Regularization for FWI with Geological Incoherence Index

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Abstract

The present work advances in the use of the Full Waveform Inversion (FWI) for generation of images based on the complete wave equation. In particular, the use of the Geological Inconsistency Index - GII was analyzed as a constraint in the process of regularization of the inverse problem. Computational experiments presented in this work show that the use of GII leads the solution to a more likely solution, with geological meaning providing more realistic images of the subsurface compared to the classic FWI and the FWI with Total Variation Regularization.

Introduction

The Full Waveform Inversion (FWI) is a well-established technique that uses the complete wave equation to estimate acoustic subsurface parameters, providing high-resolution images. The first work that introduced the FWI concept was developed by Lailly and Bednar (1983) followed by Tarantola (1984) in the beginning of the 1980s. In these works, the authors developed the classical FWI formulation, seeking to solve the inverse problem in the time domain. While many theories concerning FWI have been developed from the 1980s to 1990s, large-scale FWI employment in the oil and gas industry has only been put into the production line in the last 20 years.

In general terms, the objective of a FWI algorithm is to find the best values of properties that, using wave propagation modeling, generate a synthetic seismogram close to the seismogram recorded in the survey. However, it is known from the literature that FWI is an ill-posed problem, which fails in the criteria of uniqueness and solution stability (Aster et al., 2005; Fichtner, 2010; Virieux and Operto, 2009).

To stabilize the process of solving ill-posed problems, current works have been developed in the FWI area using regularization (Dai et al., 2017; Maharramov et al., 2016; Asnaashari et al., 2013), and it is in this context that the present work was developed. We use the Geological Incoherence Index (GII), proposed by Santos (2012), as a constraint for the inverse problem.

The use of GII in the regularization of the inverse problem seeks to encompass geological features, which are neglected in conventional FWI. GII regularization makes the problem less ill-posed and leads to more stable solutions.

Method

The FWI imaging problem is a data fit problem in which an initial model of parameters χ is iteratively updated using some optimization process and converges to a model hopefully close to the geological model of the subsurface of the area where the seismic survey was performed to provide the observed data.

It means that the goal is to find a parameter vector χ^* that minimizes the functional $F(\chi)$, given by the difference between the seismic data observed in the field and the data obtained with the computational simulation.

In the present work, the Finite Difference Method (FDM) is used to obtain the numerical solution of the forward problem, the computacional of synthetic seismograms. The wave propagation is modeled by a generalization of the Helmholtz equation, given by:

$$\frac{\omega}{\kappa(x)}u(x,\omega) + \nabla.\left(\frac{1}{\rho}\nabla u(x,\omega)\right) = s(x,\omega), \qquad (1)$$

where $u(x,\omega)$ and $s(x,\omega)$ are the components of the pressure field and source term respectively, in position x and in frequency ω, κ is the stiffness coefficient and ρ is the density.

To solve the inverse problem, in this paper, the technique known as multiscale is used. The use of this strategy assumes that the objective function for lower frequencies is smoother and consequently has few local minima. In this approach, the seismic data is divided into frequency bands, and the inversion process is used to update the model for each band, in ascending magnituder of frequencies.

For the minimization of the objective function, we use the L-BFGS method, which is a limited memory version of the BFGS method, and has been applied to solve problems with a large number of variables, such as FWI, (Nocedal and Wright, 2006).

To stabilize the inversion process, we propose the use of GII a regularization technique for the inverse problem. The GII was proposed by Santos (2012) and is used to

quantify the consistency between the interfaces of the layers and the velocity gradient of the model. To compute the GII it is necessary to define the velocity gradient vector:

$$\nabla v = \left(\frac{\partial v}{\partial x}, \frac{\partial v}{\partial z}\right), \tag{2}$$

and the angle θ_v that the gradient vector makes with the horizontal direction at each node of the numerical model:

$$\theta_v = arctan\left(\frac{\frac{\partial v}{\partial z}}{\frac{\partial v}{\partial x}}\right), \tag{3}$$

If we consider all points of the domain, it is possible to compute the field θ_v. In addition, it is necessary to calculate the angle θ_s between the normal vector at each point of the interface and the horizontal. Once these angles are calculated, their values are extrapolated along their respective layers. Thus, at the end of the process the field θ_s is supposed to be very similar to the field θ_v, for a model with low GII.

