A greed algorithm for seismic data interpolation using the approximate irregular discrete Fourier transform

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Abstract

In this paper, an alternative greed algorithm for interpolation/regularization of irregularly-sampled seismic data in the Fourier domain is described in connection with the approximate irregular discrete Fourier transform (AIDFT). The greed algorithm is used to fill in empty bins, generated by defective sampling, under statistical and/or physical constraints, so as to achieve an acceptable Fourier spetrum. Much like in other implementations, a least square norm Fourier spectrum is the input for the process. Here, this least square initial solution is provided by the AIDFT. The greed algorithm proposed is an iterative procedure that consists in, step by step, correcting for survey's footprints of main Fourier components.

Introduction

Irregular seismic surveys may imply many shortcomings in processing, imaging, and inversion of seismic data. Thus many authors have tackled the problem of regularizing defective data via interpolation. Among many different approaches, Fourier interpolation has had widespreaded use due to the wave nature of seismic data (Sacchi, et al. 1998; Duijndam et al, 1999). It generally consists in fitting irregular data with a predefined set of Fourier components $\mathbf{F}_m$, plane waves at sampling points, and using this set to predict the data in a regular survey. A least square error function

$$E = \sum_n \left\| d_n - \sum_m D_m F_{nm} \right\|^2 \tag{1}$$

is minimized with an optimum choice of the vector $\boldsymbol{D}$, the weights each plane wave $\mathbf{F}_m$ has in the decomposition of the data $\mathbf{d}$. Minimizing (1) is equivalent to solve,

$$\mathscr{F}^H \mathscr{F} \boldsymbol{D} = \mathscr{F}^H \mathbf{d} \quad , \tag{2}$$

where $\mathscr{F}$ stands for a matrix that has the plane waves $\mathbf{F}_m$ as its columns. In general, this formulation does not yield a well posed problem for finding $\boldsymbol{D}$. $\mathscr{F}^H\mathscr{F}$ is not an orthonormal matrix and it has no inverse particularly when there are missing samples. Thus, an alternative way to fill in empty bins are required.

In an iterative reweighted least square approach a set of relatively small parameters are inserted and adjusted to pursue an extra constraint, either physical or statistical, in the form,

$$\left[\mathscr{F}^H \mathscr{F} + \boldsymbol{\Lambda}^{(k)}\right] \boldsymbol{D}^{(k+1)} = \mathscr{F}^H \mathbf{d} \quad , \tag{3}$$

where $\boldsymbol{D}^{(k+1)}$ is the solution at iteration $(k+1)$, $\boldsymbol{\Lambda}^{(k)}$ is a diagonal matrix with entries $\lambda_i(\boldsymbol{D}^{(k)})$ borrowed from iteration (k). The process starts with the L2 solution of equation(2) and, after a few iterations, an acceptable solution is finally achieved as,

$$\boldsymbol{D} = \left[\mathscr{F}^H \mathscr{F} + \boldsymbol{\Lambda}\right]^{-1} \mathscr{F}^H \mathbf{d} \quad . \tag{4}$$

The solution in the equation above is generally obtained in an approximated way, often via conjugated gradient, due to the large dimensions involved. The problem has generally complexity higher than $\mathscr{O}(n^2)$[1], n is the number of non zero entries, and tends to reach a deadlock for larger problems. On the other hand, diminishing dimensions to keep feasibility of the solution often limits its representativeness.

A different class of approach, the so called greed algorithms, involves searching in the L2 solution of (4) for the "best fit" component, let us say $\mathbf{F}_{best}$, then subtracting from $\mathbf{d}$ its projection onto the corresponding subspace, this defines D_{best}. The result is then projected onto another chosen component $\bar{\mathbf{F}}_{sec}$ after an orthogonalization. The idea is to represent the data with a minimum number of Fourier components, avoiding leaking and saving computer resources. This procedure tends to push too far the assumptions made about the solution and, for a more rigorous solution, may otherwise require large memory and CPU resources. Also, the process is somewhat vague about what to do when more than one sample per bin is available.

