Preliminary Pc3-pulsations analysis at conjugate stations using continuous wavelet transform

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Abstract

In this work, Pc3-pulsation data were analyzed for pairs of conjugate stations located at low latitudes (L-shell <2). While one pair is located in the Asia-Pacific region, the other is under the SAMA influence. This choice allows interesting comparisons. We carry out the signal spectral analyses using the continuous wavelet transform and wavelet coherence to investigate dynamics process features between conjugate stations in the time-frequency domain. The Pc3 pulsations showed an amplitude enhancement at the SAMA station and the SAMA-affected conjugate-pair coherence analysis presented a significant value (> 0.8). These differences between the conjugate-station pairs may arise from the unique characteristics of the SAMA presence, a region that presents an enhanced ionospheric conductivity under geomagnetic disturbance, probably involving the process related to cavity/waveguide modes.

Introduction

Magnetic pulsations are the ground manifestation of Ultra-Low frequency hydromagnetic waves propagating in the magnetosphere. Frequencies typically range between f ~ 1 mHz and f ~10 Hz; ground amplitudes range from less than 0.1 nT to tens or hundreds of nT and generally increase with latitude up to auroral regions. In this work, we focus on magnetic pulsations Pc3 (22 - 100 mHz) for their ability to assist in the diagnosis of the electrodynamic environment between the solar wind and magnetosphere-ionosphere system (Francia et al., 2009, Kamide et al., 2007). The dominant characteristics of these pulsations are consistent with those expected of the field line resonances (FLRs), which are transverse standing Alfvén waves along geomagnetic field lines, that is, equivalent to the concept of a vibrating field line fixed between the ionospheres in opposite hemispheres (Sutcliffe et al. 2011). Pc3 pulsations happen mainly due to the transmission into the magnetosphere of interplanetary upstream waves (Greenstadt et al., 1980; Ndiitwani & Sutcliffe 2009) generated through ion-cyclotron instability by protons reflected off the bow shock along the interplanetary magnetic field (IMF) lines (Francia et al., 2009). Some earlier studies have reported that Kelvin–Helmholtz instability (KHI) at the flanks of the magnetopause was often considered as a possible mechanism of Pc3 pulsations (Yagova et al., 2017). On the other hand, the wave structure may be affected by cavity modes (Menk et al., 2000).

The ionosphere is the region defined by the highest electron density in the atmosphere, with the upper limit about 2000 km and the lower one 70 km, but the most important contribution lies in the 90 – 1000 km region Kamide et al., 2007). It presents electrical and plasma peculiarities and can modify electromagnetic waves, such as pulsations. One way to aid in understanding the role of the ionosphere is to measure the pulsations simultaneously at geomagnetically conjugate points at low latitudes. In this work, the signal analysis approach, supported on the suggestions given by Obana et al., 2005 and Saito et al., 1989, enables the mitigation of the external influence of the solar wind and magnetosphere interaction.

Basically, as a conceptual model, two points on the Earth's surface linked by the same geomagnetic field line are called conjugate points (Wescott et al.,1966). To enrich the study, this work considers conjugate phenomena as occurring simultaneously and in a symmetric manner in a conjugate area established substantially by the actual linkage of magnetic field lines between the northern and southern hemispheres (Timoçin et al., 2018). Moreover, the pulsations features are remarkably similar in conjugate stations in terms of amplitude, bandwidth, quasiperiodic wave power modulation, and polarization (Kim et al., 2017).

The most significant and largest Earth's magnetic anomaly is the South Atlantic Magnetic Anomaly (SAMA), a region that extends from the East Pacific to South Africa, covering latitudes between 15°S and 45°S. In this region the main magnetic field has a significant magnitude reduction of the order of 22,000 nT, approximately one-third compared to the maximum magnetic field value. Sanchez et al., (2020) located the SAMA minimum value currently in northern Argentina, considered its center, which has historically displaced from Southern Africa to South America over the last 300 years (Domingos et al., 2017; Hartmann et al., 2009 and Pavon-Carrasco et al., 2016).

