Least Squares Migration: Improvements in Quantitative Analysis

Bruno Pereira Dias*, Cláudio Guerra, André Bulcão & Roberto de Melo Dias (PETROBRAS)

This paper was prepared for presentation during the 16th International Congress of the Brazilian Geophysical Society held in Rio de Janeiro, Brazil, 19-22 August 2019.

Abstract

This work shows applications of the image domain least-squares migration with the concept of the point spread functions, including pre-stack and post-stack strategies and using both Reverse Time Migration (RTM) and Kirchhoff migration engines. We present two applications in 3D datasets of deepwater offshore of Brazil. Both strategies showed that least-squares migration provide an uplift in the migration amplitudes and resolution even on geologically complex models. Quantitative analysis shows the large impact that LSM can provide for 3D seismic (for exploratory objective and reservoir characterization.

Introduction

A vision for least-squares migration (LSM) has been stated by Albert Tarantola more than 30 years ago: “Imaging will not be based on principles, but on well-posed questions about the properties of the Earth’s interior” (Tarantola, 1986). By posing the right question, one understands the migration problem as an inversion problem and its solution has more correct migration amplitudes and better image resolution even on geologically complex models.

One of the earliest applications of LSM was realized by Nemeth, Wu and Schuster (1999) in the data domain. Later, Hu, Schuster and Vasalek (2001) formulated the image domain LSM, also named as migration deconvolution. In their work, the Hessian was interpreted with the concept of the Point Spread Function (PSF), ubiquitously used in the image processing, medical imaging and astronomy community to retrieve images with higher resolution. Nowadays, the image least-squares migration has been applied in large size 3D imaging projects (Fletcher et al., 2012 and Letki et al. 2015).

LSM is considered an advanced imaging tool, potentially boosting the signal quality of pre-salt to correct distortions in poorly illuminated areas, related to wave propagation in complex overburden and incomplete geometry of acquisiton (Nemeth *et al.*, 1999 and Hu *et al.*, 2001).

Method

LSM may be formulated as an inversion problem in which the objective is to find the reflectivity (or the image), $\boldsymbol{r}$, which best explains the observed data according to a least-squares objective function $E(\boldsymbol{r})$:

$$\boldsymbol{r} = argmin\, E(\boldsymbol{r}) = \frac{\mathbf{1}}{\mathbf{2}}\|\boldsymbol{u}(\boldsymbol{r}) - \boldsymbol{d}\|^{\mathbf{2}}, \quad (1)$$

where $\boldsymbol{d}$ is the observed data, $\boldsymbol{u}$ is the computed data synthesized by Born modeling ($\boldsymbol{J}$) according to

$$\boldsymbol{u}(\vec{\boldsymbol{x}}_{\boldsymbol{s}}, \vec{\boldsymbol{x}}_{\boldsymbol{r}}, \boldsymbol{h}) = \int_{\boldsymbol{\Omega}} \boldsymbol{G}_{\boldsymbol{s}}(\vec{\boldsymbol{x}}_{\boldsymbol{s}}, \vec{\boldsymbol{x}} - \boldsymbol{h})\, \boldsymbol{r}(\vec{\boldsymbol{x}}, \boldsymbol{h})\, \boldsymbol{G}_{\boldsymbol{r}}(\vec{\boldsymbol{x}}_{\boldsymbol{r}}, \vec{\boldsymbol{x}} + \boldsymbol{h}) \boldsymbol{d\Omega}, \quad (2)$$

in which, subscripts *s* and *r* denote source and receiver, respectively, G corresponds to Green´s functions, *h* is a model extension parameter, and $\vec{x}$ is a position vector (in the model space if written without subscript). Eq. 2 can be written in a more compact form as

$$\boldsymbol{u} = \boldsymbol{J}\boldsymbol{r}. \quad (3)$$

Considering Eq. 3, the least-squares solution of Eq. 1 is:

$$\boldsymbol{r} = \left(\boldsymbol{J}^{\boldsymbol{T}}\boldsymbol{J}\right)^{-\mathbf{1}}\boldsymbol{J}^{\boldsymbol{T}}\boldsymbol{d}. \quad (4)$$

