The concept of model rays in domain conversion

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Abstract

In this work we present the concept of model rays which can be seen as the dual of image rays, but traced in time domain. We present the system of first order differential equations which permit to numerically compute the model rays. Among other geometrical interpretations, the model rays can be seen as the instrument to perform the time-to-depth conversion of seismic images.

Introduction

Time migration based methods are still widely used in the industry not only because they are more robust and faster than depth domain ones, but also because shifts in technology paradigms take longer to happen in the oil industry. The conversion of seismic domains from time to depth, has been possible after the introduction of the concept of image ray (Hubral, 1977), which validity is subject to the same conditions of validity of time migration methods. Subsequently, the use of image rays was extended to the conversion of seismic images (Silva et al 2009, Filpo et al 2016) and time migration velocity models (Silva et al, 2009; Cameron et al 2007 and Iversen et al 2008).

In general, time-to-depth conversion algorithms based on image ray tracing, or equivalently based on image wavefront construction, involve mapping of values distributed along a regular grid in the time domain into another regular grid in the depth domain (Figure 1). This mapping is constructed with rays traced in depth, using velocity information in depth or either in time domain. However, no matter which approach is taken, mapping domains using image rays results in an irregular distribution of points in the depth domain. This is far for desirable because multiplicity or voids of such rays can happen, causing difficulties in the numeric interpolation.

Filpo et al (2016) presented a solution to that problem of spreading properties in the time-depth conversion with image rays that is the use of the Taylor series. Such a strategy became feasible from the formulation of time-to-depth conversion as a change of coordinates problem, where vertical lines in the time domain are mapped to image rays at depth, and the horizontal lines on originally flat wave that emerge from the datum. The required derivatives to apply the Taylor series are obtained by finite differences along the wavefronts.

The same does not happen with the use of image rays in the conversion from depth to time, where the regular grid of output is directly filled with values collected in the depth domain through a simple interpolation. Simply speaking, algorithms involving collection of values are generally much simpler than those involving scattering

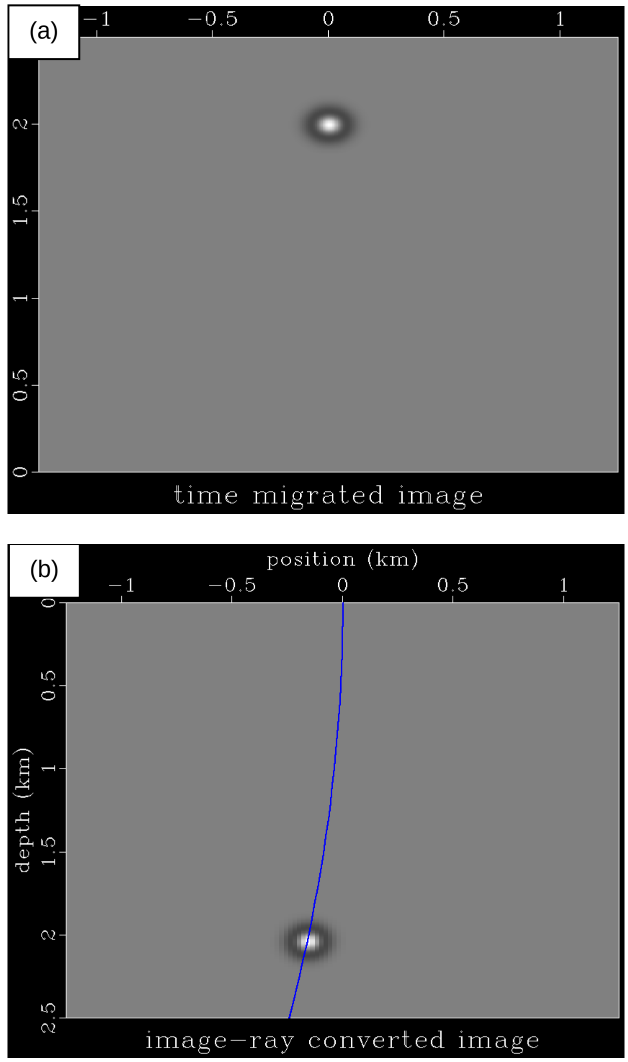
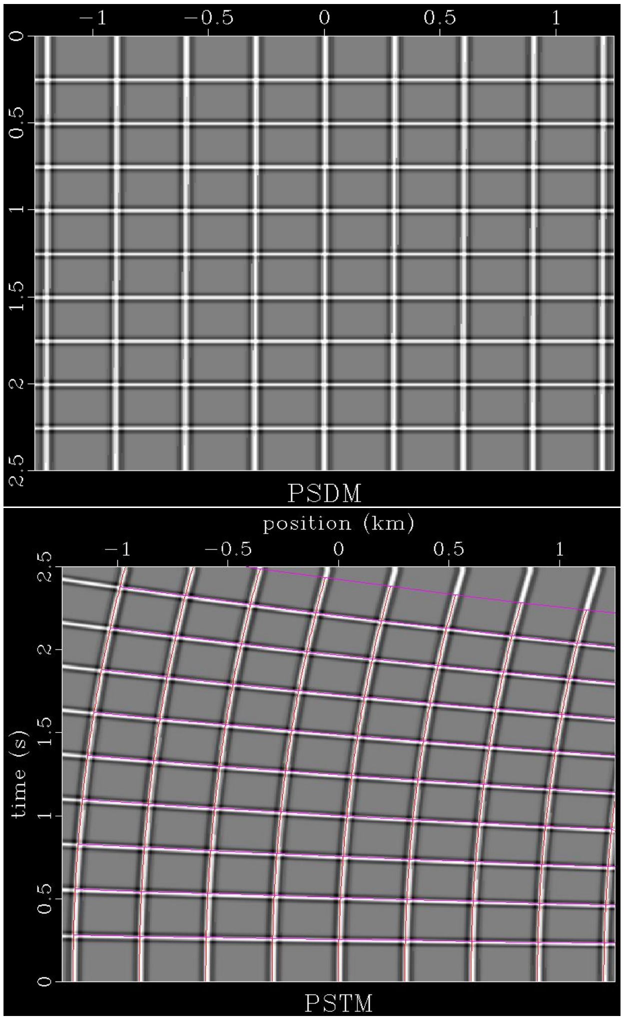