Due to the low field intensity over the SAMA region, the trapped and azimuthally drifting energetic particles originating from the inner radiation belt, precipitate deeply into the ionosphere and atmosphere (Abdu et al., 2005). The particle precipitation results in the enhanced ionospheric conductivity at the altitude of D and E layers due to an increase of ionization during both the quiet and disturbed periods suggested by observations of the ionosonde, radiometer, and very low frequency (VLF) radio propagation (Abdu et al., 2005) reported that the ionospheric conductivity tends to be more enhanced during the disturbed periods than during the quiet ones. Trivedi et al., (2005) reported that the SC amplitude during the main phase tends to be more enhanced near the center of the SAMA region than in the other regions, based on the analysis results of 37 SC events. They interpreted this observation as an intensification of ionospheric currents due to the enhancement of ionospheric conductivity associated with the precipitation of electrons and ions from the inner radiation belt into the atmosphere of the SAMA region.

For a Pc3 pulsation investigation in conjugated stations affected by the SAMA, a fundamental question arises. How much different are the spectral characteristics of Pc3 pulsations measured at conjugate stations when one of the areas is located inside the anomaly? The answer to this question could help us to improve our understanding of the behavior of the Pc3 pulsations in the SAMA region.

Our main objective is to analyze geomagnetic data from conjugate stations involving the SAMA. In this exploratory case study, the horizontal geomagnetic field (H) components were simultaneously recorded at two conjugate pair stations. We characterize the Pc3 pulsations and look for patterns by comparing the behaviors in conjugate stations. Moreover, we compare the low-latitude Pc3 characterizations for conjugate-pair stations affected and not affected by the SAMA. Composing the methodology, we use two techniques: time-frequency analysis, and coherence analysis. Initially using the continuous wavelet transform (CTW), highlights non-linear and non-stationary process conditions in conjugate-station measurements. The last allows us to evaluate the similarity of Pc3 pulsations between conjugated stations.

Geomagnetic data-sets

In this work, we use the geomagnetic horizontal (H) component. This component is more susceptible to the modulation effects and, therefore, it is more suitable for the study of Pc3 pulsations in conjugate areas (Obana et al., 2005, Tanaka et al., 2004). The record sampling rate is of 1 second for all stations. Besides, the data sampling period allows us to analyze the Pc3.

This work uses data of the period between 01:00 UT and 04:00 UT on 25th UT October 2016 which corresponds to a geomagnetically disturbed period (Kp = 3). Previous studies have shown that most cases of Pc3 occur in the range of 1 < Kp < 3.5 (Jacob 1970, Yagova et al., 2017). Fortunately, the period of the storm was close to the September equinox, which ensures equally illumination by the Sun on both Earth's hemispheres. This condition creates inter-hemispheric symmetry and similarities both in conductivities and atmospheric electric current systems between the conjugate areas (Hartinger et al., 2017, Timoçin et al., 2018). Those atmospheric characteristics enable dealing with a physical system somehow more simplified. Besides, due to a disturbed period with solar wind pressure variations, provides a unique opportunity to investigate how the electrodynamical coupling between the solar wind and the magnetosphere-ionosphere system affects the low latitudes.

Ground-Based stations

The selected stations are located at low latitudes. Theoretically, they are close to the feet of the same magnetic field line. The locations were obtained according to Altitude Adjusted Corrected Geomagnetic (AACGM) coordinates. This model uses the IGRF-13 to define the magnetic field line traces from one hemisphere to another, according to the procedure from Shepherd (2014) and Laundal et al., (2017). AACGM model also allows the calculation of the maximum altitude h_{eq} of the magnetic flux line between the stations. Stations used here belong to INTERMAGNET (International Real-time Magnetic Observatory Network) and EMBRACE MagNet (Brazilian Study and Monitoring of Space Weather) (Denardini et al., 2018). Table 1 presents the geographic and conjugate coordinates, magnetic field intensity B and L-shell values.

Table 1: Coordinates of the geomagnetic stations.

		Coordinates						
		Geographic		Conjugate				
	Stations	Lat [°]	Lon [°]	Lat [°]	Lon [°]	B [nT]	heq [km]	L [R_E]
KAK	Kakioka	36.23	140.18	-20.21	138.97	46737	2417	1.47
ASP	Alice Springs	-23.77	133.88	39.93	138.97	53139		
SJG	San Juan	18.11	-66.15	-35.2	-55.06	37284	1260	1.28
SMS	São Martinho da Serra	-29.44	-53.82	14.38	-64.4	22460		

Method

To obtain these Pc3 pulsations, we use a band-pass filter applied to the H component. Among the available techniques to filter, the infinite impulse response (IIR) band-pass filter (Balasis et al., 2016, Zanandrea et al., 2004) has been selected because it provides the best performance. The band-pass filter frequency range was 22 - 100 mHz (frequency range of the Pc3), where the filtering obtains the best identification of the pulsations of interest.