From these premises, Santos (2012) has defined the GII as being:

$$GII = \frac{\sum_{i=1}^{n}|\theta_{v_i} - \theta_{s\_i}|}{n}, \tag{4}$$

where n is the number of nodes of the numerical model.

The proposal is to incorporate the GII into the functional as a regularization term, as follows:

$$f(\chi) = \left|\left|F(\chi)\right|\right|_2^2 + \alpha_n GII, \tag{5}$$

where α_n is a regularization parameter.

Another type of regularization well known in the literature, and that will be used in this work to generate reference results, is the Total Variation Regularization (TV). The main advantage of the TV Regularization is its ability of reducing noise by preserving important features in an image, like interfaces (Alemie, 2017). Rudin et al. (1992) point out that TV regularization consists of a change in the objective function that can be described as:

$$f(\chi) = \left|\left|F(\chi)\right|\right|_2^2 + \alpha\left|\left|\chi\right|\right|_{TV}, \tag{6}$$

where the term corresponding to the TV regularization is defined by:

$$\left|\left|\chi\right|\right|_{TV} = \sum_{i=1}^{Nx}\sum_{j=1}^{Nz}\sqrt{\left|(\nabla_x\chi)_{i,j}\right|^2 + \left|(\nabla_z\chi)_{i,j}\right|^2 + \epsilon}, \tag{7}$$

where $(\nabla_x\chi)_{i,j} = \chi_{i+1,j} - \chi_{i,j}$, $(\nabla_z\chi)_{i,j} = \chi_{i,j+1} - \chi_{i,j}$ and ϵ is a constant added to ensure that $\left|\left|\chi\right|\right|_{TV}$ is differentiable.

Results

To demonstrate the performance of the proposed methodology, two velocity models are used in the experiments: the modified Amoco model and the modified Marmousi model. All models are discretized with a regular mesh of finite differences with spacing of 12 meters with 767×293 nodes.

The observed data to be inverted were generated synthetically using 188 sources spaced uniformly by a distance of 48 m between each of them. In addition, 382 receivers were spaced by a distance of 24 m from each other. The non-reflexive boundaries are implemented by a combination of the strategies presented by Reynolds (1978) and Berenger (1994).

To simulate the presence of noise in the observed data, up to 30% of Gaussian noise was inserted in the seismograms used in the experiments.

For the inversion, the frequencies 6 Hz, 8 Hz, 10 Hz, 12 Hz and 14 Hz were used, and the stop criterion for each frequency was the maximum number of iterations (10 iteration per frequency) or 10 consecutive fails in reduction of the objective function at the same iteration.

The relative error used to evaluate the results is defined as:

$$E = \frac{1}{n_x n_z}\sum_{i=1}^{n_x}\sum_{j=1}^{n_z}|\chi_{i,j} - \overline{\chi_{i,j}}|, \tag{8}$$

where $\chi_{i,j}$ and $\overline{\chi_{i,j}}$ are the velocity values at the node (i,j) of the inverted and exact models, respectively.

We also present comparative results of what we define as vertical profile average error, in which the differences between the velocities of the recovered model and the exact model for each one of the 767 vertical profiles of the model are presented.

Figure 1 presents exact models (Amoco and Marmousi), initial velocity models, inversion results obtained with GII Regularization and vertical profiles of average error and the relative error.

