Comparative analysis of optimization algorithms for converted wave (PS) events

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Abstract

Latterly, to determine the RMS velocity of a nonhyperbolic event of the top of an ultra-deep reservoir is a very common challenge for the offshore exploration, which demands a more complex seismic processing. For this reason, an inversion procedure, under an optimization criterion, was performed to calculate the RMS velocity of the reflected PS event, with eight different travel-time approximations and five different optimization algorithms. Each approximation with each algorithm were compared, and the errors between the observed seismic event and the calculated one were computed. Then, it was possible to determine the best combination of nonhyperbolic approximation and optimization algorithm, for this kind of converted PS event.

Introduction

Previous works showed how complex is to perform an inversion procedure aiming to recover information concerning the velocity analysis for a reflected conventional (PP) or converted (PS) event (Aleixo and Schleicher, 2010; Golikov and Stovas, 2012; Zuniga 2017 and 2021). For this reason, not only different nonhyperbolic multiparametric travel-time approximations must be tested in order to find out which one produces the lowest error; the selection of the most appropriate optimization algorithm is almost equally important, which makes necessary to compare each nonhyperbolic approximation among each other, and with each selected optimization algorithm.

Seven equations, exhaustively tested in previous works (Aleixo and Schleicher, 2010; Golikov and Stovas, 2012; Zuniga 2017), were used and compared to a recently proposed approximation (Zuniga, 2021) —each one tested with five different optimization algorithms— aiming to find out which one is able to recover the velocity of the reflected event with the lowest error; and therefore, to find out which one provides the most appropriate RMS velocity.

Model

The model used in this work is from Pre-salt data analyzed from Santos Basin (Figure 1), and the reservoirs are at more than 5000 meters depth, and the water depth is more than 2000 meters depth.

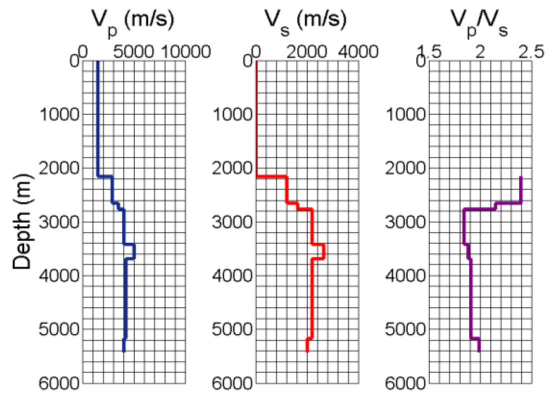

Figure 1: P-wave velocity (Vp), S-wave velocity (Vs) and Vp/Vs ratio for the Pre-salt model from Santos Basin.

Method

Several comparisons were performed considering different nonhyperbolic travel-time approximations (Zuniga, 2017); however, only recently (Zuniga, 2021), different optimization algorithms were tested to perform the inversion for different nonhyperbolic approximations.

The approximations tested in this work were previously studied also by different authors (*e.g.* Thomsen, 1986; Castle, 1988 and 1994; Tsvankin and Thomsen, 1994; Li and Yuan, 1999; Tsvankin and Grechka, 2000a and 2000b; Fomel and Grechka, 2000 and 2001; Tsvankin, 2001; Yuan and Li, 2002; Li, 2003).

For a comparison effect the hyperbola equation was used.