In Eq. 4, one can recognize the conventional migrated image $\boldsymbol{\nabla E} = \boldsymbol{J}^T\boldsymbol{d} = \boldsymbol{m}$, where $\boldsymbol{J}^T$ is the adjoint of the Born modeling and the Hessian $\boldsymbol{H} = \boldsymbol{J}^T\boldsymbol{J}$, which is the second derivatives of $E(\boldsymbol{r})$. Notice that computing the Hessian involves modeling and migration operators. Therefore, from Eq. 3, LSM can be recast in the image domain, which reads

$$\boldsymbol{Hr} = \boldsymbol{m} \quad \Rightarrow \quad \boldsymbol{r} = \boldsymbol{H}^{-\mathbf{1}}\boldsymbol{m}. \quad (5)$$

A remarkable difference between the traditional migration and the LSM is the presence of the Hessian. The Hessian is a spatially variant operator, which encodes:

- Illumination from the source and the receiver wavefield due to incomplete acquisition and velocity model variations.
- Resolution associated with the band-limited nature of the seismic signal.

- Wavelet signature employed on the migration and modeling.

Neglecting these effects can plague the final quality and reliability of the seismic images delivered to the interpreters.

The explicit computation and storage of the Hessian (as a matrix, for example), however, is computationally infeasible, since the number of its elements is the square of the number of parameters in the model. This is the reason why it is imperative to interpret the Hessian as an operator. In addition, for the sake of decreasing the computational effort, it is important to take advantage of the Hessian structure (Valenciano et al, 2009), which is diagonal dominant and almost locally invariant.

In the formulation of the image-domain LSM with PSFs, one evaluates the Hessian in predefined points of the model by the Born modeling of unit scattering followed by a traditional migration. A PSF is the output of this scheme and describes the response of a seismic acquisition/imaging system to a point reflectivity. More importantly, PSFs explicitly encode the illumination and blurring caused by this system. The computational cost to obtain one grid of PSFs is equal to a direct modeling and a traditional migration. Figure 1 indicates what is necessary for the evaluation of the PSF: (1) to setup a unit scattering grid, balancing compactness and interference from neighboring PSFs, (2) to estimate the source signature (wavelet), and (3) to provide a velocity model for the Born modeling and migration. In addition, acquisition geometry should be provided in order to assess the effects caused by an incomplete acquisition.

The PSFs, or equivalently, the operator $\boldsymbol{H}$ acts as a multidimensional spatially variant convolution-like operator. Ideally, this operation on the true reflectivity mimics the migrated image, m:

$$m(\vec{x}) = \int_{\Omega} PSF(\vec{x},\vec{x}\,')\, r(\vec{x}')\, d\Omega \ , \qquad (6)$$

This concept can be further generalized to pre-stack image gathers, which can be extended either to the surface offset domain (h), subsurface angle domain (θ) (or any akin domain):

$$m(\vec{x},h) = \int_{\Omega} PSF(\vec{x},\vec{x}',h)\, r(\vec{x}',h)\, d\Omega$$

$$or \ \ m(\vec{x},\theta) = \int_{\Omega} PSF(\vec{x},\vec{x}',\theta)\, r(\vec{x}',\theta)\, d\Omega, \qquad (7)$$

After retrieving the PSFs around a neighborhood of the original points and considering them as a good approximation of the Hessian, we apply an iterative optimization algorithm to reduce the difference between $\boldsymbol{Hr}$ and $\boldsymbol{m}$. A traditional approach for image domain LSM is to consider the L_2-norm objective function (Valenciano et al., 2009), aiming at retrieving the reflectivity $\boldsymbol{r}$, such that,

$$\boldsymbol{r} = argmin\ E(\boldsymbol{r}) = \frac{1}{2}\left\|C_d^{-\frac{1}{2}}[\boldsymbol{Hr} - \boldsymbol{m}]\right\|^2 \qquad (8)$$

That is, $\boldsymbol{r}$ is the reflectivity which minimizes the L_2-norm of the difference between the migrated image, m, and the reflectivity response to the Hessian or its approximation using the PSFs. The term $C_d^{-1/2}$ is a preconditioner to the inversion problem, and one popular choice is to use the pseudo-Hessian (Shin et. al, 2001), which corresponds to the compensation for the illumination by the source wavefield.