Time-frequency analysis

Our time-scale analysis consists in the computation of the wavelet coefficients obtained from the continuous wavelet transform (CWT) of the filtered signal (x, Pc3 pulsations). CWT is a powerful mathematical tool for time-frequency domain analysis of stationary and nonstationary time series. The wavelet coefficients $\mathcal{W}_x^{\psi}(a,b)$ provided by the CWT are defined by:

$$\mathcal{W}_x^{\psi}(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}x(t)\,\overline{\psi(\frac{t-b}{a})}dt \qquad a>0\,,$$

Where "*a*" is scale, "*b*" denotes translation and ψ_0 is the analyzing wavelet function, and is defined on the open time and real scale (*a*, *b*) half-plane (Daubechies 1992, Antoine et al., 2004). The analyzing wavelet function used in this work is the Morlet function defined by:

$$\psi_0(t)=\pi^{-1/4}\exp(i\omega_0)\exp(-t^2/2)$$

where ω_0 = 5 is a non-dimensional frequency parameter of the Morlet function, which is adapted to satisfy the wavelet admissibility condition, meaning that, for this value, the integration of the analyzing wavelet is near zero. Our purpose is to discriminate the distribution of signal energy per scale along the time, which plot helps to obtain the maximum value location and their temporal occurrence explicitly.

Coherence Analysis

In principle, coherence analysis can quantify any probable (co-relational) relationship between signals (Bortel et al., 2007). This part of the methodology evaluates if the Pc3 pulsations are coming practically from the same source region, *i.e.*, if they are observed simultaneously at the conjugate stations. Any dissimilarity can also identify disturbances caused both by locally morphological processes and asymmetrical effects from external sources.

The cross-wavelet transform is a wavelet analysis tool that enables us to deal with the time-scale dependencies between two-time series, defined as:

$$\mathcal{W}_{x,y}^{\psi}(a,b)=\mathcal{W}_x^{\psi}(a,b)\;\mathcal{W}_y^{\psi}(a,b),$$

Based on this cross-transform, we can compute the wavelet coherence $\mathcal{C}_{\mathcal{W}}^2(a,b)$ and a phase measure. They provide linear-quantitative estimators of the degree the relationship by scales between the two signals (Labat 2005, Torrence et al., 1998). Here, the wavelet coherence is computed using the following expression:

$$\mathcal{C}_{\mathcal{W}}^2(a,b)=\frac{\left|S(a^{-1}\mathcal{W}_{x,y}^{\psi}(a,b))\right|^2}{S\left(a^{-1}\left|\mathcal{W}_x^{\psi}(a,b)\right|^2\right)\;S\left(a^{-1}\left|\mathcal{W}_y^{\psi}(a,b)\right|^2\right)}$$

Where $\mathcal{C}_{\mathcal{W}}^2(a,b)$, as defined, ranges between 0 and 1, and "*S*" is a smooth operator in time-scale domain. We adopted the operator "*S*" as a convolution of the transform coefficients with a smoothing Gaussian moving average, in both time and scale directions (Grinsted et al., 2004). This smoothing process is used to avoid unnecessary noise due to the amplification provided by the product. We plot these coefficients in a colormap graph similar to the scalogram used in the wavelet transform. The wavelet coherence phase $\theta_{x,y}$, here called just as phase, is obtained by the real and imaginary coefficients:

$$\theta_{x,y}=\tan^{-1}\left(\frac{\Im\left\{S(a^{-1}\mathcal{W}_{x,y}^{\psi}(a,b))\right\}}{\Re\left\{S(a^{-1}\mathcal{W}_{x,y}^{\psi}(a,b))\right\}}\right)$$

This measurement yields information on the delay of one signal considering the other as a function of time and scale. Implementation of the WTC and wavelet coherence were based on a library of MATLAB functions provided by Grinsted et al. 2004.

Results and discussions

Figure 1 shows the Pc3 pulsations of each conjugate pairs (from top to bottom, each pair is indicated by red and blue). The time interval was chosen considering spaced significant Pc3-pulsation amplitudes during the magnetic disturbed period.