Equation 1 - Dix (1955), the hyperbola equation.

$$t = \sqrt{t_0^2 + \frac{x^2}{v^2}} \tag{1}$$

The nonhypeebolic multiparametric travel-time approximations used were:

Equation 2 - Malovichko (1978).

$$t = t_0^2\left(1-\frac{1}{S}\right)+\frac{1}{S}\sqrt{t_0^2+\frac{Sx^2}{v^2}} \qquad (2)$$

Equation 3 - Alkhalifah and Tsvankin (1995).

$$t = \sqrt{t_0^2+\frac{x^2}{v^2}-\frac{2\eta x^4}{v^2[t_0^2v^2+(1+2\eta)x^2]}} \qquad (3)$$

Equation 4 - Ursin and Stovas (2006).

$$t = \sqrt{t_0^2+\frac{x^2}{v^2}-\frac{(S-1)x^4}{4v^4\left(t_0^2+\frac{(S-1)}{2}\frac{x^2}{v^2}\right)}} \qquad (4)$$

Equation 5 (Blias, 2009).

$$t = \frac{1}{2}\sqrt{t_0^2+\frac{1-\sqrt{S-1}}{v^2}x^2}+\frac{1}{2}\sqrt{t_0^2+\frac{1+\sqrt{S-1}}{v^2}x^2} \qquad (5)$$

Equation 6 - Muir and Dellinger (1985).

$$t = \sqrt{t_0^2+\frac{x^2}{v^2}-\frac{f(1-f)x^4}{v^2(v^2t_0^2+fx^2)}} \qquad (6)$$

Equation 7 – Li and Yuan (2001).

$$t = \sqrt{t_0^2+\frac{x^2}{v^2}-\frac{(\gamma-1)}{\gamma v^2}\frac{(\gamma-1)x^4}{4t_0^2v^2+(\gamma-1)x^2}} \qquad (7)$$

Equation 8 – Zuniga (2021), the most recently approximation developed, which is proposed to be tested for several optimization algorithms.

$$t = \sqrt{t_0^2+\frac{x^2}{V^2}+\frac{-(\gamma-1)^2\,x^4\left(1+\frac{z_{WD}\,V_{WD}}{t_0\,V^2}\right)^4}{\gamma\,V^2\left[4t_0^2\,V^2+(1-\gamma)\,x^2\left(1+\frac{z_{WD}\,V_{WD}}{t_0\,V^2}\right)^2\right]}} \qquad (8)$$

Where, for the approximations used in this work, t is the travel-time, x is the offsets, t_0 is the time for zero-offset and v is the velocity of reflected wave. The S parameter is the heterogeneity parameter. The η parameter is the one which quantifies the nonhyperbolicity concerning the anisotropy. The f parameter is the anellipitical parameter. The γ parameter considers the effects of wave conversion, anisotropy and heterogeneity. For Eq. 8, the z_{WD} *and* V_{WD}, are *a priori* parameters, which are, respectively, the water depth and the velocity of the P-wave traveling through the water.

The four optimization algorithms used here with the multi-start procedure were:

FMINSEARCH (Find Minimum Search) is a MATLAB implementation of the Nelder-Mead algorithm (Nelder and Mead, 1965). This local search algorithm is focused in the use of a simplex, a polytope of n+1 vertices in n dimensions with edges of the same size, and it is useful for unconstrained optimization problems.

SID-PSM (Simplex Derivative - Pattern Search Method) is a local search algorithm implemented in MATLAB, based on a pattern search method with the pool step guided by simplex derivatives. This kind of algorithm was created to solve unconstrained and constrained problems, and each search step is based on the optimization of quadratic surrogate models (Custódio *et al*., 2010).

MCS (Multilevel Coordinate Search) is a MATLAB implementation of the algorithm for global optimization of bound-constrained problems (Neumaier *et al*., 2005). This algorithm is based on performing the partition of the search space into boxes with an evaluated base point.

The TOMLAB/LGO is a TOMLAB solver implemented in MATLAB. This kind of solver provides access to several derivative-free optimization solvers (Holmström *et al*., 2008). The LGO (Local and Global Optimization) solver used is a combination of global and local nonlinear solvers that implements a combination of Lipschitzian-based branch-and-bound with deterministic and stochastic local search.

It was computed the differences between the observed and the calculated curve as a percentual travel-time error, information which allow us to compare the accuracy among each approximation with each optimization algorithm.

Results

In a general manner, the hyperbola equation (Dix, 1955) showed the worst results, as it was expected, due to the complexity of the reflection events.

