Estimation of the crustal extension through airborne magnetic data processing: application to the Ponta Grossa dike swarm, southern Brazil

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Abstract

In this work we present a method for estimating the crustal extension from magnetic anomaly profiles extracted from aeromagnetic data on a dike swarm located in the region known as Ponta Grossa Arch(PR, southern Brazil). By estimating the width of the dikes within a profile taken over the swarm, we can estimate how much the crust was stretched. We identify the position of each dike by the maximum of the analytic signal amplitude (ASA) of the magnetic signal, delimited by two minima. We also estimate their widths using formulas recently proposed in the literature. The input values for the formulas are obtained from the magnetic field derivatives. The methodology from the extraction of the profile to the calculation of the extension is implemented automatically through computer routines written in Matlab/Octave.

Introduction

Dikes are magnetic geological structures that, when occur in swarms, can be associated with the tectonic process of crustal extension, since they originate from the intrusion of magma into the fractures generated in the extensional process. This work aimed at estimating the crustal extension using the magnetic profiles of a dike swarm located in the Ponta Grossa Arch.

The width estimates are performed through the Signum transform method (Oliveira et al., 2017) that uses as input the roots of magnetic anomaly derivatives combinations. We decompose the vertical derivative of magnetization into even and odd parts, and turn the last into an even function by applying the Hilbert transform. Then we use a weighted average of the transformed functions to evaluate the parameters for each dike. The maximum between two minima of the ASA profile are used to identify the anomalies of each dike in the swarm (de Souza et al., 2019). We developed an algorithm written in Matlab/Octave language to automatically perform the separation, transformation and parameter estimation. After summing estimated widths we calculate the minimum crustal extension for the area.

Theory

Magnetic anomaly due to a dike and its derivatives

In the reference Oliveira et al. (2017) the authors present the equation of magnetized dike, its derivative, along with the Signum transform. The magnetic anomaly of a rectangular prism, can be simplified for the case where the dimensions relative to the thickness and length are at least 10 times greater than the width (Dentith and Mudge, 2014), as it is the case of most diabase dikes. In this case the terms relative to thickness and length are either too small due to attenuation or cancel each other out. The magnetic anomaly of a dike, centered on $x = x_0$, can be written as the sum of a symmetric function and an antisymmetric function (McGrath and Hood, 1970):

$$f(x) = A[\cos Q f^s(x) + \frac{1}{2}\sin Q f^a(x)]. \tag{1}$$

The functions $f^s(x)$ and $f^a(x)$ can be expressed by:

$$f^a(x) = \tan^{-1}\frac{X+a}{h} - \tan^{-1}\frac{X-a}{h} \tag{2}$$

and

$$f^s(x) = \ln\frac{(X+a)^2+h^2}{(X-a)^2+h^2} \tag{3}$$

where $X = x - x_0$, a is half-width of the dike and h is the depth to the top of the dike. The amplitude A is given by

$$\begin{cases} A = 2Jbc\sin\phi; \\ b = \sqrt{\sin^2 i + \cos^2 i \cos^2 d}; \\ c = \sqrt{\sin^2 I + \cos^2 I \cos^2 D}, \end{cases} \tag{4}$$

where J is the magnetization intensity, (i,d) are the resulting magnetic inclinations and declination, and (I,D) are the inclinations and declines of the Earth's magnetic field. The parameter ϕ represents the geological dip of the dike. In addition to the amplitude (equation 4), the geological dip is present in the definition of the effective dip angle Q

$$Q = \lambda + \psi - \phi - 90^o, \tag{5}$$

where λ and ψ are given by: the angles of inclination and declination: of the dike and the magnetic field of the Earth.

$$\tan\psi = \frac{\tan i}{\cos d}; \qquad \tan\lambda = \frac{\tan I}{\cos D}. \tag{6}$$

