The effect of preconditioning on the LSRTM

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Abstract

Least-squares reverse time migration (LSRTM) aims to improve the quality of seismic images by fitting the reflection wavefield with a linearized inverse scattering model, using conjugate gradient (CG) iterations. Due its high computational cost, it is important to accelerate the convergence rate in LSRTM. With this objective we investigate a preconditioner for LSRTM inspired by the asymptotic inverse scattering expression for RTM. The proposed preconditioner can significantly reduce in the number of CG iterations in LSRTM than the more conventional laplacian preconditioner.

Introduction

The least-squares migration method (LSM) is recognized as an important process for estimating earth's reflectivity, then producing images with a better quality than those produced with conventional migration (Zhang et al., 2014). The LSM theory was first derived by Nemeth et al. (1999) for application in Kirchhoff migration, then, with the development of computational power, applications of LSM to reverse-time migration were developed. Nowadays, reverse-time migration is the state-of-the-art imaging method for complex subsurface structures, and when it is applied as an inverse least-squares inverse problem, it can reduce not only the acquisition footprint but also the artifacts in the RTM image, resulting in an image with enhanced resolution (Dai and Schuster, 2013).

In the least-squares inversion, model updates are calculated through minimizing the objective function. Therefore, a mandatory intermediary step in the LSM method is the evaluation of the gradient of the objective function. However, the gradient formed by cross-correlation suffers from geometrical spreading effects, which results in poor amplitudes for deep reflectors and a slow convergence rate (Huang et al., 2016). Xu and Sacchi (2017) addressed this issue applying a preconditioning to the objective function which compensated for the source wavefield energy, they obtained an acceleration in convergence and a better spacial resolution. Huang et al. (2016) used an approximation of the Hessian matrix as a preconditioning operator to compensate geometrical spreading effects and improve the performance of the least-squares inversion.

In this study, we analyze the effects which different preconditioning might have on the least-squares inversion results. First, we analyzed the influence of using the conventional Laplacian operator as a preconditioning, for it is well known its property of enhancing high-frequency signal, while attenuating low-frequency noise. Secondly, we studied an illumination compensate preconditioner which was inspired on the inverse asymptotic of the Born modeling derived by Op't Root et al. (2012).

Method

Least-squares migration assumes one can estimate the subsurface reflectivity from the recorded reflected wavefield considering only single scattering in an otherwise smooth background media. For the acoustic wave equation,

$$\frac{1}{c(\mathbf{x})^2}\frac{\partial^2 p(\mathbf{x},t;\mathbf{x}_s)}{\partial t^2} - \nabla^2 p(\mathbf{x},t;\mathbf{x}_s) = s(t;\mathbf{x}_s)\,, \qquad (1)$$

the single scattering, or Born approximation, assumes that the velocity model, $c(\mathbf{x})$, can be separated into a smooth background component, $c_0(\mathbf{x})$, and a perturbation component, $\delta c(\mathbf{x})$, i.e.,

$$c(\mathbf{x}) = c_0(\mathbf{x}) + \delta c(\mathbf{x})\,. \qquad (2)$$

Correspondingly, the pressure field, $p(\mathbf{x},t;\mathbf{x}_s)$, can be decomposed as

$$p(\mathbf{x},t;\mathbf{x}_s) = p_0(\mathbf{x},t;\mathbf{x}_s) + \delta p(\mathbf{x},t;\mathbf{x}_s)\,, \qquad (3)$$

in which $p_0(\mathbf{x},t;\mathbf{x}_s)$ is the wavefield propagating in background velocity model, which therefore obeys

$$\frac{1}{c_0(\mathbf{x})^2}\frac{\partial^2 p_0(\mathbf{x},t;\mathbf{x}_s)}{\partial t^2} - \nabla^2 p_0(\mathbf{x},t;\mathbf{x}_s) = s(t;\mathbf{x}_s)\,. \qquad (4)$$

for a source pulse $s(t;\mathbf{x}_s)$ injected at position $\mathbf{x}_s$.