Figure 1 shows that the Equation 8 is more accurate for the first algorithm used.

Equation 7 showed very good results very similarly to Equation 8, while Equation 5 is just a little less accurate than the other two nonhyperbolic approximations.

Equations 2 and 6 showed less accurate results, even though their quality of the accuracy still being high. Equations 3 and 4 showed a higher error than the other approximation.

In Figure 2, it is possible to observe that the SID-PSM optimization algorithm showed a strong enhancement concerning the quality of the accuracy; while, in Figure 3, MCS algorithm showed another significant increase concerning the accuracy.

Figure 4 showed that for all approximations, none of them showed a significant increase concerning the accuracy when the TOMLAB/LGO optimization algorithm was used in comparison to MCS algorithm.

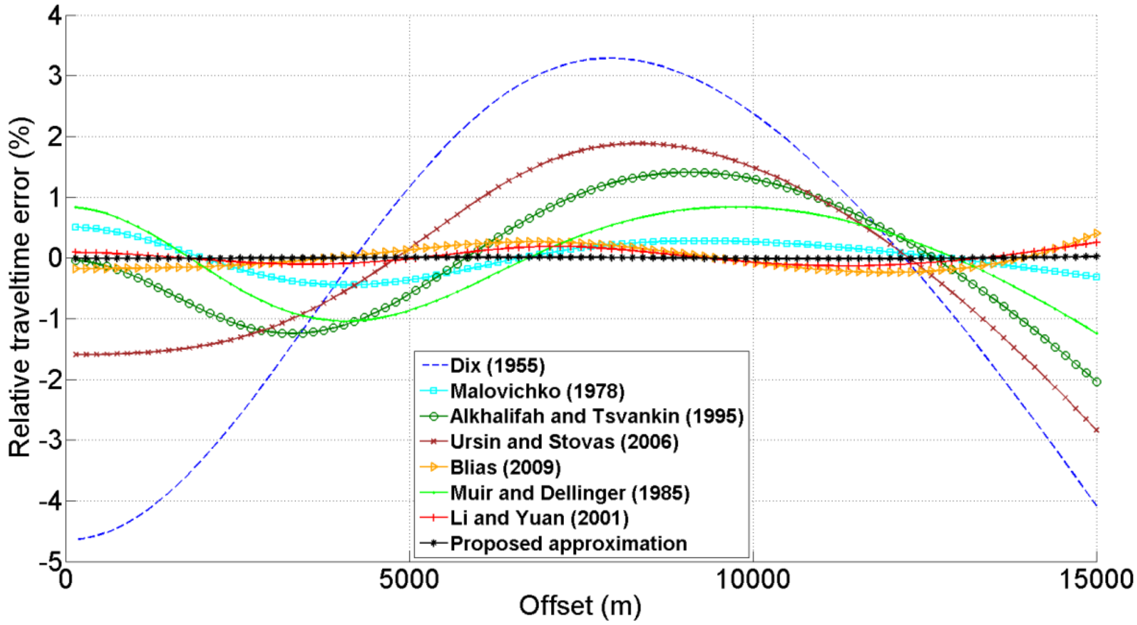

Figure 2: Relative error in travel-time between the observed curve and the calculated curve with each approximation of the PS reflection event with FMINSEARCH optimization algorithm.