The vertical derivative of magnetization of the dike having induced magnetization only, in $z = 0$ and $x_0 = 0$ is:

$$f_z(x) = 2A\frac{a(a^2+h^2-x^2)}{(a^2+2ax+x^2+h^2)(a^2-2ax+x^2+h^2)}, \tag{7}$$

while the derivative in x is

$$f_x(x,z) = 2A\frac{-2ahx}{(a^2+2ax+x^2+h^2)(a^2-2ax+x^2+h^2)}. \quad (8)$$

Sine and Cosine Transforms

The cosine and sine transforms are defined, according to de Souza et al. (2019), respectively, as:

$$F_{cos}(k) = \mathscr{F}_{cos}[f(x)] = \int_{-\infty}^{\infty} f(t)\cos(kx)dx, \quad (9)$$

$$F_{sen}(k) = \mathscr{F}_{sen}[f(x)] = \int_{-\infty}^{\infty} f(t)\sin(kx)dx. \quad (10)$$

The inverse transformation can be written as:

$$f(x) = \mathscr{F}_{cos}^{-1}[F_{cos}(k)] = \int_{0}^{\infty} F_{cos}(k)\cos(kx)dk, \quad (11)$$

$$f(x) = \mathscr{F}_{sen}^{-1}[F_{sen}(k)] = \int_{0}^{\infty} F_{sen}(k)\sin(kx)dk. \quad (12)$$

We can write any function $f(x)$ as a sum of an even part and an odd part,

$$f(x) = f_e(x) + f_o(x), \quad (13)$$

where $f_o(-x) = -f_o(x)$ and $f_e(-x) = f_e(x)$ are odd and even functions, respectively. It can be shown from the properties of the sine and cosine transforms that

$$f_e(x) = \mathscr{F}_{cos}^{-1}[F_{cos}(k)], f_o(x) = -\mathscr{F}_{sen}^{-1}[F_{sen}(k)] \quad (14)$$

It follows from equation 1 that the vertical derivative of the anomaly profile $f(x)$ can be written as

$$f_z(x) = A[\cos Q f_z^e(x) + \sin Q f_z^o(x)], \quad (15)$$

where f_z^e and f_z^o represent the even and odd parts of the vertical derivative. Applying Hilbert's transform to the odd part we get

$$f_z^e = H[\sin Q f_z^o] = \sin Q f_z^e. \quad (16)$$

The reconstructed vertical derivative, *i.e.*, the function of a vertical dike, located on the magnetic pole and without remnant magnetization is given by any of the following equations

$$f_z^{cos}(x) = \frac{\mathscr{F}_{\cos}^{-1}[\mathscr{F}_{\cos}[f_z(x)]]}{\cos Q} = Af_z^e, \quad (17)$$

$$f_z^{\sin}(x) = \frac{H[\mathscr{F}_{\sin}^{-1}[\mathscr{F}_{\sin}[f_z(x)]]]}{\sin Q} = Af_z^e. \quad (18)$$

The dip angle Q of equation 5 according to Murthy (1985), is obtained by

$$Q = \tan^{-1}\frac{f_x(x_0)}{f_z(x_0)}. \quad (19)$$

Equations 17 and 18 are equivalent to the equation 15, when the dike has vertical dip angle $Q = 0^o$ and the transformations 17 and 18 are energy preserving.

For $Q \approx 0^o$ or $Q \approx 180^o$, the vertical derivative is dominated by $\cos Q$, while for $Q \approx 90^o$ or $Q \approx 270^o$it is dominated by $\sin Q$. Therefore, in the first case equation 17 produces more accurate results while in the second one, equation 18 yields better results. Using a simple average of the even and odd parts, $(f_z^e/\cos Q + f_z^o/\sin Q)/2$, would avoid manually choosing either the even or the odd part to reconstruct the function. However, this average is still problematic due to division by zero in one of its terms when $Q = 0^o$ or $Q = 90^o$. These singularities are avoided when performing a weighted average with a weight given by $w = |\cos Q|$. In this case, the singularities will be always canceled because $w = 0$ and $(1-w) = 1$ when $Q = 0^o$, while $w = 1$ and $(1-w) = 0$ when $Q = 90^o$:

$$f_z^{rec}(x) = wf_z^{\cos} + (1-w)f_z^{\sin}. \quad (20)$$

Equation 20 is our reconstructed vertical derivative z.