The discrete Fourier spectrum of an irregularly sampled multidimensional set of data can be estimated with a $N \log N$ complexity algorithm in an approximated way (Oliveira, 2017, 2018). The approximation is achieved with a series expected to converge and exist only when sampling is not defective, no data losses.

The purpose of this paper is to discuss an alternative greed algorithm that could help moving from a spectrum obtained with the AIDFT, under the hypothesis of null samples where sampling is defective, to a solution similar to the one it would be obtained solving equation (4).

The L2 solution and the approximated discrete irregular Fourier transform

The AIDFT expresses $\left[\mathscr{F}^H \mathscr{F} + \boldsymbol{\Lambda}\right]^{-1} \mathscr{F}^H$ in a series that converges generally as fast as the Fourier kernel's ($e^{i\theta}$) Taylor series does for small θ. The idea is to choose

[1] The number of null samples here is negligible.

an embedded grid where all samples' positions are close enough to some grid point so that the deviations are kept small. Given a set of samples, the choice of the embedded grid not only determines the expected Fourier spectra range but also defines over or under sampled problems. Over sampled problems, that where grid points may have many close samples, yields a typical overdetermined problem, while under sampled problems have grid points without neighbors. The AIDFT exists only for over/even sampled problems. Under sampled problems are treated as if a null sample was associated to any empty bin, a grid point with no neighbors. At this point, for simplicity, let us define $\mathscr{F}_a = [\mathscr{F}^H \mathscr{F} + \mathbf{\Lambda}]^{-1} \mathscr{F}^H$ and $\mathscr{F}_s = \mathscr{F}$, with subscripts a and s, respectively, standing for *analysis* and *synthesis*, as in frame theory. An extension of Parseval's theorem that holds for frames allows one to conclude that $\|\mathbf{D}\|^2 \propto \|\mathbf{d}\|^2$ which means that if $\mathbf{D}$ is obtained as in (4) it is also a L2 norm solution to the problem[2] if $\mathbf{d}$ has its empty bins filled with zeros.

Defective sampling operators

Defective data $\mathbf{d}_-$ may be represented by $\mathbf{d}_- = \mathbf{d}\,\mathbf{h}$, with $\mathbf{h}$ a "broken comb" operator. In the Fourier domain this is written as $\mathbf{D}_- = \mathbf{D} * \mathbf{H}$, where $\mathbf{H}$ is a burring operator (see figure (1). The blurring operator is expected to distort the Fourier spectrum but not to prevent anomalous amplitudes recognition.

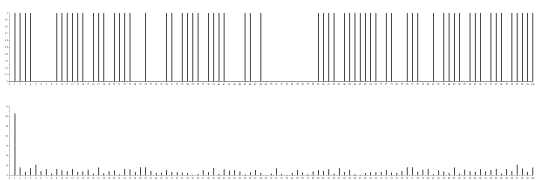

Figure 1: *Defective sampling operator, in the space domain (above), and in wave-number domain(below). About* 40% *missing data positions randomly set in this case.*

A greed like algorithm to fill in empty bins

Unlike other greed approaches to interpolate missing data, here each initial Fourier component is altered without the orthogonalization requirement. The main goal here is to fill in missing data bins, reverting the effects of a defective/irregular sampling operator $\mathbf{h}$ under the assumption that its Fourier spectrum $\mathbf{H}$ is sufficiently sparse to allow the recognition of signal's main components even after the associated blurring effect.

The blurring effect results from the convolution of $\mathbf{H}$ with the data's Fourier spectrum. Given that $\mathbf{H}$ typically has its biggest amplitude at the wave-number $k = 0$, that is $H(0)$, and all neighboring samples are considerably smaller[3], one can still select the anomalous components in the data's spectrum $\mathbf{D}$. After blurring, any component of $\mathbf{D}$ is the sum of its original values times $H(0)$ and smaller contributions from neighboring components. Defining $\mathbf{H}_1$ as the result of $\mathbf{H} - H(0)\boldsymbol{\delta}(k)$ one can write,

$$\overline{\mathbf{D}} = \mathbf{D}H(0) + \mathbf{H}_1 * \mathbf{D} \ . \tag{5}$$

[2]When there are grid points with many samples, one can say that $\mathbf{D}$ is L2 only if $\mathbf{\Lambda} = \lambda \mathbf{I}$, λ a constant.

