Simulating attenuation in saturated sandstones with Dvorkin-Mavko model

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Abstract

Seismic methods are often used for reservoir characterization aiding structural delineation as well as rock property inference. Seismic attenuation is sensible to porosity, fluid saturation and permeability, such way the monitoring of that property can be very useful for understanding the differences of rock properties and fluid distribution along the reservoir. Inverse quality factor is direct related to seismic attenuation and can be estimated using several theoretical models. In this work, we used the Dvorkin-Mavko model for simulating P and S inverse quality factors (Q) of brine saturated sandstones and compare to experimental results. The results showed a relationship between inverse Q and porosity exhibiting a linear trend similar to what was observed in the experimental estimates.

Introduction

Seismic attenuation is one of the rock properties intensively studied for reservoir characterization. This property brings the link between elastic velocities, pore space and transport properties. Early works like Biot (1956) addressed such relationship and provided the first attenuation models. Subsequently works as Mavko and Jizba (1991) and Dvorkin *et al.* (1994) showed that not only the global flow affects the seismic wave attenuation, but local (squirt) flow issues are also important especially at high-frequencies. Dvorkin and Mavko (2006) introduced a model for estimating both P- and S- wave quality factors (Q_p^{-1} and Q_s^{-1} respectively) in rocks exhibiting partial or full saturation. This model was applied for estimating the inverse quality factor of sandstone rocks and compare to measurement estimates obtained at ultrasonic frequencies.

Methodology

The Dvorkin - Mavko model was used to simulate the quality factor of the P wave in brine saturated sandstones. To obtain all the necessary properties when using the Dvorkin - Mavko model, the Gassmann, Biot, and Geertsma-Smit models were also used. Next, all the models used will be detailed.

Theoretical models

Gassmann model

Gassmann's (1951) model correlates petrophysical properties of dry rock and the fluid contained within it with the incompressibility of saturated rock.

In addition, based on the premise that the saturating fluid has low viscosity, it considers that the saturation of the rock will not alter the modulus of rock shear. Mathematically, it is defined as:

$$\frac{K_{sat}}{K_{min} - K_{sat}} = \frac{K_{dry}}{K_{min} - K_{dry}} + \frac{K_{fl}}{\emptyset(K_{min} - K_{fl})}; \; G_{sat} = G_{dry}. \quad (1)$$

Where, K_{sat}, K_{min}, K_{dry} and K_{fl} are compression module of saturated rock, mineral matrix, dry rock, and saturating fluid, $\emptyset$ are the porosity and G_{dry} and G_{sat} shear modulus of dry and saturated rock.

Equation 1 calculates the Gassmann theoretical compressibility module of the rock saturated by a fluid. With these values of the saturated rock and the new average rock density it is possible, to calculate the wave propagation velocities P and S for the saturated rock:

$$V_{psat} = \sqrt{\frac{K_{sat} + \left(\frac{4}{3}\right) G_{sat}}{\rho_{sat}}}, \quad (2)$$

$$V_{ssat} = \sqrt{\frac{G_{sat}}{\rho_{sat}}}, \quad (3)$$

$$\rho_{sat = \rho_{dry} + \emptyset \rho_{fl}}. \quad (4)$$

Where the V_{psat} and V_{ssat} are the P and S wave velocity to saturated rock, ρ_{sat} is the bulk density of the saturated rock, ρ_{dry} is the dry density, and ρ_{fl} is the fluid density.

Biot's velocity relations

Biot (1956) derived theoretical formulas for predicting the frequency-dependent velocities of saturated rocks in terms of the dry-rock properties. That formulation incorporates some, but not all, of the mechanisms of

viscous and inertial interaction between the pore fluid and the mineral matrix of the rock (Mavko et al., 2009).

An important observation to be made is that, like the Gassmann model, the Biot model also considers that the change in the fluid contained within the pores of the rock does not change the shear modulus of the rock. Thus, the shear modulus for dry rock is equivalent to the shear modulus of saturated rock (Makarynska et al., 2010).

