Backpropagation-based redatuming

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Abstract

Redatuming is used to relocate sources and receivers to a new depth level. This aim can be achieved by different techniques, for example the interferometric method or the wavefield continuation. In this work, we propose to use redatuming by backpropagating the seismic records. To apply this methodology, we need to know the medium above the redatuming level. To validate the theory discussed, we used 2D synthetic examples with different complexities in the overburden, and we compare the results to those of correlation-based redatuming. It was possible to note that the backpropagation-based redatuming presented similarities to the correlation-based one, regarding the events positioning and in the addition of artifacts. We also note that both techniques presented almost the same sensibility to an inexact velocity model. Despite the similarities, the method based on the wavefield continuation required significantly less computation time when we compare it to the interferometric technique.

Introduction

Seismic redatuming is used to correct distortions in seismic records caused by irregular acquisition topography, the weathering zone, strong lateral velocity variations in the subsurface, or large distances to the exploration target. Redatuming corresponds to moving, virtually, sources and receivers to a new depth level, called datum. This procedure allows to simulate a seismic experiment closer to the target (Wapenaar et al., 1992).

Over the years, several redatuming techniques have been proposed. The first one was wave-equation redatuming. This technique was presented by Berryhill (1979) for post-stack data and later extended to prestack data (Berryhill, 1984). It is based on the Kirchhoff integral and considered to be very accurate. However, this approach requires good information about the velocity model of the overburden as prior information. More recently, interferometric techniques have been presented. Based on the reciprocity theorems (Wapenaar and Fokkema, 2006), these techniques are guided by the data. Redatuming by crosscorrelation is a classical type of interferometric technique. In this technique, the virtual data are estimated from the original data by crosscorrelating with measured or modeled Green's functions in the overburden (Vasconcelos et al., 2009).

In this work, we use the interferometric formulas of der Neut et al. (2015) to introduce a new version of backpropagation-based redatuming. To evaluate its performance, we compare its results with those obtained by correlation-based redatuming. Our implementation carries out the backpropagation by means of a finite-difference solution of the acoustic wave equation. We used an exact and smooth overburden model to evaluate the performance in a more realistic scenario, where the exact velocity model between the acquisition surface and the datum level is not avaliable.

Theory

Throughout this work, we express the wavefield in the space-frequency domain as $\hat{P}(\mathbf{x},\omega)$. In this notation, $\mathbf{x}$ denotes the 3D space coordenates (x,y,z) and ω refers to the angular frequency. We consider a 3D acoustic medium with variable density $\rho(\mathbf{x})$ and propagation velocity $c(\mathbf{x})$.

We consider two independent acoustic states A and B, defined by medium parameters $\rho^{A,B}(\mathbf{x})$ and $c^{A,B}(\mathbf{x})$ and source terms $\hat{F}^{A,B}$. If inside a volume V enclosed by a surface S with an outward pointing normal vector $\mathbf{n}$, the medium parameters are the same for state A and B, we can use the Gauss' theorem and relate these two states as (Bleistein et al., 2001)

$$\oint_S \frac{1}{\rho(\mathbf{x})}(\hat{P}^B\nabla\hat{P}^A - \hat{P}^A\nabla\hat{P}^B)\cdot\mathbf{n}dS = \iiint_V \frac{1}{\rho(\mathbf{x})}\left(\hat{P}^A\hat{F}^B - \hat{P}^B\hat{F}^A\right)dV. \quad (1)$$

The medium parameter outside volume V can be different for states A and B. If all sources are situated inside volume V, the volume integral on the right side does not change is value when the integration domain is extended to infinity. However, in this situation the surface integral on the left side vanishes under Sommerfeld's radiation conditions (Bleistein et al., 2001). Thus, it must vanish independently of the shape of V. If we take V to be an infinite layer between the acquisition surface S_0 and the datum S_1, we can conclude that

$$\iint_{S_0} \frac{1}{\rho(\mathbf{x})}(\hat{P}^B\nabla\hat{P}^A - \hat{P}^A\nabla\hat{P}^B)\cdot\mathbf{n}_0 dS = -\iint_{S_1} \frac{1}{\rho(\mathbf{x})}(\hat{P}^B\nabla\hat{P}^A - \hat{P}^A\nabla\hat{P}^B)\cdot\mathbf{n}_1 dS, \quad (2)$$

which is known as the reciprocity theorem of the convolution type.The integrals are carried out over depth levels S_0 and S_1, with outward pointing normal vectors $\mathbf{n_0}$ and $\mathbf{n_1}$, respectively.