[3]Loosely stating that the signal is well represented by the available data.

Further splitting $\mathbf{D} = \mathbf{D}_{main} + \mathbf{D}_{rest}$ so that anomalous components are called $main$, after blurring the main components will be given by,

$$\overline{\mathbf{D}}_{main} = \mathbf{D}_{main} H(0) + \mathbf{H}_1 * \mathbf{D}_{main} + \mathbf{D}_{rest} H(0) + ... \tag{6}$$

Given that $\mathbf{H}_1$ and $\mathbf{D}_{rest}$ are, by definition, smaller, ideally one has,

$$\overline{\mathbf{D}}_{main} = \mathbf{D}_{main} H(0) + \text{smaller terms} \ , \tag{7}$$

Then, under certain conditions, it is possible to devise an iterative procedure that approximately reverts the blurring effect.

The procedure is made up of a few steps:

1. Select the biggest samples in the current spectrum until a given percentage of the total energy is reached and zero out the remaining components;
2. Apply the blurring operator to the spectrum obtained in step (1) and estimate how much the selected amplitudes decrease;
3. Divide the spectrum from step (1) by its rate of decrease so as to recover their expected actual amplitudes;
4. Apply the blurring operator to the result of step (3) and subtract from the current spectrum;
5. Go back to step (1) if it was not achieved a negligible level of residual energy.

The algorithm above is to be applied in the Fourier domain. The blurring operator application at the Fourier spectrum of an irregularly sampled data (step (2)) turns out to be a sequence of operations as $\mathscr{F}_a \mathbf{h} \mathscr{F}_s \mathbf{D}$. In other words, the operator is applied in the space domain and transformed back to the Fourier domain. Notice that, should the data sampling be regular with empty bins, $\mathscr{F} \to FFT$ and the proposed algorithm goes as much like described above.

It might be necessary to adapt the greed algorithm proposed if the data's "degree of sparseness" is not high. Other hypothesis, if available, must be considered to reach a reasonable outcome. A good example would be a limitation on the expected bandwidth. A bandpass filter could easily be applied at step (1) to the selection of components to work with. Figure 2 have a few illustrating pictures where 4 events characterized by 4 dominant wave-numbers $k = (12, 5, 3, 20)$, amplitudes $A = (1, 1, 0.5, 1.5)$, and phases $\phi = (.1, .3, .5, -.2)\pi$ are found at positions $x_0 = (1, 5, 7, 8)$, their wavelets are represented by $\Theta(x - x_0) A \mathrm{e}^{-\alpha x} \sin(2\pi k x + \phi)$, with $\alpha = -.005$, $\Theta(x)$ the Heaviside function, and space interval set to 1. From figure 2, it can be seen that the procedure relatively succeeded in predicting a reasonable function, although not reproducing smaller details of the expected spectrum. Also, it can be seen (fifth picture from top to bottom) that the original data was not fully honored. The role of the wavelet used is to reduce the suitability of the data to the sparseness hypothesis, otherwise the prediction would be perfect, the example a little unfair.

The proposed procedure is heavily based on the assumption that sampling problems occur in a rather non-systematic way. Should sampling failures occur systematically, the corresponding Fourier operator $\mathbf{H}$ would not look like that in figure 1. The simple case where every other sample is lacking in an evenly spaced survey, $\mathbf{H}$ would have