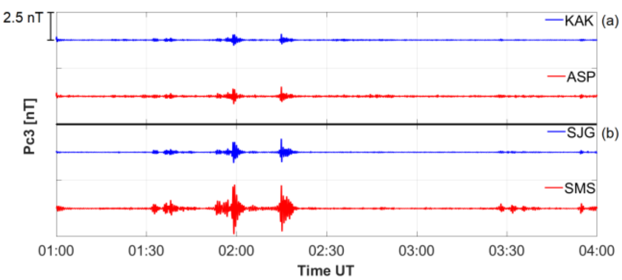

Figure 1: **Filtered Pc3 pulsations at conjugate stations. (a) KAK-ASP and (b) SJG-SMS.**

By means of a visual inspection, the plots in Figure 1 reveal consistent propagation patterns, *i.e.*, Pc3 wave packets, that are simultaneous and similar between the conjugate stations, especially between conjugate stations KAK-ASP. On the other hand, the SMS station shows pulsations more intense in amplitude than the pulsations in its conjugate station (SJG). In SMS there is an enhancement of Pc3 pulsations amplitudes at a location close to the center of the SAMA, compared to another station further away, as mentioned by Trivedi et al., (2005), (2004).

Figures 2 and 3 show the wavelet scalograms and their coherence for each conjugate station, respectively KAK-ASP and SJG-SMS, respectively. In the wavelet scalograms, the horizontal axis represents time in hours, and each vertical axis represents the signal intensity (in nT) and the wavelet scale (in mHz), respectively. The color bar has the same range (nT2/mHz) for all conjugate stations.

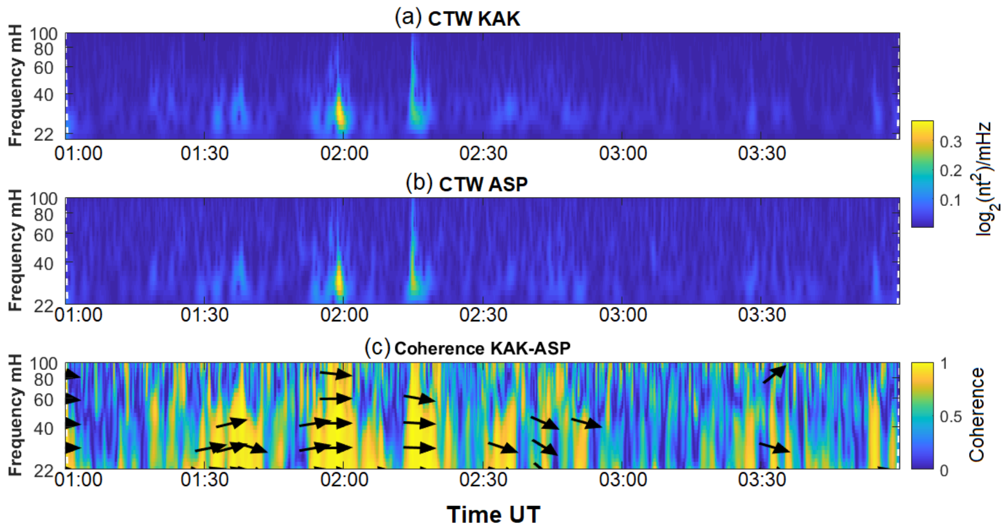

Figure 2: Squared-wavelet coefficients scalograms for each pair of conjugate stations: (a) KAK e (b) ASP. (c) Color-map of wavelet coherence.