A computationally affordable solution to the problem (8) is to apply an iterative algorithm, such as the steepest-descent scheme:

$$r_{n+1} = r_n - \alpha C_d^{-1} \boldsymbol{H^T}(\boldsymbol{H} r_n - m) \ , \qquad (9)$$

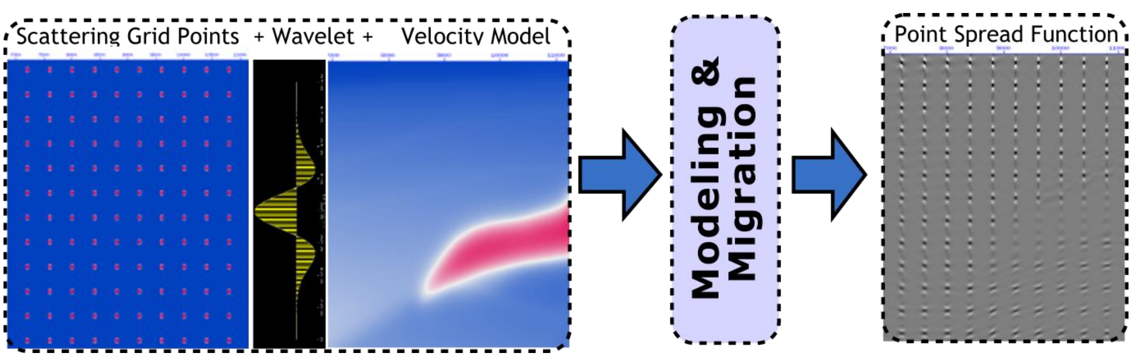


Figure 1*: Scattering grid points, wavelet and velocity model (left) are necessary to estimate the PSFs by wavefield modeling. One can observe the distortion on the deepest PSFs due to the modeling and migration apparatus and velocity model variations.*

where n is the iteration number and α is a scale-factor. It is known that linear iterative algorithms based on L_2-norm improve the image resolution and correct for illumination effects, but the bandwidth of the outcome is restricted to be the same as that of the input seismic image (Rosa, 2018).

The objective function (Eq. 8) can be augmented in several ways. For instance, sparsity promotion (Pereira-Dias, et al. 2017) provides a super-resolution depth-imaging technique even if the input was a bandlimited image plagued with illumination issues. In addition, the presence of noise or undesired events such as multiples in the migrated image may reduce the quality of the least-squares migration result. In this case, other constraints such as structure-oriented filtering (Hale, 2009) might be imposed and interpreted as an additional geophysical constraint to inversion problem. In the prestack approach, it is also possible to include a regularization in the offset domain, imposing continuity of the amplitudes.

Results

We show how the method performs on two approaches and different datasets:

- 3D nodes data in the Santos Basin, Brazil. Water depth is about 2,200 m. PSFs are generated by an acoustic two-way extrapolator for the Born modelling and imaging with RTM limited to 45 Hz cut frequency. Figure 2 shows the results for the 3D PSFs, RTM and post-stack LSM. The main contribution concerning illumination relates to the edge of the survey. In the target area, the resolution gain provided better stratigraphic details, and the maintenance of events continuity was ensured by using structure oriented filtering.
- Conventional streamer (narrow azimuth and 8km cable length) of a 3D survey at the Espírito Santo Basin, Brazil. Water depth is about 2,000 m. PSFs are generated by two-way Born modelling and Kirchhoff depth migration in surface offset domain. Figure 3 show the results for 3D prestack PSFs, Kirchhoff and LSM. It is clear that PSFs capture the amplitude distribution along the surface offset direction when compared to the migrated gather. That is why LSM, in addition to improve resolution, better balances amplitudes in that direction. Worth to mention that no regularization was used in this application.

Conclusions

We presented two applications of the image domain least-squares migration with the concept of the point spread functions, including pre-stack and post-stack strategies and using both Reverse Time Migration (RTM) and Kirchhoff migration engines. Both strategies showed that least-squares migration provides an uplift in the migration amplitudes and resolution even on geologically complex models. The most suitable approach, in imaging projects, should be pondered according to each geological context in order to provide the interpreters reliable images.

Acknowledgments

The authors thank PETROBRAS for authorizing this publication.

References

Dai, W., Fowler, P. and Schuster, G. T., 2012, *Multi-source least-squares reverse time migration*: Geophysical Prospecting, 60: 681–695.

Fletcher, R.P., Archer, S., Nichols, D., Mao, W., 2012, *Inversion after depth imaging*: Expanded Abstracts, SEG 82nd Annual Meeting, Las Vegas.

Hu, J., Schuster, G. T. and Valasek, P. A., 2001, *Poststack migration deconvolution*: Geophysics, **66**, 939-952.

Letki, L., Tang. J., Inyang, C. Du, X., Fletcher, R. 2015, *Depth domain inversion to improve fidelity of subsalt imaging: a Gulf of Mexico case study*: First Break, **33**, pp. 81-85.