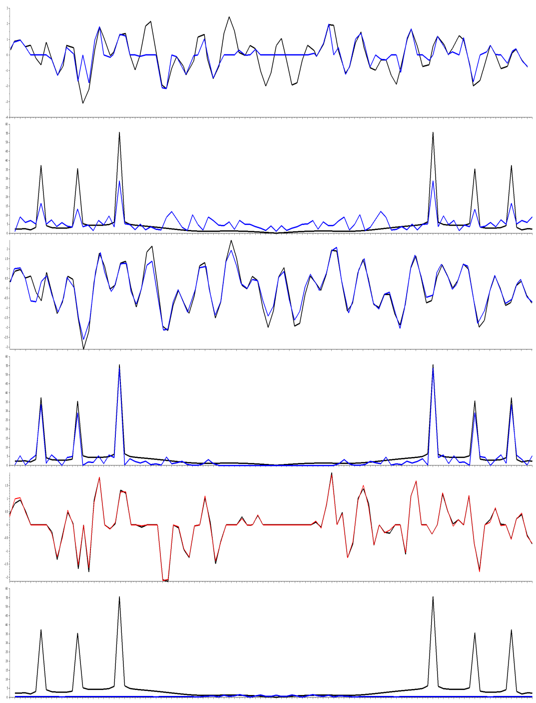

Figure 2: *From top to bottom, odd pictures in the space domain, even pictures in the wave-number domain; First and second pictures, the full data in black, the irregular/defective data in blue; Third and fourth pictures, the full data in black, the recovered/interpolated data in blue; Fifth picture, the regular/defective data in black, the recovered/interpolated data at sampling positions in red; Sixth picture, the spectrum of the full data in black, the residual spectrum in blue.*

just two main peaks at $k=0$ and $k=0.25\Delta x$ (Δx the space interval), making it difficult to identify higher wave-number anomalous components (aliasing), see, for instance, Hennenfent et al (2007). In order to apply the proposed greed algorithm in these cases, another *a priori* information must be added. Many authors have covered these cases with different approaches like prediction in $f-x$ domain, semblance related $priors$ to guide inversion, and many others. Here, we make use of the well known relationship between plane waves in the space domain and origin crossing straight lines in the $f-\mathbf{k}$ domain to define a signal indicating function $\mathscr{S}(f,\mathbf{k})$ that replaces actual amplitudes $\mathbf{D}$. The proposed procedure changes only in that now is the anomalous $\mathscr{S}(f,\mathbf{k})$ locations that points to where to get $\mathbf{D}_{main}$.

Sinthetic seismic data example

In geophysical problems, very often the solution is expected to be sparse in the wave-number domain, that is, it is assumed that a limited subset of $\mathbf{D}$ is sufficient to describe $\mathbf{d}$. Hyperbolic events are expected to spread around the wave-number domain, making sparseness a weaker hypothesis. However, locally, small curvatures (higher velocities) are approximately seen as a set of tangent lines.

Defective sampling often occur in a systematic way. Thus, correcting for defective surveys ideally should take into account different means to handle inherent limitations on signal recognition.

A synthetic 2D seismic data set over plane reflectors is used to test the algorithm proposed. The model contains 3 plane reflectors, dips are $0°$, $20°$, and $10°$, velocities 1500m/s, 2500m/s, and 3500m/s. Shots are irregularly positioned around a regular 40 meter spaced grid and traces are irregularly placed so as to keep an average trace distance of 20 meters. The purpose is to regularize shots and trace positions, and also to reduce the average shot spacing to 20 meters. This test covers randomly and systematically acquired data to interpolate. Temporal frequencies are controlled by a Ricker wavelet with 25Hz peak.

To formulate the problem of interpolating shots and traces at once, subsets of 24 "contiguous" shots were treated. Coordinates were all converted to the regular intended grid with trace positions converted into offsets with respect to corresponded shot positions. Figure 3 shows two input shots before (top) and after (bottom) regularization. An interpolated shot, also at regular positions is shown.

Conclusions

An alternative greed approach to regularize and interpolate relatively sparse seismic data was shown. The approach is applied in conjunction with an approximate estimate of the Fourier spectrum for irregularly sampled multidimensional functions to populate defective acquisition bins initially taken as if a null sample had been recorded. The proposed process was not tested against random noise. However, it is expected that moderate noise can helplessly prevent weak signal recovery.

The algorithm can and should be combined with other *priors* to enhance the ability to predict candidate samples for signal recovery.