As can be seen in Figures 1 (g) and 1 (h), the results obtained with the use of GII regularization presented smaller errors, compared to FWI without regularization and FWI with Total Variation Regularization. This improvement in the results with the use of GII is more
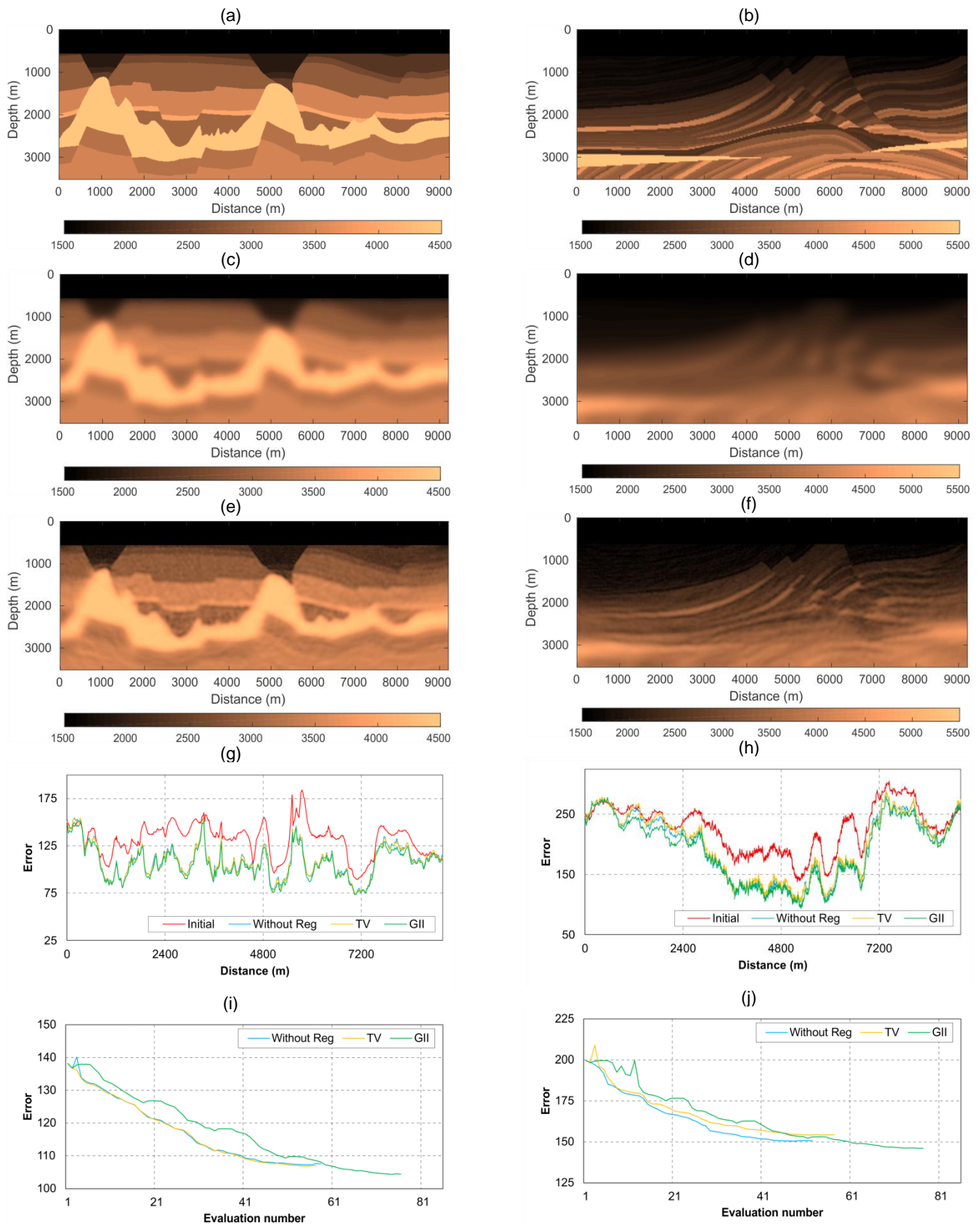
Figura 1 – Exact models (a,b), initial velocity models (c,d), inversion results obtained with GII Regularization (e,f), vertical profile average error (g,h) and vertical profile relative errors (I,j).

noticeable mainly in the regions farthest from the center of the model, with low illumination.

Figures 1 (I,j) show the relative errors of the models during the objective function evaluations in the inversion. It is noted that the use of GII as a regularization term presents better results than the other methodologies presented here, but the computational cost of this approach is somewhat higher, because GII regularization requires more evaluations of the objective function during the inversion.

Conclusions

In this work, we evaluated the use of the Geological Incoherence Index - GII, proposed by Santos (2012), as a FWI regularization, in order to make the problem less ill-posed and more stable. The use of this methodology sought to encompass geological characteristics, which are neglected in conventional FWI.

Although GII regularization requires more evaluations of the objective function during the inversion process, the use of GII has presented better results than the other strategies in the numerical experiments performed with Amoco and Marmousi models.

Acknowledgments

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