic field at low magnetic inclinations (low latitudes) generates instability in the RTP and produces elongated artifacts along the direction of the magnetic declination. On these cases, the standard RTP is not suitable and another technique with an improvement is necessary. We applied the RTP from datasets in Brazil from south to north, encompassing mid to low latitudes. We successfully performed magnetic interpretation of the two datasets in mid latitudes, however, for the case where the dataset is close to the magnetic equator RTP failed, as expected. This work points the risks and success of RTP filtering in real data from Brazil, a huge country with rocks that have remanent magnetizations and that encompass a broad range of latitudes, and thus, magnetic inclinations.

Acknowledgments

F.F. Melo was supported by a Phd scholarship from FAPERJ (grant E-26/200.532/2018). V.C.F. Barbosa was supported by fellowships from: CNPQ (grant 307135/2014-4) and FAPERJ (grant E-26/203.091/2016). The authors thank M. F. Bitencourt, L. M. Florisbal, S. P. Oliveira and F. J. F. Ferreira for the geological insights. The authors thank CPRM for permission to use the aeromagnetic data set.

References

Baranov, V., 1957, A new method for interpretation of aeromagnetic maps: Pseudo-gravimetric anomalies: Geophysics, 22, 359-383.

Bhattacharyya, B. K., 1965, Two-dimensional harmonic analysis as a tool for magnetic interpretation: Geophysics, 30, 829-857.

Biondi, J. C., 2005, Brazilian mineral deposits associated with alkaline and alkaline-carbonatite complexed, *in* P. Comin-Chiaramonti, C. B. and Gomes, eds., Mesozoic to Cenozoic Alkaline Magmatism in the Brazilian Platform.

Blakely, R. J., 1996, Potential theory in gravity and magnetic applications. Cambridge University Press.

CPRM, 2004, Project: Aerogeophysical survey of Goiás state, first phase: Survey and data processing: Final report, Rio de Janeiro, SGM/MME/CPRM and SIC/SGM/FUNMINERAL/Goiás State, Brazil, vol. 22 and maps (in Portuguese).

CPRM, 2011, Project: Aerogeophysical survey Paraná-Santa Catarina: Survey and data processing: Final report, Rio de Janeiro, Brazil, vol. 22 and maps (in Portuguese).

CPRM, 2015, Project: Aerogephysical survey of Carajás: Survey and data processing: Final report, Rio de Janeiro, Brazil, vol. 22 and maps (in Portuguese).

Chulliat, A., S. Macmillan, P. Alken, C. Beggan, M. Nair, B. Hamilton, A. Woods, V. Ridley, S. Maus, and A. Thomson, 2014, The US/UK World Magnetic Model for 2015–2020: NOAA National Geophysical Data Center.

Dutra, A. C., Y. R. Marangoni, and T. C. Junqueira-Brod, 2012, Investigation of the Goiás Alkaline Province, Central Brazil: Application of gravity and magnetic methods: Journal of South American Earth Sciences, 33, 43–55.

Dutra, A. C., Y. R. Marangoni, and R. I. F. Trindade, 2014, Aeromagnetic and physical-chemical properties of some complexes from Goiás Alkaline Province: Brazilian Journal of Geology, 44, 361–373.

Grainger, C. J., D. I. Groves, F. H. Tallarico, and I. R. Fletcher, 2008, Metallogenesis of the Carajás mineral province, southern Amazon craton, Brazil: Varying styles of Archean through Paleoproterozoic to Neoproterozoic base-and precious-metal mineralisation: Ore Geology Reviews, 33, 451–489.

Gunn, P. J., 1975, Linear transformations of gravity and magnetic fields: Geophysical Prospecting, 23, 300-312.

Hansen, R. O., and R. S. Pawlowski, 1989, Reduction to the pole at low latitude by Wiener filtering: Geophysics, 54, 1607–1613.

Hartmann, L. A., M. F. Bitencourt, J. O. S. Santos, N. J. McNaughton, C. B. Rivera, and L. Betiollo, 2003, Prolonged Paleoproterozoic magmatic participation in the Neoproterozoic Dom Feliciano belt, Santa Catarina, Brazil, based on zircon U-Pb SHRIMP geochronology: Journal of South American Earth Sciences, 16, 477–492.

Kanasewich, E. R., and R. G. Agarwal, 1970, Analysis of combined gravity and magnetic fields in wavenumber domain: Journal of Geophysical Research, 75, 5702-5712.

Li, Y. and D. W. Oldenburg, 2001, Stable reduction to the pole at the magnetic equator: Geophysics, 66, 2, 571-578.

Li, Y., M. Nabighian and D. W. Oldenburg, 2014, Using an equivalent source with positivity for low-latitude reduction to the pole without striation: Geophysics, 79, 6, J81–J90.

Marangoni, Y. R., and M. S. M. Mantovani, 2013, Geophysical signatures of the alkaline intrusions bordering the Paraná Basin: Journal of South American Earth Sciences, 41, 83–98.

Marangoni, Y. R., H. Zhang, and H. J. Ferreira, 2016, Gravity and magnetic integrated data interpretation of the Córrego dos Bois complex, Goiás Alkaline Province, central Brazil: Revista Brasileira de Geofísica, 33, 599–610.

Melo, A. T., J. Sun, and Y. Li, Geophysical inversions applied to 3D geology characterization of an iron oxide copper-gold deposit in Brazil: Geophysics, 82, 5, K1-K13.

Melo, F. F., V. C. F. Barbosa, L. Uieda, V. C. Oliveira, Jr., and J. B. C. Silva, 2013, Estimating the nature and the horizontal and vertical positions of 3D magnetic sources using Euler deconvolution: Geophysics, 78, no. 6, J87–J98.

Melo, F. F., and V. C. Barbosa, 2018, Correct structural index in Euler deconvolution via base-level estimates: Geophysics, 83, 6, J87-J98.

Mendonca, C. A., and J. B. C. Silva, 1993, A stable truncated series approximation of the reduction-to-the-pole operator: Geophysics, 58, 1084–1090.

Niccoli, M., 2014, How to evaluate and compare color maps: The Leading Edge, 33, 910–912.

Oliveira, V. C., Jr., D. P. Sales, V. C. F. Barbosa, and L. Uieda, 2015, Estimation of the total magnetization direction of approximately spherical bodies: Nonlinear Processes in Geophysics, 22, 215–232.

Scheibe, L. F., S. M. A. Furtado, P. Comin-Chiaramonti and C. B. Gomes, 2005, Cretaceous Alkaline Magmatism from Santa Catarina state, southern Brazil, *in* P. Comin-Chiaramonti, C. B. and Gomes, eds., Mesozoic to Cenozoic Alkaline Magmatism in the Brazilian Platform.

Silva, J. B. C., 1986, Reduction to the pole as an inverse problem and its application to low-latitude anomalies: Geophysics, 51, 369-382.

Swain, C.J., 2000. Reduction-to-the-pole of regional magnetic data with variable field direction, and its stabilisation at low inclinations. Exploration Geophysics 31, 78–83.

Uieda, L., V. C. Oilveira Jr., and V. C. F. Barbosa, 2013, Modeling the Earth with fatiando a terra. In: Proceedings of the 12th Python in Science Conference (SciPy 2013), pp. 96-103.

Zhang, H., D. Ravat, Y. R. Marangoni, G. Chen, and X. Hu, 2018, Improved total magnetization direction determination by correlation of the normalized source strength derivative and the reduced-to-pole fields: Geophysics, 83, 6, J75-J85.