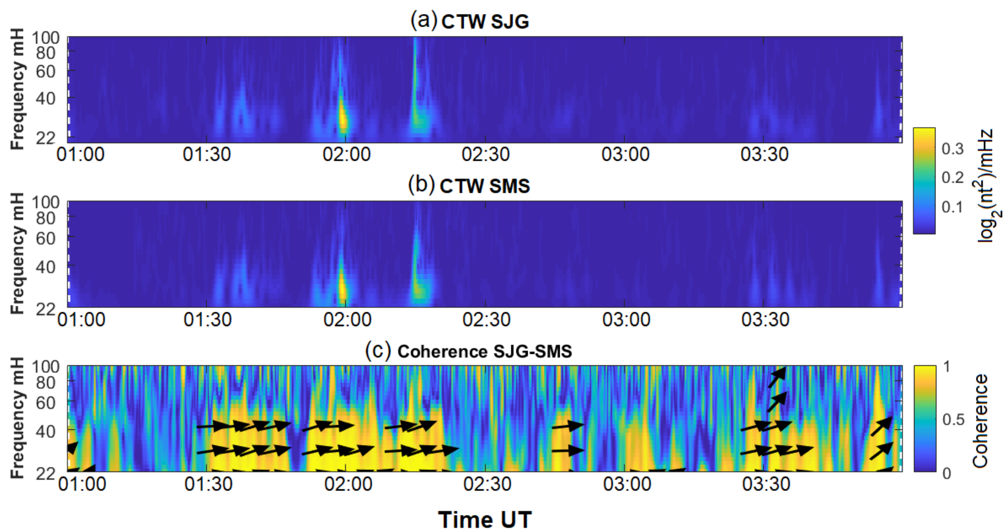

Figure 3: Squared-wavelet coefficients scalograms for each pair of conjugate stations: (a) SJG e (b) SMS. (c) Color-map of wavelet coherence.

In general, figure 2 (panel "a" and "b") shows that wavelet coefficients were found in association with Pc3 pulsations for each station, whose features were similar in amplitude, duration, and scale (converted to central-frequency) for the conjugate pairs. In most cases, wavelet coefficients show similar energy patterns between KAK-ASP conjugate pairs. Besides, the main frequencies are between 22-40 mHz. Furthermore, the colormap plot (panel C) shows that these stations present strong coherence (> 0.8) and that the pulsations are in phase, which is represented by arrows pointing right. Those characteristics could corroborate that the Pc3 pulsations observed are propagated from the same source region and that the stations are located on the same magnetic induction line.

Figure 3 (panel "a" and "b"), the maximum values of wavelet coefficients are indicated by the yellow color, which coincides with the more intense Pc3 wave packets (Figure 1, B). Similarity feature interpretation corresponds to an integration view of amplitude, duration, and scale (converted to central-frequency) for the stations, particularly the conjugate pairs. In most cases, wavelet coefficients show similar energy patterns between conjugate pairs. Besides, the primary spectra are between frequencies 22-40 mHz. On the other hand, the colormap plot shows the Pc3 pulsations are in phase, and also shows that these stations present strong coherence (> 0.8).

Conclusions

In this study was analyzed the horizontal (H) component data of the geomagnetic field from two conjugate station pairs located at low latitudes (L-shell < 2), but in two different regions, American-SAMA and Asia-Pacific. We search for comparison patterns of the Pc3 pulsations at conjugate stations, to characterize the pulsations behaviors at the conjugate stations.

In general, Pc3 pulsations between conjugate-pair stations present simultaneous and similar patterns in the occurrence interval. Besides, Pc3 pulsation occurrences present strong coherence and they are in phase in both regions. In the Asia-Pacific region, we observed that the Pc3 pulsations were similar in amplitude, wave structure and duration. Similarly, in the America-SAMA region. Otherwise, the SMS station (inside SAMA) showed a generalized amplification in the amplitude and spectral power (PSD) compared with its conjugate station (northern hemisphere). Furthermore, a comparison of the amplitudes of the Pc3 pulsations over the different stations clearly shows their enhancement in the station SMS, located close to the SAMA center, concerning other stations outside from the SAMA. On the other hand, the wavelet coefficients show similar energy patterns between the conjugate stations, besides the wavelet coefficients allowed to locate structures with the highest energy values and to indicate that they were coincident in the pulsations Pc3 during a geomagnetically disturbed period (Kp = 3).

We suggest that Pc3 pulsations, reported here, could have been driven by Kelvin-Helmholtz instability at higher latitudes and propagate to lower latitudes through magnetic flux lines to the conjugate stations. However, the differences between conjugate stations (SJG-SMS) may arise from the unique characteristics of this sector, with a strong longitudinal variation of the magnetic field and precipitation of energetic particles from the inner Van Allen belts, due to the presence of the SAMA. We also can speculate that the SMS station, where the magnetic flux line is contained within the ionosphere (L-shell < 1.3), and near the center of the anomaly, the ionosphere exerts a more substantial influence on the amplification and the propagation of the Pc3 pulsations, probably connected/driven somehow by cavity/waveguide modes.

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