# The concept of model rays in domain conversion

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## Abstract

**In this work we present the concept of model rays which can be seen as the dual of image rays, but traced in time domain. We present the system of first order differential equations which permit to numerically compute the model rays using a velocity field in time domain. We demonstrate with a synthetic example that model rays traced with this system of equations are identical to others traced with two different methods. Among other geometrical interpretations, the model rays can be seen as the instrument to perform the time-to-depth conversion of seismic images. Time-to-depth conversion algorithms which make use of model rays are much simpler than those one based on image rays. Image rays are more suitable to perform depth-to-time conversion.**


## Introduction

Time migration based methods are still widely used in the industry not only because they are more robust and faster than depth domain ones, but also because shifts in technology paradigms take longer to happen in the oil industry. The conversion of seismic domains from time to depth, has been possible after the introduction of the concept of image ray (Hubral, 1977), which validity is subject to the same conditions of validity of time migration methods. Subsequently, the use of image rays was extended to the conversion of seismic images (Silva et al 2009, Filpo et al 2016) and time migration velocity models (Silva et al, 2009; Cameron et al 2007 and Iversen et al 2008).

In general, time-to-depth conversion algorithms based on image ray tracing, or equivalently based on image wavefront construction, involve mapping of values distributed along a regular grid in the time domain into another regular grid in the depth domain. This mapping is constructed with rays traced in depth, using velocity information in depth or either in time domain. However, no matter which approach is taken, mapping domains using image rays results in an irregular distribution of points in the depth domain. This is not the ideal because multiplicity or voids of can happen and also.

Figure 1 shows a velocity model in depth superimposed by a set of image rays and its corresponding wavefronts. This curves forms a curvilinear grid directly associated with a cartesian grid in time domain, being seismic traces mapped into image rays and time slices into wavefronts. The velocity spatial distribution in this model is given by the equation:

$$V(x,z) = 1.95 + 0.1z + 0.15x\,,$$

being $x$ and $z$ in kilometers and $V$ in kilometers per second.

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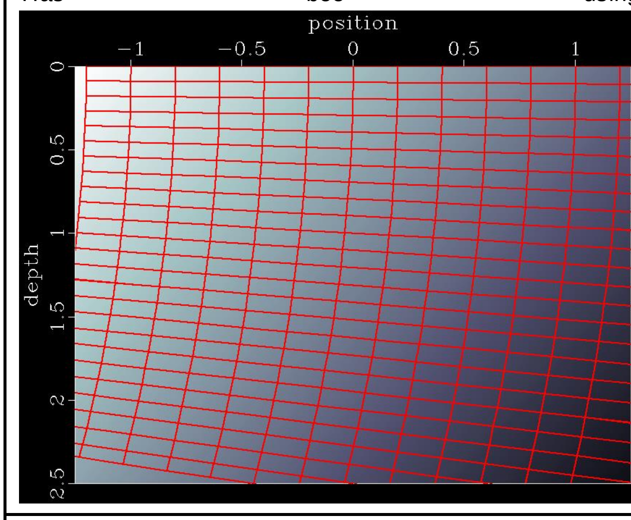


***Figure 1 - Velocity model in depth superimposed by a set of rays and its corresponding wavefronts.***

Filpo et al (2016) presented a solution to that problem of spreading properties in the time-depth conversion with image rays that is the use of the Taylor series. Such a strategy became feasible from the formulation of time-to-depth conversion as a change of coordinates problem, where vertical lines in the time domain are mapped to image rays at depth, and the horizontal lines on originally flat wave that emerge from the datum. The required derivatives to apply the Taylor series are obtained by finite differences along the wavefronts, For simplicity, we refer to this domain conversion process just as spreading algorithm. Remember that to trace image rays a velocity model in depth is required.

Figures 2-a and 2-b show an example of the spreading algorithm application. The input section is an artificial time migrated section which contains just a circular feature (time axis is upwards). The velocity model used to trace image rays and wavefronts is the same of the Figure 1. Note that there are no caustics and neither shadow zones in this model, that is a necessary condition for the perfect working of the spreading algorithm, which is based in a coordinate transformation approach.

The same kind of trouble does not happen with the use of image rays in the conversion from depth to time, where the regular grid of output is directly filled with values collected in the depth domain through a simple interpolation. This kind of transformation is referred just as gathering algorithm in this text. For example, to convert a velocity model from depth to time, we just trace image rays with a desired sample rate and collect velocity values along them.

