A strategy for high resolution elastic inversion for layer properties using a three-term AVO formulation

Uilli O. Freitas[1], Fernando S. Moraes[1,2] and Sérgio A. M. Oliveira[1,2]. [1]Grupo de Inferência de Reservatório da Universidade Estadual do Norte Fluminense, Macaé, Brazil; [2]Instituto Nacional de Ciência e Tecnologia de Geofísica do Petróleo (INCT-GP/CNPq), Salvador, Brazil

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Abstract

We develop a strategy for high resolution AVO inversion of reservoir elastic properties. The objective is to obtain a more accurate model of P-wave, S-wave velocity and density, which is well known to be an unstable problem. The formulation consists of solving, initially, the AVO inverse problem using a Bayesian formulation with a trivariate Cauchy a priori distribution, which stabilizes and incorporate sparsity information on the reflectivity and well-log information. Then, we solved a coupled constrained impedance inversion using mixed norm, L2 and total variation, to obtain the layer properties. We demonstrate the applicability of this strategy using the North Viking Graben open data set. The results show that all three elastic parameters are satisfactorily recovered, approximating our solution to a full waveform inversion solution on the same input data set.

Introduction

In AVO inversion we try to retrieve the elastic parameters from a seismic data set. However, as the major part of inverse problems, this problem is ill-posed, i.e., cannot be solved with a unique answer. As a result, it is necessary to impose some constrains on the problem. One of the attempts is the reduction of three to two terms approximations, proportioning more stability to the inversion problem but at the expense of introducing theoretical error (Downton, 2005). Another point related to this reduction in the number of parameters is that more often density is the omitted parameter, despite its importance for reservoir characterization.

Using a three-term approximation in the conventional AVO inversion formulation, the density estimation is particularly difficult, and the reason is that using small incidence angles the approximated reflection coefficient has low sensitivity with respect to density variations and the use of large incidence angle is controversial to the assumptions of the approximations (Mallick, 2007). For this reason, different techniques have been proposed to improve the solution of the three-term AVO inverse problem (Downton, 2005; Alemie and Sacchi, 2011), allowing to obtain better density estimates.

The regularization consists of using constraints to estimate a unique and stable parameter and depending on the regularization choice some specific characteristics are imposed in the solution. Quadratic regularizations are well-known and used in inversion geophysics problems, are associated with Gaussian distribution and impose smallest, flattest and smoothest characteristics to the model. However, it is well known from well log data that the amplitudes of the reflection coefficients are non-Gaussian (Walden and Hosken, 1986) and, as such, can be related to sparse structures. Therefore, in order to estimate sparse models, non-quadratic norms have been used, not only in AVO inversion problem but also in deconvolution (Oldenburg et al., 1983), least square migration (Wang and Sacchi, 2006), etc.

Considerable work have been developed using the Bayesian inversion approach, and in this framework, the solution is constrained by a priori information. One common assumption is the Gaussian prior distribution of the model, but this distribution does not reach a sparse solution. Thus, we propose the use of probabilities distributions with long tails, which are associated with non-quadratic norms.

Forward Problem

For this work we use the Aki and Richards linear approximation (Eq.1) to calculate the reflection coefficients:

$$R_{PP}(\theta,t) = \; a\, r_\alpha + b\, r_\beta + c\, r_\rho, \tag{1}$$

where the coefficients a, b and c depend on the angle of incidence and the ratio between S and P-wave velocities and $r_x = (x_2 - x_1)/(x_2 + x_1)$, where x represents α (P-wave velocity), β (S-wave velocity) and ρ (density).

Considering the earth structure as a series of N horizontal layers of constant properties separated by planar interfaces, the reflectivity in a time interval and for a set of reflection angles can be written in a matrix form as:

$$\mathbf{r} = \mathbf{Lm}, \tag{2}$$

where $\mathbf{L}$ is a $Nx3N$ sparse matrix that contains the coefficients dependent of incidence angle and $\mathbf{m}$ is a vector of dimension $3N$ that contains the AVO reflectivity (r_x).