Figure 2:

Figures 2-a and 2-b show an example where a time image containing only a circular feature centered at a given point is converted to depth through this algorithm.

As we saw earlier, the inverse transformation from depth to time can be easily performed from gathering amplitudes along image rays. The concept of model rays arose from the observation of results of this transformation.

Figure 3a shows an image in depth domain with horizontal and vertical events, while Figure 3b shows the same converted to time domain. The observation of the converted section clearly shows that the vertical lines in depth are transformed into curves in the time domain with a behavior very similar to the image rays.



The search for equations to trace these curves led us to find

$$\left(\frac{\partial z}{\partial \xi}\right)^2 + \frac{1}{v^2}\left(\frac{\partial z}{\partial \tau}\right)^2 = 1 \qquad (1)$$

which is the first-order non-linear partial differential equation that governs the wavefronts propagation in time domain. In this equation ξ is the CMP coordinate, τ is time, v is velocity and $z = z(\xi, \tau)$ is the wavefront in time domain. This equation is of the eikonal type and the ray theory says that the rays are characteristic curves of an eikonal equation. This is why led us to name the curves as model rays.

The first step is to write the eikonal equation (1) in the form of a Hamiltonian:

$$H(\xi, q) = \frac{1}{2}\left[q_\xi^2 + \frac{1}{v^2} q_\tau^2 - 1\right] \qquad (2)$$

where H is the Hamiltonian, $\boldsymbol{q} = (q_\xi, q_\tau)$ is the slowness vector, $\boldsymbol{\xi} = (\xi, \tau)$ is the position vector in time domain.

Following Cerveny (2001), given any Hamiltonian, a solution can be found using the method of characteristics, which are the solutions of the system of first-order differential equation

$$\begin{cases} \dfrac{dx}{ds} = \nabla_p H \\ \dfrac{dp}{ds} = -\nabla_x H \\ \dfrac{dt}{ds} = p^T \nabla_p H \end{cases} \qquad (3)$$

where H is a real valued function, called the Hamiltonian, $\boldsymbol{p} = (p_x, p_y)$ is the slowness vector, $\boldsymbol{x} = (x, y)$ is the position vector in time domain, t is the traveltime and s is an integration parameter. Therefore, the model ray equations are

$$\begin{cases} \dfrac{dq_\xi}{dz} = \dfrac{1}{v^3}\dfrac{\partial v}{\partial \xi} q_\xi^2 \\ \dfrac{dq_\tau}{dz} = \dfrac{1}{v^3}\dfrac{\partial v}{\partial \tau} q_\tau^2 \\ \dfrac{dt}{dz} = \dfrac{1}{v^2} q_\tau \\ \dfrac{d\xi}{dz} = q_\xi \end{cases} \qquad (4)$$

where t is the traveltime and z (depth) is the integration parameter.

Although the concept of image ray has been designed to solve the problem of time-depth conversion, its dual, the model ray, is more suitable because for each point in depth domain there is only one model ray in time domain. In this way the amplitude in time can be uniquely transferred to converted position in depth.

In this case, time-depth conversion is summarized in the mapping of a distributed property in a Cartesian grid in the time domain in a curvilinear grid in the depth domain, where

Conclusions

We showed the system of differential equations that can be used to trace model rays.

Acknowledgments

The authors would like to thank Petrobras and Halliburton for the support.

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