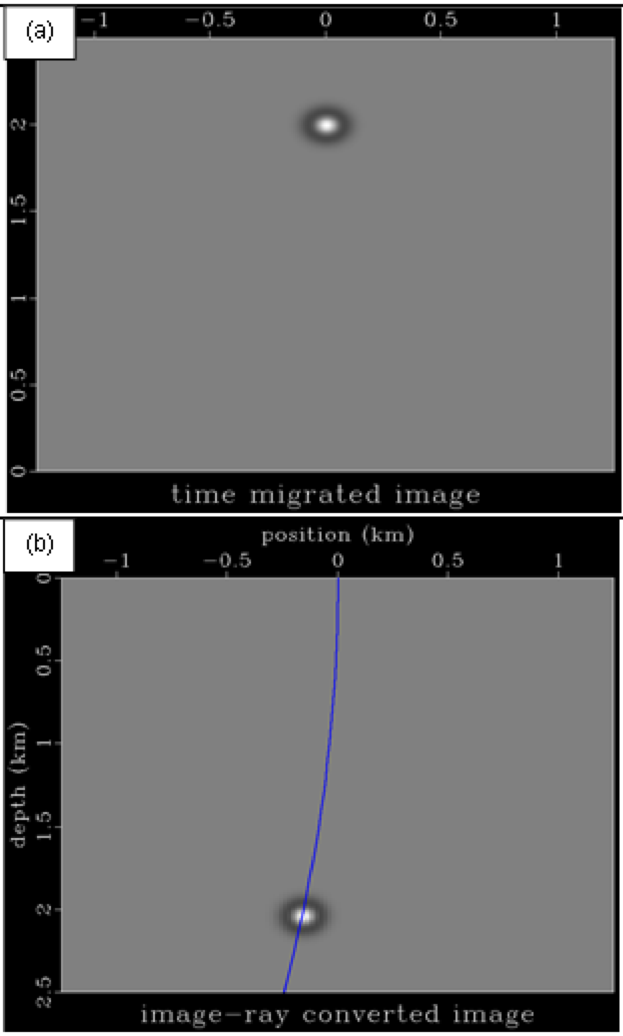


***Figure 2: a) Artificial time-migrated image. b) Depth-converted image using the image-ray spreading algorithm.***

The concept of model rays arose when we start to observe some features of seismic images converted from depth to time by collecting amplitudes along image rays. Figure 3a shows an image in depth domain with horizontal and vertical events, while Figure 3b shows the time-converted image by means of the gathering algorithm. The observation of the converted section clearly shows that the vertical lines in depth are transformed into curves in the time domain with a behavior very similar to the image rays. Note that the time axis is upwards in the converted image.

The similarity between the curves defined by the converted features and the image rays instigate us to understand them and to figure out if they are rays (Silva et al, 2009). The name model rays was given because they are associated with vertical lines in the model space (depth domain), while image rays are associated with vertical lines in the image space (time domain).

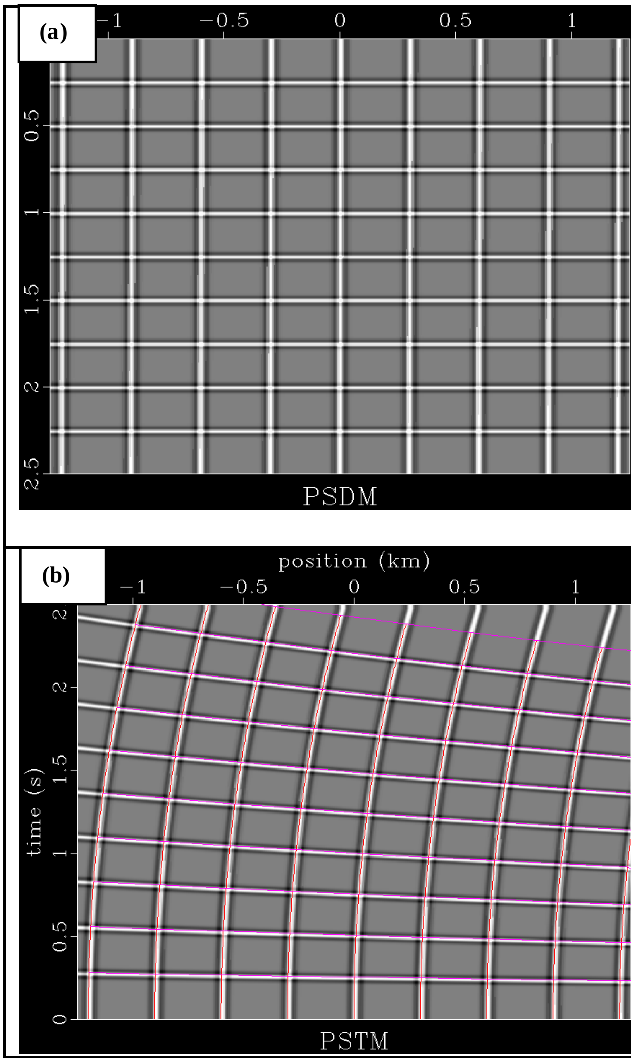


***Figure b: a) Artificial seismic image in depth. b) Time-converted image using the image ray collecting algorithm.***

## Equations

The search for equations to trace these curves led us to find

$$\left(\frac{\partial z}{\partial \xi}\right)^2 + \frac{1}{v^2}\left(\frac{\partial z}{\partial \tau}\right)^2 = 1 \qquad (1)$$

which is the first-order non-linear partial differential equation that governs the wavefronts propagation in time domain. In this equation $\xi$ is the CMP coordinate, $\tau$ is time, $v$ is velocity and $z = z(\xi,\tau)$ is the wavefront in time domain. This equation is of the eikonal type and the ray theory says that the rays are characteristic curves of an eikonal equation. This is why led us to name the curves as model rays.