Substituting the expressions 2 and 3 in equation 1, and keeping only first-order terms, we obtain the linearized wave equation:

$$\frac{1}{c_o(\mathbf{x})^2}\frac{\partial^2 \delta p(\mathbf{x},t;\mathbf{x}_s)}{\partial t^2} - \nabla^2 \delta p(\mathbf{x},t;\mathbf{x}_s) = \frac{r(\mathbf{x})}{c_o^2(\mathbf{x})}\frac{\partial^2 p_o(\mathbf{x},t;\mathbf{x}_s)}{\partial t^2}\,, \qquad (5)$$

where $r(\mathbf{x}) = 2\delta c(\mathbf{x})/c_o(\mathbf{x})$ represents the reflectivity.

Formally, the finite difference solution of equation (5) can be cast in matrix notation as

$$\mathbf{Lm} = \mathbf{d}\,, \qquad (6)$$

where $\mathbf{L}$ stands for the Born modeling operator, $\mathbf{m}$ is the reflectivity model and $\mathbf{d}$ represents the single scattered wavefield $\delta p(\mathbf{x},t;\mathbf{x_s})$, or Born data, used to fit the recorded reflection wavefield.

The LSRTM consists of solving the linearized least-square problem

$$\underset{\mathbf{m}}{\operatorname{argmin}} \left\|\mathbf{Lm}-\mathbf{d}^{\mathrm{obs}}\right\|_2^2 \tag{7}$$

where $\mathbf{d}^{\mathrm{obs}}$ is the observed seismic reflection data. This problem is solved using conjugate gradient (CG) iterations. In order to implement these iteration we need the gradient objective function (7), which corresponds to the migration operation (Claerbout, 1992). Using the adjoint-state method (Plessix, 2006) we can compute the gradient at each CG iteration by first solving the adjoint equation backward in time,

$$\frac{1}{c_o(\mathbf{x})^2}\frac{\partial^2 q(\mathbf{x},t;\mathbf{x}_s)}{\partial t^2} - \nabla^2 q(\mathbf{x},t;\mathbf{x}_s) = e\left(\mathbf{x}_g,t;\mathbf{x}_s\right), \tag{8}$$

where $q(\mathbf{x},t;\mathbf{x}_s)$ is the adjoint wavefield and $e\left(\mathbf{x}_g,t;\mathbf{x}_s\right)$ represents the misfit at each iteration. Having computed the adjoint wavefield the gradient of the objective function can be computed

$$\mathbf{L}^T\mathbf{e} = \sum_{\mathbf{x}_s}\sum_{t} \nabla^2 p_0(\mathbf{x},t;\mathbf{x}_s)\cdot q(\mathbf{x},t;\mathbf{x}_s), \tag{9}$$

Preconditioning

The convergence of conjugate gradient method may suffer from acquisition footprints and low wavenumber artifacts in the gradient of the objective function. In order to improve the conditioning of linear iteration we can use preconditioning by model reparameterization (Claerbout and Fomel, 2008),

$$\mathbf{m} = \mathbf{Pa} \tag{10}$$

where $\mathbf{P}$ represents a preconditioning operator and $\mathbf{a}$ the model reparameterization. Ideally, operator $\mathbf{P}$ should be designed to filter the low wavenumbers artifacts, accelerate the convergence rate of CG iterations and improve the quality of the reflectivity model. In order to filter low wavenumber artifacts, operator $\mathbf{P}$ can not be positive and in order to warrant CG convergence we need to add regularization term to $\mathbf{a}$, resulting in the following regularized least-squares linear system:

$$\begin{pmatrix} \mathbf{LP} \\ \mu\mathbf{I} \end{pmatrix}\mathbf{a} = \begin{pmatrix} \mathbf{d} \\ \mathbf{0} \end{pmatrix}, \tag{11}$$

where μ is the regularization parameter and $\mathbf{I}$ is the identity matrix.