Nemeth, T.,Wu, C., and Schuster, G. T., 1999, *Least-squares migration of incomplete reflection data*: Geophysics, **64**, 208–221.

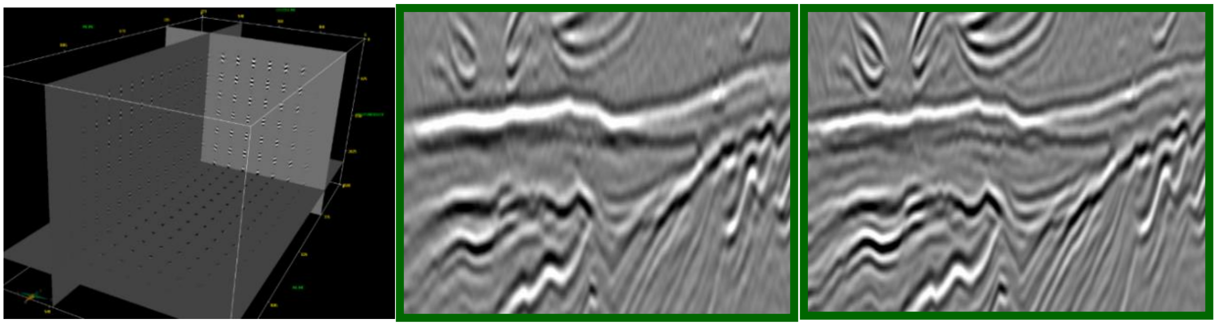
Pereira-Dias, B., Bulcão, A., Soares Filho, D. M., Santos, L. A., Dias, R. M., Loureiro, F. P., and Duarte, F. S., 2017, *Least-Squares Migration in the Image Domain with Sparsity Constraints: An Approach for Super-Resolution in Depth Imaging*: 15th International Congress of the Brazilian Geophysical Society & EXPOGEF, Rio de Janeiro, Brazil, pp. 1213-1218.

Rosa, A. L. R., 2018, *The Seismic Signal and Its Meaning*: Geophysical References Series, v. 23, Society of Exploration Geophysicists.

Shin, C., Jang, S., Min, D.-J., 2001, *Improved amplitude preservation for prestack depth migration by inverse scattering theory*: Geophysical Prospecting, v. 49, pp. 592-606.

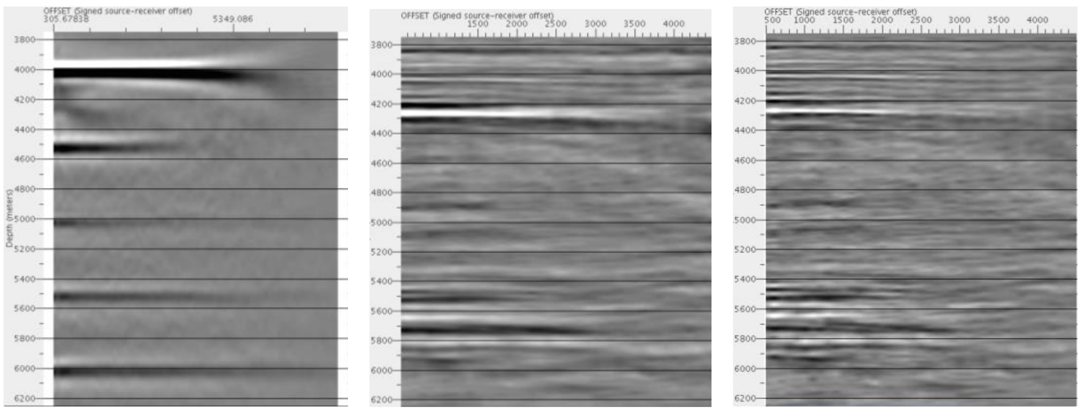
Tarantola, A., 1986, *A strategy for nonlinear elastic inversion of seismic reflection data*: Geophysics, **51**, no. 10, pp. 1893-1903.

Valenciano, A. A., Biondi, L. B., Clapp, R. G, 2009, *Imaging by target-oriented wave-equation inversion*: Geophysics, **74**, no. 6, pp. WCA109-WCA120.



(a) PSF **(b) RTM** **(c) Post-LSM**

Figure 2: *(a) 3D Point spread function grid. (b) Reverse Time Migration (c) Post-stack image domain LSRTM.*



(a) PSF **(b) KDM** **(c) Pre-LSM**

Figure 3: *Surface offset gathers of (a) Point spread functions. (b) Kirchhoff Depth Migration (c) Pre-stack LSM.*