The first step is to write the eikonal equation (1) in the form of a Hamiltonian:

$$H(\xi,q)=\frac{1}{2}\left[q_{\xi}^{2}+\frac{1}{v^{2}}q_{\tau}^{2}-1\right] \qquad (2)$$

where $H$ is the Hamiltonian, $\boldsymbol{q}=(q_{\xi},q_{\tau})$ is the slowness vector, $\boldsymbol{\xi}=(\xi,\tau)$ is the position vector in time domain.

Following Cerveny (2001), given any Hamiltonian, a solution can be found using the method of characteristics, which are the solutions of the system of first-order differential equation

$$\begin{cases}\dfrac{dx}{ds}=\nabla_{p}H\\ \dfrac{dp}{ds}=-\nabla_{x}H\\ \dfrac{dt}{ds}=p^{T}\nabla_{p}H\end{cases} \qquad (3)$$

where $H$ is a real valued function, called the Hamiltonian, $\boldsymbol{p}=(p_x,p_y)$ is the slowness vector, $\boldsymbol{x}=(x,y)$ is the position vector in time domain, $t$ is the traveltime and $s$ is an integration parameter. Therefore, the model ray equations are

$$\begin{cases}\dfrac{dq_{\xi}}{dz}=\dfrac{1}{v^{3}}\dfrac{\partial v}{\partial \xi}q_{\xi}^{2}\\ \dfrac{dq_{\tau}}{dz}=\dfrac{1}{v^{3}}\dfrac{\partial v}{\partial \tau}q_{\tau}^{2}\\ \dfrac{dt}{dz}=\dfrac{1}{v^{2}}q_{\tau}\\ \dfrac{d\xi}{dz}=q_{\xi}\end{cases} \qquad (4)$$

where t is the traveltime and z (depth) is the integration parameter.

As we saw earlier, the inverse transformation from depth to time can be easily performed from gathering amplitudes along image rays. In the same way, time-to-depth conversion can be carried out by means of collecting values along model rays. Simply speaking, algorithms involving collection of values are generally much simpler than those involving scattering. Another advantage of using model rays is that they require velocity field in time domain, which can be obtained directly from time-migration velocity analysis.

**Experiment**

Figure 4 summarizes the results of a simple experiment designed to validate the system of model ray equations. The idea is to compare the trajectory of a model ray traced by solving the system 4, with other two different methods. The first method tracks the model ray trajectory by tracing several image rays, whose initial conditions are defined by paraxial theory. The second method uses a finite-difference algorithm to solve an Huygens type equation to trace pseudo wavefronts and its orthogonal lines, which corresponds to model rays. Both methods are described in Silva et al, 2009. Note that there are no significant differences between the three plotted curves. It is possible to observe that exist more than one curve just because they are plotted with different line width.

The first step to solve the model ray tracing system is to create a velocity field in time domain. In this case, we just convert from depth to time the velocity model of Figure 1 with the image ray collecting algorithm. Then, the model ray is traced, step by step, solving the system 4 by means of the fourth order Runge Kutta method.

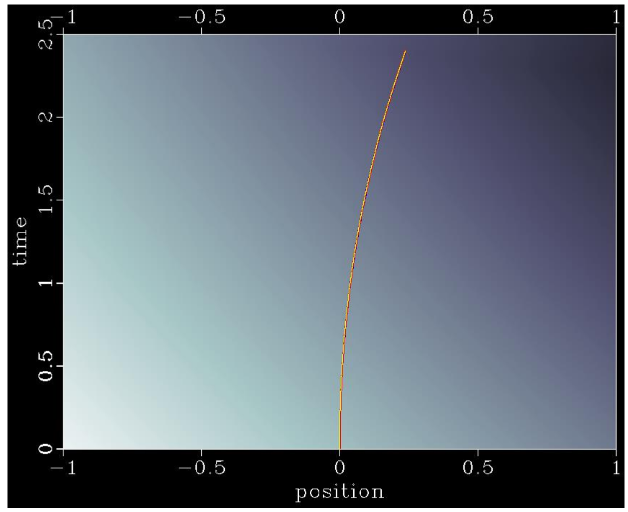


***Figure 4 - Velocity model in time superimposed by three model rays traced with different methods: 1) two-point image rays tracing, the wider line plotted in blue, 2) wavefront construction using Huygens principle, regular line plotted in red, and 3) the Runge Kutta method, the thinner line plotted in yellow. Note that time axis is upwards.***

**Conclusions**

Starting from a first-order non-linear partial differential equation in time, we use the zero-order ray theory to derive a kinematic ray tracing system for model rays.

We validate the equations system by comparing its trajectories with others obtained with two different methods.

We demonstrate that model rays are more suitable than image rays to perform time-to-depth conversion, because it permits the use of gathering algorithms which are much simpler than the spreading types. In this approach, amplitudes in time can be uniquely transferred to converted position in depth.

Another advantage of the use of model rays is that they make use of velocity field in time, which can be obtained directly from MVA for time migration.

**Acknowledgments**

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