High resolution AVO formulation

Generally, the inverse problem is formulated as an optimization of an objective function $E(\mathbf{m})$, which has two main terms, the first one, misfit, related to the observation and corresponding to the measure of data fidelity and, the second one, the model norm, that is a penalty function related to the parameters which objective is to stabilize the inverse problem.

$$E(\mathbf{m}) = \ misfit + \mu(model\ norm) \qquad (3)$$

where μ is the regularization parameter. In the Bayesian inversion approach, our task is to specify the likelihood function and the prior information. After that, we need to select a model parameter that maximizes the posterior distribution function, i.e., to maximize $p(\mathbf{m}|\mathbf{d}) \propto p(\mathbf{d}|\mathbf{m})p(\mathbf{m})$. Now, considering the AVO linear problem, i.e., $\mathbf{d} = \mathbf{Lm} + \mathbf{n}$, and assuming Gaussian and independent noise, the likelihood function $p(\boldsymbol{d}|\boldsymbol{m})$ for the observed seismic data $\boldsymbol{d}$, given the parameter $\boldsymbol{m}$ is:

$$p(\mathbf{d}|\mathbf{m}) \propto \ exp\left[-\frac{1}{2}(\mathbf{d}-\mathbf{Lm})^T\mathbf{C}_d^{-1}(\mathbf{d}-\mathbf{Lm})\right], \qquad (2)$$

where $\boldsymbol{C}_d$ is the noise covariance matrix. It was assumed independent noise, then $\boldsymbol{C}_d = \sigma_d^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and σ_d^2 is the data noise variance.

As first strategy to achieve better elastic properties estimation we use the Trivariate Cauchy distribution as prior information. This distribution is capable of imposing sparsity in the AVO parameters and, furthermore, is capable of imposing correlation information among the parameters.

The inversion is carried out maximizing the follow posterior distribution function:

$$p(\mathbf{m}|\mathbf{d}) \propto exp\left[-\frac{1}{2}(\mathbf{d}-\mathbf{Lm})^T\mathbf{C}_d^{-1}(\mathbf{d}-\mathbf{Lm})\right]* \\ exp\left[-2\sum_{i=1}^{N}\ln\left(1+\mathbf{m}^{\mathrm{T}}\boldsymbol{\varphi}^{\mathrm{i}}\mathbf{m}\right)\right], \qquad (3)$$

Solving Eq. 3 we get:

$$(\mathbf{L}^T\mathbf{L} \ + \ \mu\mathbf{R})\mathbf{m} = \mathbf{G}^T\mathbf{d}, \qquad (4)$$

where $\mathbf{R}$ is a $3Nx3N$ matrix of regularization which elements are given by

$$R_{kn} \ = \sum_{i=1}^{N}\frac{2\varphi_{kn}^{i}}{1+\mathbf{m}^{\mathrm{T}}\boldsymbol{\varphi}^{\mathrm{i}}\mathbf{m}}, \qquad (5)$$

Although our problem is a linearized form of the AVO inverse problem, the regularization term is parameter dependent, thus our system of equation becomes nonlinear and we solve it by IRLS (Iterative Reweighted Least Squares).

Solving Eq. 4 we obtain pseudo sections of P-wave, S-wave velocity and density. Pseudo sections means that we still have the wavelet effect in our parameters, so we apply another inversion with the objective of take the wavelet effect out as well as try to obtain the sparsity that the first inversion was not able to.

We use the previously described approach to do the inversion and obtain the elastic parameters, but we differ the prior information. The inversion used is the proposed by Oliveira (2011), which the prior information is a weighted mixed norm composed by L2 norm and Total Variation and the input data is the pseudo sections previously estimated.

Results

We demonstrate the applicability of the proposed strategy by usage of the North Viking data set. It is an open data set created for evaluating and comparing seismic inversion methods, which contains a marine 2-D unprocessed seismic line and two well-logs measurements with key elastic parameters over depth intervals that span reservoir rocks (Keys and Foster, 1998). The seismic data was processed and migrated using an amplitude preserving PSTM workflow and then subject to preconditioning with the role of residual multiples elimination and residual move out correction. The image gathers were transformed to the angle domain in a range of five to forty degrees.

We restricted this study to approximately 600 angle image gathers and for a time interval which the well A is inserted in (Fig.1). This well contains three reservoirs that are Jurassic age clastic sediments, which depositional environments range from fluvial to deltaic and shallow marine. They are separated by deep water shales and the hydrocarbon traps are usually fault-bounded structures.

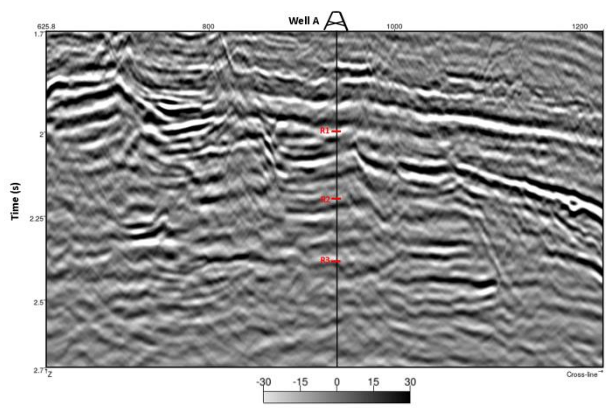

Figure 1: Seismic section interval with the well A. The well A contains three reservoirs, demarcated as R1, R2, and R3.