The calculation of velocities for Biot's high frequencies (infinite frequencies) is done using the equations below (Mavko et al., 2009):

$$
\begin{aligned}
v_{P\infty} = &\left\{\frac{1}{\rho_0(1-\emptyset)+\emptyset\rho_{fl}(1-\gamma^{-1})}\left[\left(K_{fr}+\frac{4}{3}G_{fr}\right)\right.\right. \\
&\left.\left.+\frac{\emptyset\frac{\rho}{\rho_{fl}}\gamma^{-1}+\left(1-\frac{K_{fr}}{K_0}\right)\left(1-\frac{K_{fr}}{K_0}-2\emptyset\gamma^{-1}\right)}{\left(1-\frac{K_{fr}}{K_0}-\emptyset\right)\frac{1}{K_0}+\frac{\emptyset}{K_{fl}}}\right]\right\}^{1/2},
\end{aligned}
\tag{5}
$$

$$
v_{s\infty} = \left(\frac{K_{fr}}{\rho-\emptyset\rho_{fl}\gamma^{-1}}\right)^{1/2},
\tag{6}
$$

$$
\rho = \rho_0(1-\emptyset)+\rho_{fl}\emptyset.
\tag{7}
$$

Where K_{fr}, G_{fr}, are the effective bulk and shear moduli of the rock frame, K_0 is the bulk modulus of the mineral material making up the rock, K_{fl} is the effective bulk modulus of the pore fluid, $\emptyset$ is the porosity, ρ_0 is the mineral density, ρ_{fl} is the fluid density, γ is the tortuosity parameter.

Geertsma–Smit approximations of Biot's relations

Biot's theoretical formulas predict the frequency-dependent velocities of saturated rocks in terms of the dry-rock properties. Low and middle-frequency approximations (Geertsma and Smit, 1961) of his relations may be expressed as:

$$
V_p^2 = \frac{V_{p\infty}^4+V_{p0}^4\left(\frac{f_c}{f}\right)^2}{V_{p\infty}^2+V_{p0}^4\left(\frac{f_c}{f}\right)^2}.
\tag{8}
$$

Where V_p is the frequency-dependent P-wave velocity of saturated rock V_{po} is the Biot–Gassmann low-frequency

limiting P-wave velocity, $V_{p\infty}$ is the Biot high-frequency limiting P-wave velocity, f is the frequency and f_c s Biot's reference frequency, which determines the low-frequency range, $f \ll f_c$, and the high-frequency range, $f \gg f_c$, given by

$$
f_c = \frac{\emptyset\eta}{2\pi\rho_{fl}k}.
\tag{9}
$$

Where $\emptyset$ is porosity, ρ_{fl} is the fluid density, η is the viscosity of the pore fluid and k is the absolute permeability of the rock.

Dvorkin–Mavko attenuation model

Dvorkin and Mavko (2006) present a theory for calculating the P- and S-wave inverse quality factors. The basis for the quality factor estimation is the standard linear solid model that links the inverse quality factor Q^{-1} to the corresponding elastic modulus M versus frequency f dispersion as (Mavko et al., 2009):

$$
Q^{-1}(f) = \frac{(M_\infty-M_0)(\frac{f}{f_c})}{\sqrt{M_0}M_\infty\left[1+\left(\frac{f}{f_c}\right)^2\right]}.
\tag{10}
$$

Where M_0 and M_∞ are the low- and high-frequency limits of the modulus M and f_c is the critical frequency.

Dataset

The core samples used in this modeling were taken from He et al., (2010) (Figure 1 and Table 1). Are sandstone from the W formation of the WXS Depression and covered low to nearly high porosity and permeability ranges.

In this work, laboratory data taken from Fidelis (2017) were used in this work to test the modeled data. Sandstones came from Utah, Ohio and Tennessee formation. Table 2 refers to the porosity and Qp of sandstones saturated with brine (1.3455 ppm of NaCl). The frequency range used by Fidelis (2017) was 1.3MHz.

Core	*W-7	W-10	W-11	W-13
Well	1212B	1214	1214	114N2
Depth (m)	2347.17	2478.35	2658.33	1868.6
P_{diff} (MPa)	28.7	30.7	33.5	21.6
Quartz (%)	~ 87	~ 80	~ 85	~ 75
Feldspar (%)	~ 5	~ 15	~ 10	~ 10
Clay (%)	< 5	~ 1	< 5	< 15
Calcite (%)	~ 1.5	< 4	< 0.1	< 0.1
Dolomite (%)	~ 1.5	0	0	0
ρ_f (g/cm³)	2.6497	2.6487	2.6445	2.6375
K_t (GPa)	37.608	41.359	38.0036	35.8677
G_t (GPa)	39.1073	39.7104	38.1724	31.5108
Grain size (mm)	0.07-0.2	0.1 - 0.25	0.25 - 0.5	0.25 - 1
D (mm)	25.72	25.64	25.7	25.48
L (mm)	40.3	49.82	41.21	45.94
ϕ (%)	16.46	13.55	17.87	23.73
k (mD)	3.872	2.495	63.552	424.2
ρ_{dry} (g/cm³)	2.13	2.24	2.05	1.86
Vp_{dry} (m/s)	3699.0	3875.0	3833.0	3388.0
Vs_{dry} (m/s)	2362.5	2563.5	2503.0	2177.5
γ	0.0055	0.0037	0.0057	0.0116

Figure 1 – Data used in modeling. P_{diff} is the differential pressure between confining pressure and pore fluid pressure at core collection depth; ρ_f, K_f and G_f are the estimated density, the bulk and shear moduli of solid matrix based on the thin section under the optical microscope; D is the diameter; L is the length; is the effective porosity; k is the permeability; ρ_{dry} is the dry density calculated after cleaning and oven-drying; V_{pdry} and V_{sdry} are the measured compressional and shear velocities at dry conditions; and η is the estimated microfracture aspect ratio (He et al., 2010).