We investigate the effect of two types of preconditioners in LSRTM. The first preconditioner is a 2D Laplacian operator, which has the property of filtering the low wavenumber components (Guitton et al., 2006). In this case, the equation (10) takes the form

$$\mathbf{m} = \left[\nabla^2\right]\mathbf{a}. \tag{12}$$

We propose a second preconditioner which was inspired by the inverse asymptotic of the Born modeling operator derived by Op't Root et al. (2012). The operator applies the Laplacian operator and compensates for the source illumination. In this case $\mathbf{P}$ takes the form:

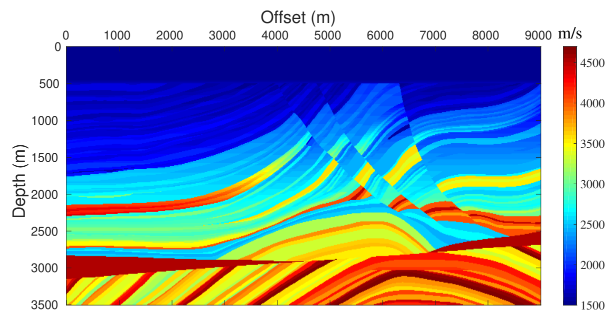
$$\mathbf{m} = \left[\frac{\nabla^2}{\mathscr{I}}\right]\mathbf{a}, \tag{13}$$

where $\mathscr{I}$ is the source wavefield illumination.

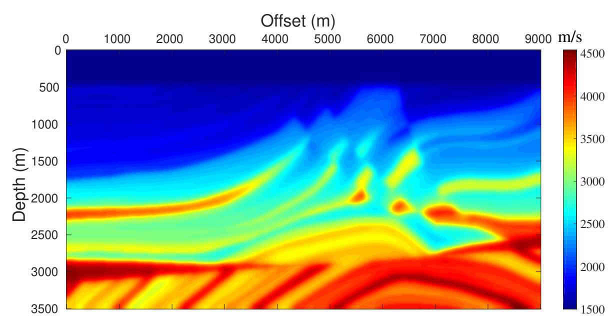
Numerical Examples

The synthetic data was obtained using a portion of the Marmousi2 model (Martin et al., 2002). The original velocity model was modified to be the size of $9km$ x $3.5km$ with a $10m$ grid interval (Figure 1a). The smoothed background velocity model, $c_o(\mathbf{x})$, appears in Figure 1b. In the experiments, 240 shots were evenly distributed on the surface, the first and last source positions are, respectively, $3000m$ and $8975m$, and the shot interval was $25m$. For each shotgather, the minimal offset was $200m$ and maximum offset was $2575m$, with 96 receivers spread evenly with a distance of $25m$ from one another. The point source injection and point receiver records were implemented by using a band limited function, which gives accurate measurements in arbitrary position of the grid (Hicks, 2002).

The data were generated using the finite difference method with a PML boundary condition. As in the LSRTM inversion we are only interested on the primaries reflection data, the input data were calculated as the difference of the synthetic data generated using the true velocity model (Figure 1a) and the data generated using the smoothed velocity model (Figure 1b). The total record length is 3.5 s with a 0.004 s sampling rate.



(a)



(b)

Figure 1: *(a) True velocity model. (b) Smoothed background velocity model.*

Based on the true reflectivity model (Figure 2a) we analyze the numerical experiments results: 1) conventional RTM with Laplacian filter, 2) LSRTM using Laplacian preconditioning after 25 iterations, 3) LSRTM using illumination compensation preconditioning after 25 iterations.

Figure 2b shows the RTM result after applying a Laplacian filter, it is observed low wavenumber artifacts in the shallow parts, specifically in the region above $1500m$, besides the relative amplitudes are not well recovered in the deep parts of the model.

By comparing with RTM image, LSRTM using Laplacian preconditioning (Figure 2c) suppresses the low wavenumber artifacts in the shallow region of the model, enhances the image resolution, and additionally, it attenuates the relative amplitudes of shallow reflectors, which is more consistent with the true reflectivity model. Likewise, LSRTM using illumination compensation preconditioning (Figure 2d) reduces the low wavenumber artifacts and improves image resolution, it produces better balanced amplitudes for deep parts of the model, since relative amplitudes in this region are increased.