Firstly, we realized the inversion procedure in the image gather where the Well A is located. This is a good choice because it is possible to determine the best values to the regularization parameters and thereafter use them to all data set. We observed that the best strategy was to use a singular value to the trade-off parameter related to the Trivariate Cauchy regularization term and then reemphasize the sparsity by a minimal value to the trade-off parameter related to the Total Variation regularization term.

The results obtained by the proposed strategy are reasonably well for P-wave and S-wave velocity, correlation coefficient equal 0.92, and for density we observe an acceptable adjustment, correlation coefficient equal 0.74 (Fig.2). This is a good estimation for density since we are using a linear relationship to the reflection coefficient and density has less sensitivity if compared to the other two parameters.

Analyzing the well-logs in the reservoirs R1, R2 and R3 (Fig.3), which tops are in approximately 2000 ms, 2200 ms and 2400 ms, respectively, we can see that it is not possible to distinguish them from P-wave and S-wave velocity, while it is perceptive in the density by reasonable decrease of the property.

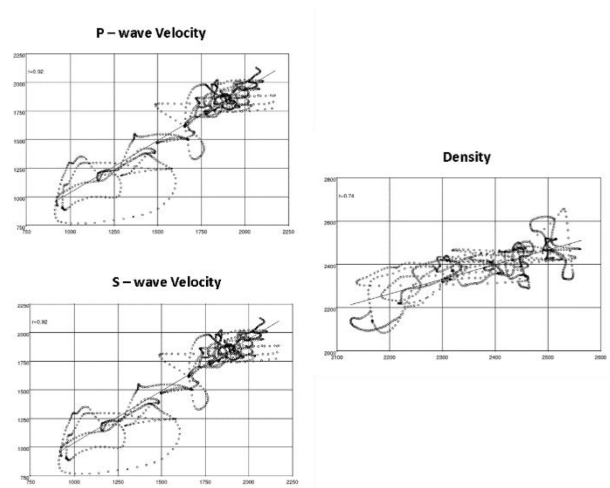

Figure 2: Correlation coefficient of inverted parameters and respective well-logs.

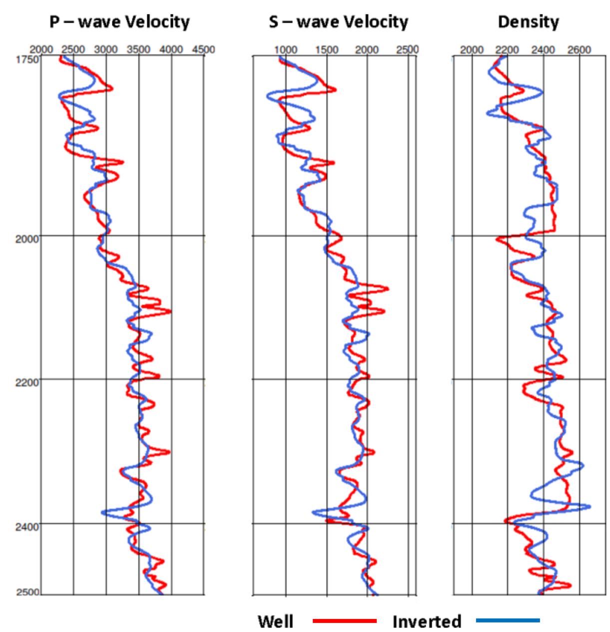

Figure 3: Comparison between P-wave velocity, S-wave velocity, and density well-logs upscaled to the seismic scale (red curves) and the inverted results (blue curves).

Comparing the inverted elastic parameters and their respective well logs in Fig. 3 we reaffirm the agreement between the estimated and true parameters for P-wave and S-wave velocities and from the inverted sections (Fig. 4) we clearly observe the recovery of the layers discontinuities and the faults that bound the reservoirs. For density, considering Fig. 3, the first top reservoir is not well identified but its continuity is considerable. The second one can be rarely distinguished and the top of the third one has better adjustment, allowing the reservoir identification. Considering the density inverted section, we clearly distingue the first and third reservoir, although the resolution and continuity is compromised.

Conclusions and recommendations

AVO inversion methods, which are generally constrained by taking the density term out or using L2 norm (smooth) regularizations, provide uncompleted reservoir information, since key elastic properties are not well estimated or even not estimated. Applying the proposed strategy we proved its feasibility, showing the good results for P-wave and S-wave velocity and an acceptable estimation for density, which is very important for reservoir characterization and for the most of conventional approach it is not estimated nor has no quality.