The amplitude of the analytic signal (ASA)

The analytic signal (ASA) of $f(x)$ is defined as follows:

$$a(x) = f(x) - iH[f(x)] \quad (21)$$

where the second term is the Hilbert transform of $f(x)$. This concept was applied by Nabighian (1972) to potential field data. The vertical derivative f_z can be expressed in terms of the Hilbert transform (De Souza et al., 2020)

$$f_z = -H[f_x], \quad (22)$$

then we can express the analytic signal as follows

$$a(x) = f_x - iH[f_x] = f_x + if_z, \quad (23)$$

The magnitude of the analytc signal (ASA) is given by

$$|a(x)| = ASA = \sqrt{{f_x}^2 + {f_z}^2}. \quad (24)$$

Depth and width estimations

The Signum transform of a function f is defined as follows:

$$ST[f(x,z)] = \begin{cases} \dfrac{f(x,z)}{|\ f(x,z)\ |}, & f(x,z) \neq 0; \\ 1, & f(x,z) = 0, \end{cases} \quad (25)$$

where x, z are spatial Cartesian coordinates. The $ST[f]$ values of -1 or +1. In theory the $ST[f]$ values are 1 over the magnetic bodies and -1 out of them. The points where $f_z = 0$, termed x_v is

$$x_v = \pm\sqrt{a^2+h^2}. \quad (26)$$

The roots of $f_z - |\ f_x\ |$ are given by x_{hv-}

$$x_{hv-} = \pm(h - \sqrt{a^2+2h^2}). \quad (27)$$

To obtain the depth of the dike h, we combine x_v and x_{hv-}:

$$h = \frac{x_v^2 + x_{hv-}^2}{2x_{vh-}}. \quad (28)$$

Using the depth 28 we find the expression for the parameter a:

$$a = \sqrt{x_v^2 - h^2}. \quad (29)$$

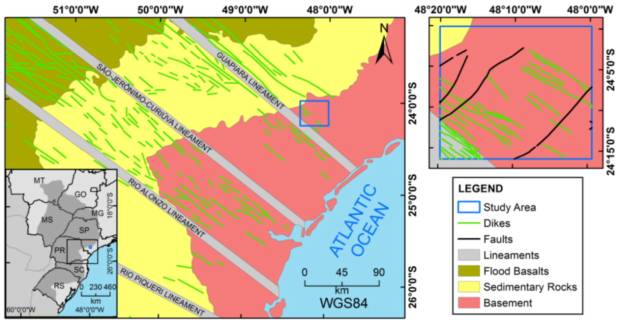

Figure 1: Simplified geologic map of the Ponta Grossa Arch. Source: Gomes et al. (2018).

Study area

As an example we prepare a map of magnetic anomalies of the study region, located at Ponta Grossa Arch, southern Brazil (Figure 1). The data were collected in an airborne survey by CPRM (the Brazilian Geological Survey), over an area of approximately 1020 km^2, with sapling spacing of approximately 500 m, covering an area of 34 km for 30 km. The testing profile was taken approximately perpendicular to the dike swarm main direction.

Results and discussion

In the section, numerical methods were used for computational calculations written in Matlab/Octave language. The profile shown in the figure 1 was taken in a direction approximately perpendicularly to the dike swarm.