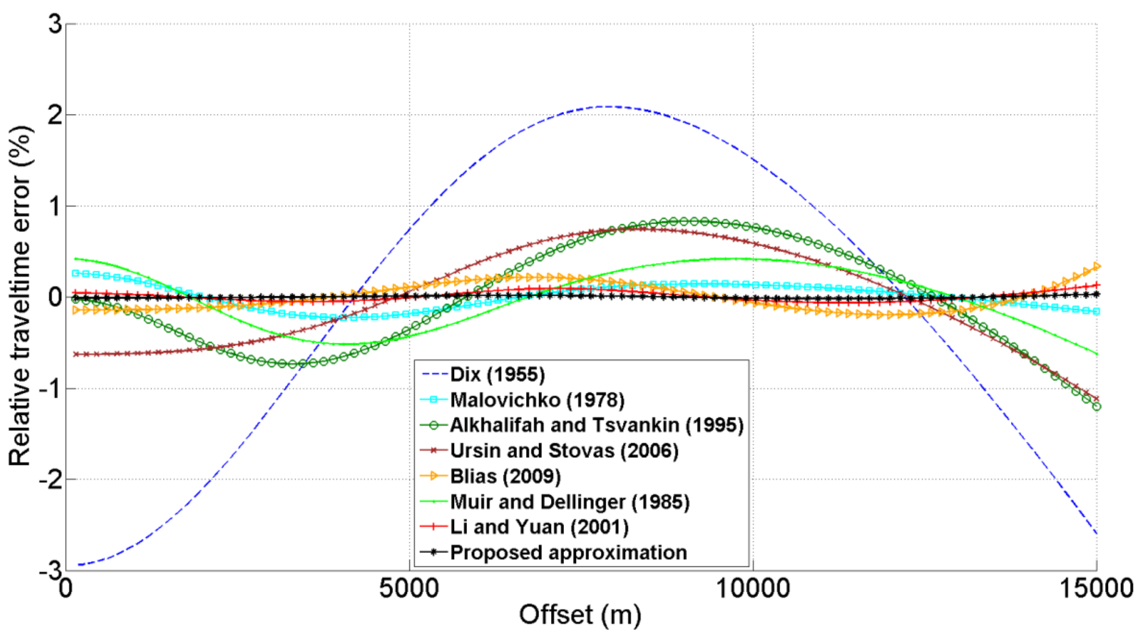

Figure 4: Relative error in travel-time between the observed curve and the calculated curve with each approximation of the PS reflection event with SID-PSM optimization algorithm.