Comparing the two LSRTM results (Figure 2c and 2d), we clearly note that the two preconditioning yields different reflectivity amplitudes. The desired effect of the illumination preconditioning on LSRTM is to preserve relative amplitudes in the whole model. However, this was only achieved for deep portions, which is due to the compensation of source illumination.

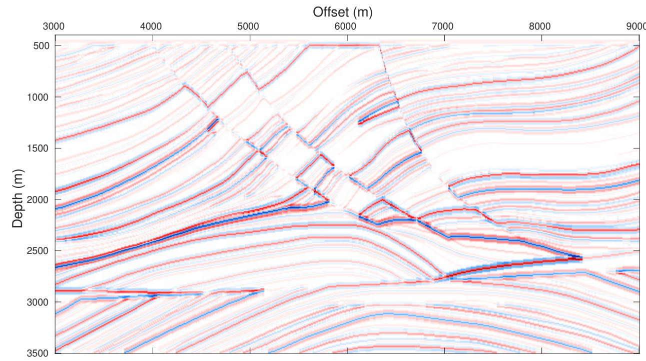
In order to evaluate the convergence of LSRTM experiments, we use the normalized data residual. The convergence curves for the two preconditioner operator are shown in Figure 3. We note a significant difference between the convergence rates. The illumination compensation preconditioning presents better convergence, after 25 iterations its data misfit was reduced to approximately 1.5%. While for Laplacian preconditioning, after 25 iterations, the misfit was reduced to approximately 55%.

In the above LSRTM experiments we used a regularization parameter $\mu = 0.1$. Once the use of illumination preconditioning accelerates the convergence, we also studied the influence that μ value has on the convergence rate for this particular preconditioner. Figure 4 depicts the convergence curves for $\mu = 0.1$ (blue curve), and $\mu = 0.01$ (red curve). It is observed that regularization term has a significant influence on the convergence rate. The inversion using $\mu = 0.1$ presents better convergence, as mentioned, its misfit was reduced to approximately 1.5%. Whereas for $\mu = 0.01$, after 25 iterations, the misfit was reduced to approximately 40%.

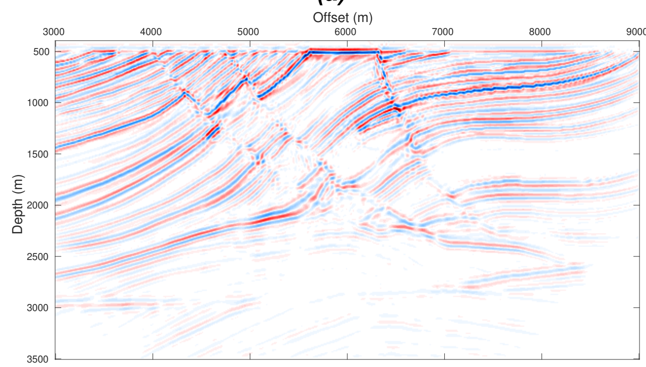
Conclusions

In this work we investigated the effect of the preconditioning on the LSRTM. We studied a preconditioner operator inspired by the asymptotic inverse scattering and the conventional Laplacian preconditioner. We also compare the LSRTM results with conventional RTM image.

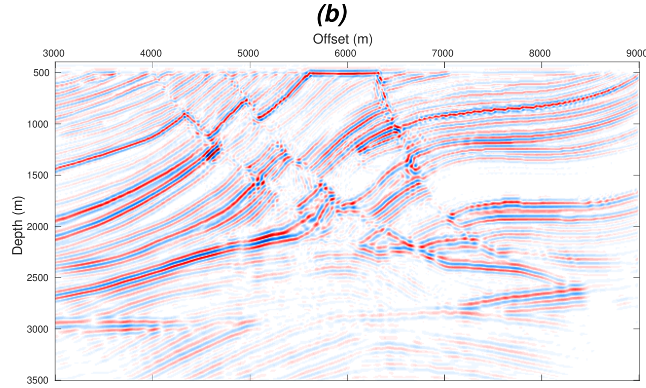
The experiments demonstrated that, for both