Scanning the x axis of the profile 4, the anomaly corresponding to each dike is separated such as its limits are between two minima with a maximum between them and the maximum (dike's center). A cut line ran from bottom to top over the vertical axis that makes the anomaly as symmetrical as possible with respect to the maximum. The algorithm discards anomalies that due to interference from the neighboring body or noise do not allow a reliable parameters estimates. Thus, instead of calculating the value of the crustal extent, its percentage is calculated, using the anomalies that produce realistic results

In Figure 2 the black lines represent the intensity of the profile anomalies, their maxima are in the center of each dike, and their minima represent the midpoint of one dike to another. The colors magenta and green do not encompass the entire anomaly because the program finds the point of greatest symmetry in relation to the maximum point of the dike.

Figure 3 illustrates (using Anomaly number 28 from Fig. 2) how the calculation of a and h is performed for each dike, where the x_v are the points where the x axis is cut into $f_{z,(x=x_v)} = 0$. In red we have the vertical magnetization f_z, and the blue points is the derivative of the magnetization on the x axis.

Firstly, the anomalies are interpolated and transformed through upward continuation filter, which simulates the sampling of the data in a higher altitude, attenuating the noise (which is equivalent to cutting high frequencies). Secondly, we perform the amplitude of the analytic signal of the profile (ASA). For the profile of the figure 4 (bottom) the

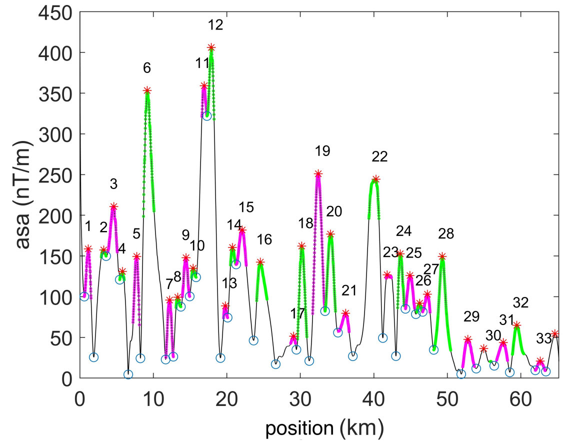

Figure 2: Analytic signal amplitude of the selected profile. The portion of the data actually used in the estimation of the parameters of each dyke is represented by the alternating magenta and green colors for a better visualization. The maximum points marked with stars * are the maximum intensity points of each dike (center of the dike) and the minima points are identified by ∘.

Table 1: Table with the parameters of the dikes for which it was possible to estimate the parameters.

Dike	a (km)	h (km)	$\bar{A}$ (km)
3	0.3501	0.3683	1.8396
5	0.2838	0.2442	1.6368
6	0.3516	0.4053	3.4184
15	0.3988	0.3873	2.3610
16	0.4078	0.3853	2.9984
18	0.2751	0.3143	1.7382
19	0.3645	0.4292	2.1293
21	0.4952	0.4020	2.0134
22	0.3984	0.3237	4.0703
24	0.2645	0.2445	1.2747
25	0.3232	0.3340	1.4050
28	0.1231	0.6936	3.7226
29	0.3847	0.4062	2.0279
31	0.4258	0.3771	2.1003
32	0.3021	0.4123	3.4619
Sum	5.1487	//	36.1979

amplitude of the analytic signal (ASA) is shown in the figure 4 (top). Subsequently, we find the maxima and minima of the ASA. Each anomaly associated with a dike will be defined by a maximum (the center) and its limits will be the minima on the left and right, respectively.

For each anomaly defined by a maximum between two minima, the depth and width parameters are calculated, using the equations 28 and 29. Adding all the calculated widths for each anomaly, an estimate of the minimum crustal extension for the studied area is obtained. The anomalies that yields nonphysical results, as widths greater than anomaly or imaginary values, are rejected.