Some important aspects must be take into account for this strategy. The first one is the build of scale matrix, which has the key role in incorporate geological information to the inversion, so a usage of an incorrected scale matrix will provide unreasonable pseudo sections. The second one is the weights to the trade-off parameters, because in the first step we already impose sparsity by a Trivariate Cauchy norm and in the second one we try to retrieve the discontinuities that the first norm was not able to. Thus, if we use high values in the two steps we super emphasize the sparsity of the data, forcing an artificial aspect of few layers.

Thus, the proposed strategy is capable to retrieve a more accurate elastic model, where more structures and layers can be defined as well as better density estimation.

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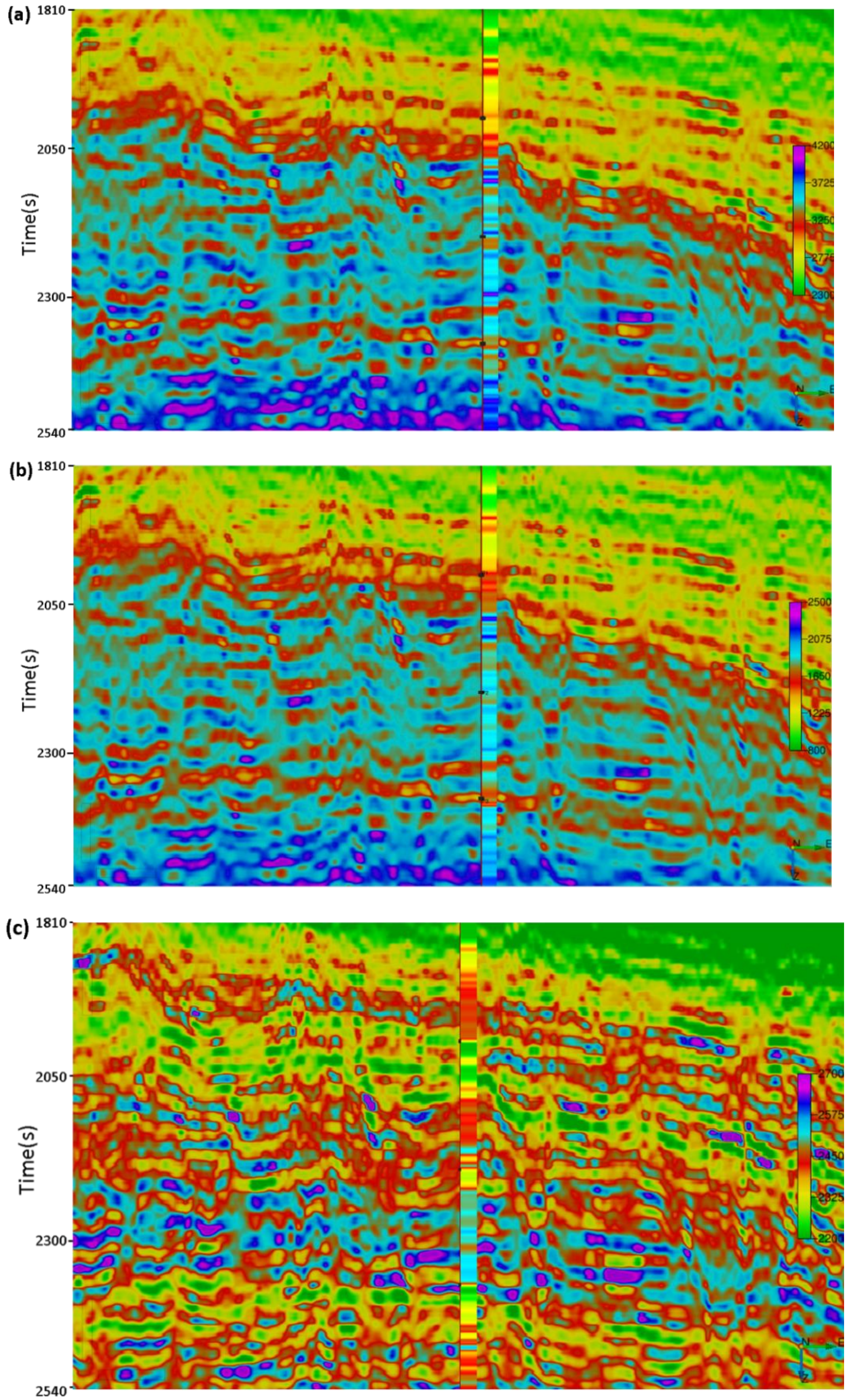

Figure 4: Inverted elastic sections. (a) P-wave velocity, (b)S-wave velocity, (c)Density.

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