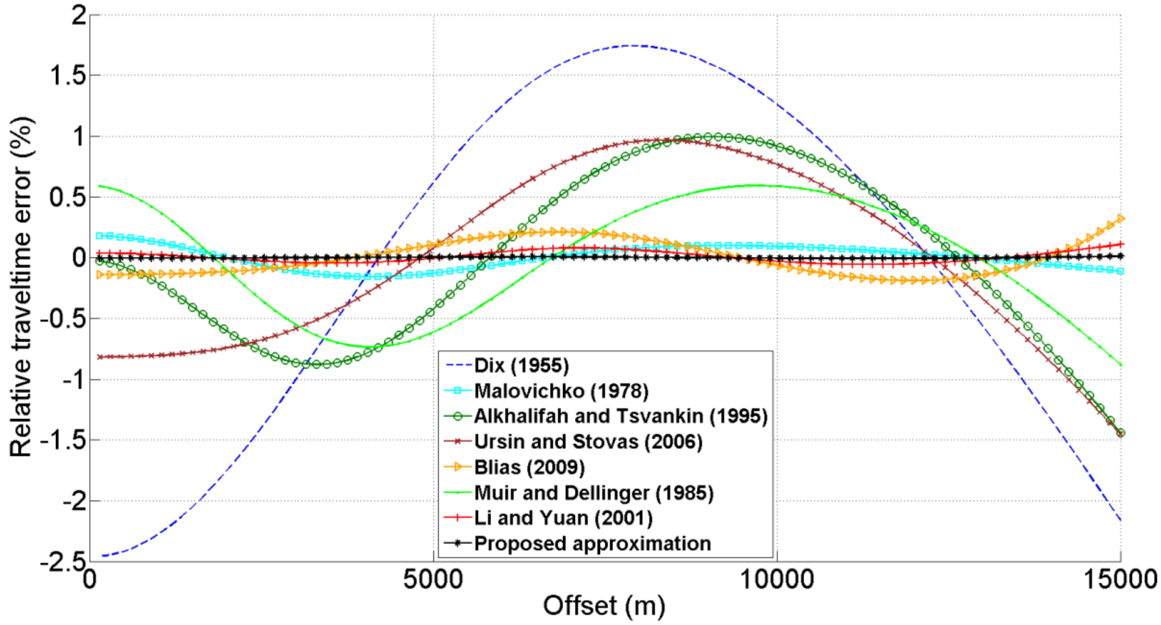

Figure 5: Relative error in travel-time between the observed curve and the calculated curve with each approximation of the PS reflection event with MCS optimization algorithm.

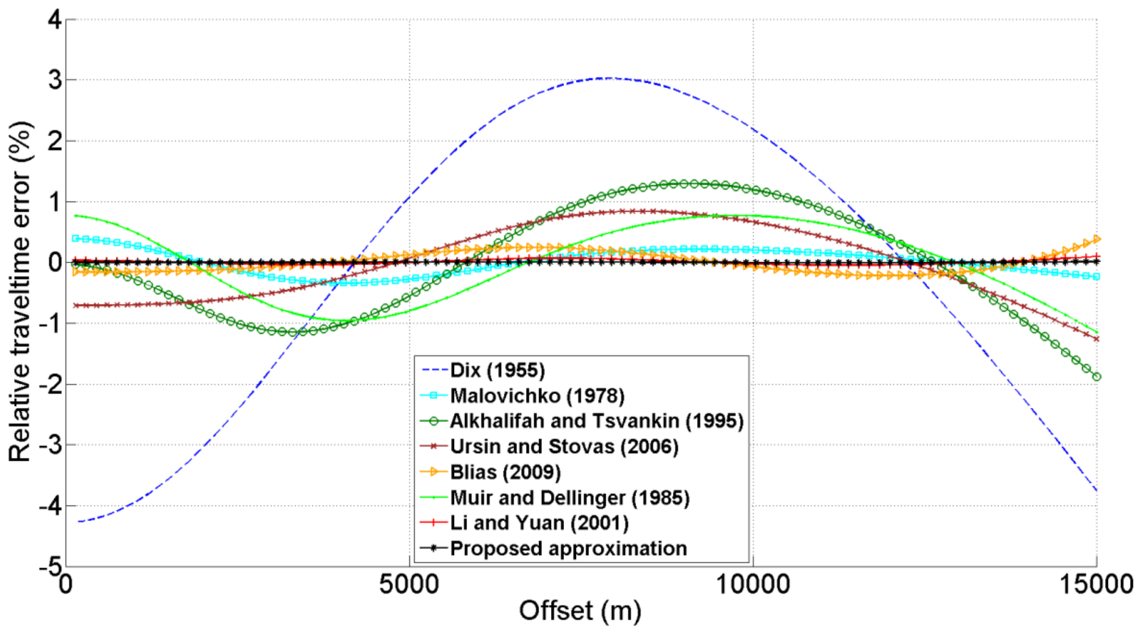

Figure 5: Relative error in travel-time between the observed curve and the calculated curve with each approximation of the PS reflection event with TOMLAB/LGO optimization algorithm.

Conclusions

Considering the results found by Zuniga (2021), the FMINSEARCH optimization algorithm presented to be 68% mor accurate, the SID-PSM algorithm showed a mean increase of 78%, the MCS algorithm presented to be 83% more accurate, and TOMLAB/LGO optimization algorithm showed to be the most accurate with 85% of increase concerning the accuracy. However, the processing time of the TOMLAB/LGO algorithm is almost twice higher than the MCS (Zuniga, 2021), which makes the MCS algorithm, the best option for this kind of problem, even though being 2% less accurate.

The Equation 8 (Zuniga, 2021) presented the best results for all optimization algorithms for this kind of reservoir. However, more tests must be performed for different structures, since this approximation is very recent and it has not been tested for a significant diversity of models.

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