The algorithm also provides as a product the sum of the dikes' half width a. In a total profile length of 65.2 km, 33 dikes were automatically identified, of which 15 have been

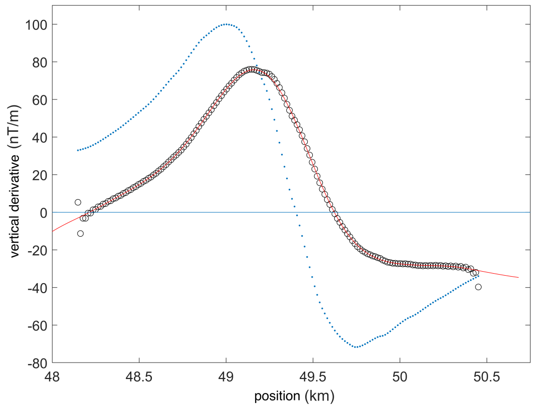

Figure 3: Vertical derivative (blue dots) of Anomaly number 28 (see Fig. 2), together with the reconstructed function (black circles), given by the equation 20, and a function fitted to the reconstructed curve (red curve).

used for the width estimates. It corresponds to $36,2$ km. The sum of the estimated dike widths is $5,15$ km, which corresponds to a minimum crustal extent of $14,2\%$.

Conclusion

In this work we introduced a methodology for estimating of the crustal extension from anomalies of the Ponta Grossa Arch dike swarm. The methodology was implemented through algorithms written in Matlab/Octave language. We obtained a minimum crustal extension of $14,2\%$, *i.e.*, the area of the arch was stretched, at least, by this value during its extensional process. The main drawback of the method is the difficulty in handling data with a high level of noise and interference, a common feature of dike swarms. Improving this characteristics could avoid reject too many anomalies. This improvement could be accomplished using synthetic models, enabling the method as tool for the interpretation of magnetic data from dike swarms, with potential applications in tectonics, hydrocarbon prospection, mining and hydrogeology.

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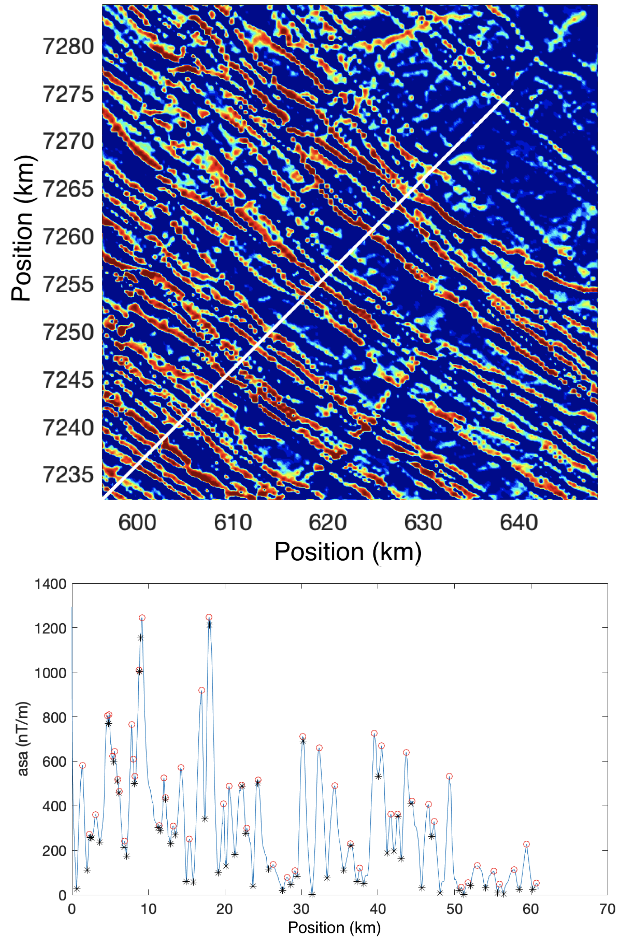

Figure 4: Top: Map of the first vertical derivative of aeromagnetic data for a region in the Ponta Grossa Arch. The white line indicates the location of the profile used in the experiment. Bottom: ASA profile along the white line

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