Table 1- Fluid property (He et al., 2010).

Fluid type	Brine (NaCl 26000ppm)
Temperature (⁰C)	15
Bulk moduli od the fluid (GPa)	2.27
Viscosity (Pa.s)	0.00114

Table 2 - Porosity and P wave quality factor (Qp) data taken from Fidelis (2017).

Qp	Porosity (%)
13.57	15.10
11.97	15.76
15.29	10.12
16.21	10.25
16.45	7.71
16.73	7.92

Results

As indicated in Equation 10, the quality factor varies with frequency, given that, for testing purposes, the quality factor was modeled for two random frequency bands, 0.7 MHz and 1.3 MHz. According to Biot's, the critical frequency was classified in the high-frequency range for the modeled data , $f \gg f_c$.

Figure 2 shows the quality factor of the P wave, varying with porosity. As can be seen, the results for the frequency of 1.3 MHz were higher than for the frequency of 0.7 MHz, and with the increase in porosity, the quality factor drops, according to the literature.
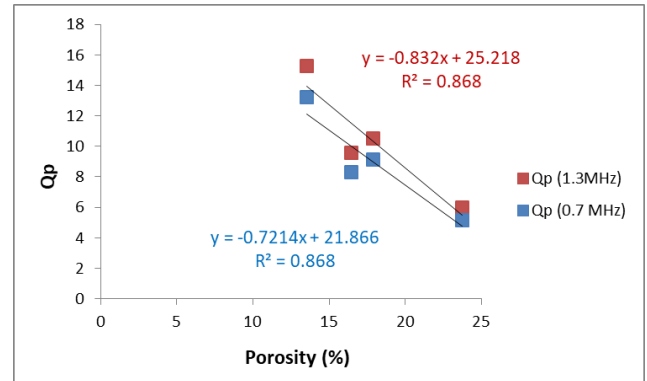

Figure 2- P wave quality factor versus porosity. Red squares refer to the frequency of 1.3 MHz and blue squares refer to the frequency of 0.7 MHz.

To test the modeling of this work, a set of attenuation data analyzed in the laboratory from Fidelis (2017) was compared with the results of the modeling.

Only data of the same frequencies were used for comparison, that is, 1.3 MHz.

Figure 3 presents the results of the data modeled in this work and the data taken from Fidelis (2017). Employing this figure, it can be said that the theoretical models were able to estimate very well the attenuation in sandstones saturated with brine since the modeled data were in the same trend as the laboratory data.
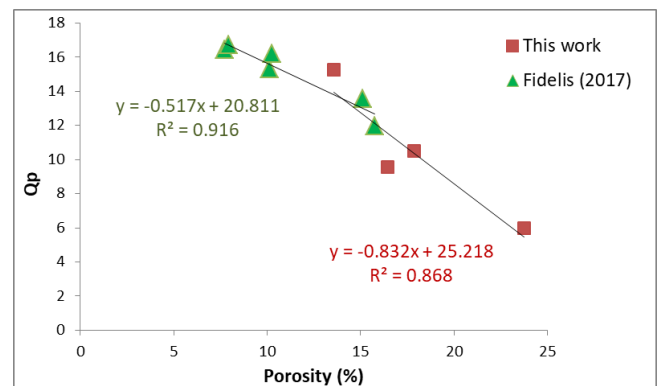
Figure 3- P wave quality factor versus porosity. Red squares refer to the modeling of this work and the green triangles given by Fidelis (2017). The correlation coefficients for the data in this work and the Fidelis (2017) data were respectively 0.868 and 0.916.

Conclusions

The theoretical models used in this work were able to estimate the attenuation in sandstones saturated with brine effectively, in addition, they presented a linear tendency with porosity, showing a correlation coefficient of 0.868.

When comparing the results of theoretical models with laboratory results, both exhibited the same trend, indicating that the model was accurate for predicting the inverse Q of the studied rocks.

Acknowledgments

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Available at: https://doi.org/10.1190/1.3360313