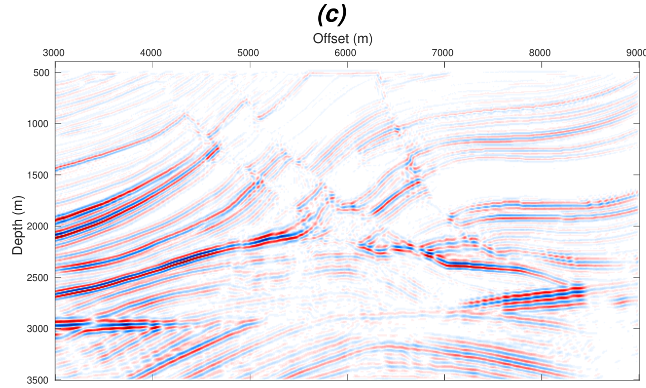
(a)



(b)



(c)



(d)

Figure 2: *(a) True reflectivity model for the Marmousi2 model, (b) RTM result using Laplacian filtering, (c) LSRTM result with Laplacian preconditioner after 25 iterations, (d) LSRTM result with illumination compensation preconditioner after 25 iterations.*

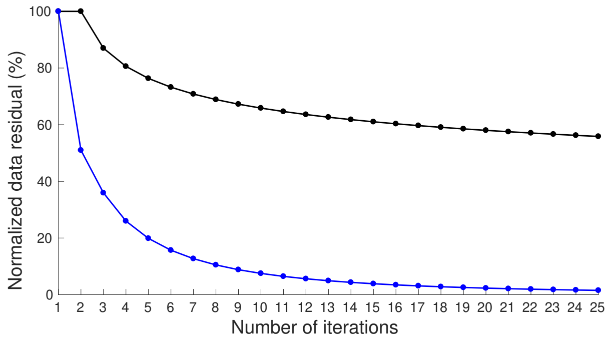


Figure 3: *Convergence curves for the LSRTM with Laplacian preconditioning (black curve) and illumination compensation preconditioning (blue curve). For these experiments we used a regularization parameter $\mu = 0.1$. The data residual were normalized by the initial value.*

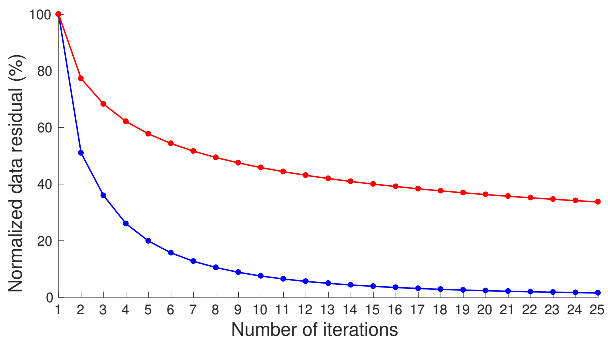


Figure 4: *Convergence curves for the LSRTM with illumination compensation preconditioning using different values of regularization parameters. The blue curve is for $\mu = 0.1$, and the red curve is for $\mu = 0.01$. The data residual were normalized by the initial value.*

preconditioning, the LSRTM improves the image quality in comparison with conventional RTM, since it produced images with better resolution and reduced low wavenumber artifacts.

The two preconditioning studied do not effectively equalize the amplitudes when compared with the true reflectivity model, which indicates that for the proposed preconditioner the effect of illumination compensation needs to be better analyzed to further improve the amplitudes in the whole model.

The illumination compensation preconditioner can accelerate significantly the convergence rate of LSRTM, in comparison with the Laplacian preconditioner. Furthermore, we also observed that the regularization parameter can influence considerably in the convergence. The computational cost of an iteration using the two investigated preconditioning is essentially the same of a ordinary LSRTM. Nonetheless, the use of the proposed preconditioner can reduce the number of CG iterations. Therefore, the cost of LSRTM can be reduced considerably.

Acknowledgments

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