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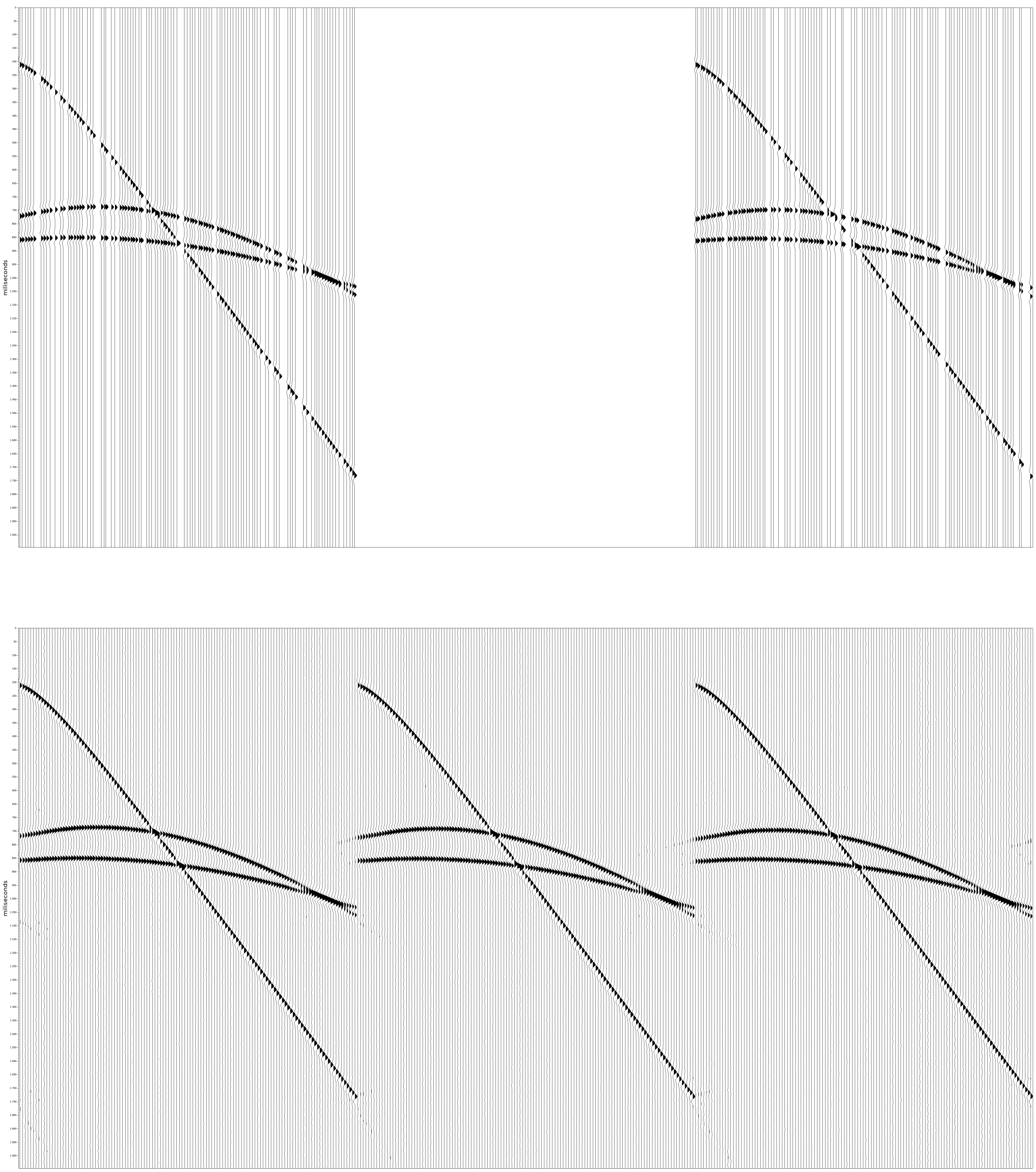

Figure 3: *Top, irregularly sampled and with missing shots. Bottom, the sequence of regularized and interpolated shots. The event of greater hyperbolic moveout corresponds to a plane reflector. The other two events comes from dipping reflectors. The trace interval is ideally 20 meters between traces and shot points. The original shot spacing is irregularly varying around 40 meters.*