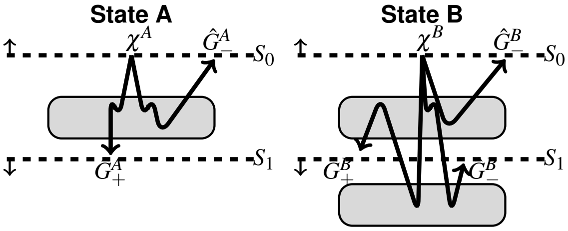


Figure 1: Illustration of acoustic states A and B. Surfaces S_0 and S_1 are infinitely long boundaries. In state A, the medium is reflection free below S_1, whereas in state B, it is identical to the physical medium below this surface.

Upon decomposition of the wavefield $P(\mathbf{x},\omega)$ into its up- and downgoing components and consideration of the stationary contribuitions to the integrals in a local high-frequency approximation, equation (2) can be recast into the form

$$\iint_{S_0} \frac{1}{\rho(\mathbf{x})}(\hat{P}_+^B \nabla \hat{P}_-^A + \hat{P}_-^B \nabla \hat{P}_+^A)\cdot \mathbf{n}_0 dS \approx \\ -\iint_{S_1} \frac{1}{\rho(\mathbf{x})}(\hat{P}_+^B \nabla \hat{P}_-^A + \hat{P}_-^B \nabla \hat{P}_+^A)\cdot \mathbf{n}_1 dS. \quad (3)$$

where the plus sign denotes the downgoing wavefield and the minus sign relates to the upgoing wavefield. Equation (3) is the one way reciprocity theorem of the convolution type. This equation forms the basis for our backpropagation-based redatuming method.

Backpropagation-based redatuming

To make use of equation (3) for redatuming, we specify states A and B as schematically illustrated in Figure 1. In both states, we consider the same arbitrary medium beetween surfaces S_0 and S_1. In state A, we consider homogeneous halfspaces without a free surface above S_0 and below S_1. We refer to this as the truncated medium (see also Barrera et al., 2021). In state B, we we also consider a homogeneous halfspace without a free surface above S_0. However, below S_1, we consider the physical (possibly unknown) medium, for which we want to redatum reflection data from S_0 to S_1.

In state A, we place a point source at $\mathbf{x}^A$ on S_0, creating a downgoing wavefield $\hat{P}_+^A = \hat{G}_+^A(\mathbf{x},\omega;\mathbf{x}^A)$. According to Wapenaar et al. (2014), its vertical derivative at a point $\mathbf{x}$ on S_0 can be expressed as $\partial_z \hat{G}_+^A = \frac{1}{2}\delta(x-x^A)(y-y^A)$. The upgoing wavefield $\hat{P}_-^A$ recorded at S_0 is produced by scattering at medium inhomogeneties between S_0 and S_1. If the truncated medium is sufficiently smooth, we can consider $\hat{P}_-^A \approx 0$. At a point $\mathbf{x}'$ on S_1, the upgoing wavefield $\hat{P}_-^A$ vanishes and the downgoing wavefield is $\hat{P}_+^A = \hat{G}_+^A(\mathbf{x}',\omega;\mathbf{x}^A)$. Correspondingly, in state B, a point source at $\mathbf{x}^B$ creates a downgoing wavefield $\hat{P}_+^B = \hat{G}_+^A(\mathbf{x},\omega;\mathbf{x}^B)$, with its vertical derivative given by $\partial_z \hat{G}_+^A = \frac{1}{2}\delta(x-x^B)(y-y^B)$. The upgoing wavefield recorded at S_0 for this state is the recorded seismic data, described by $\hat{P}_-^B = \hat{G}_-^B(\mathbf{x},\omega;\mathbf{x}^B)$. At S_1, we have the down- and upgoing wavefields given by $\hat{P}_\pm^B = \hat{G}_\pm^B(\mathbf{x}',\omega;\mathbf{x}^B)$. Inserting these expressions for the wavefields in equation (3), we obtain

$$\hat{G}_-^B(\mathbf{x}^A,\omega;\mathbf{x}^B) \approx 2\iint_{S_1} \hat{G}_-^B(\mathbf{x}',\omega;\mathbf{x}^B)\partial_z \hat{G}_+^A(\mathbf{x}',\omega;\mathbf{x}^A)dS. \quad (4)$$

Equation (4) is an approximation of an equation previously derived by der Neut et al. (2015) and Barrera et al. (2021). We recognize that the recorded wavefield at x^A due to a point source at x^B at the original acquisition surface S_0 can be represented, in the space-time domain, by a convolution between the upgoing wavefield at the datum level in the physical medium and the vertical derivative of the transmitted wavefield between the datum level and the acquisition level in the truncated medium. This convolution describes the propagation of $\hat{G}_-^B(\mathbf{x}',\omega;\mathbf{x}^B)$ from the datum level to the acquisition level. Thus, retrieving the wavefield at the datum level can be achieved by a numerical backpropagation of the recorded wavefield down to the desired receiver positions at the datum. As usual in seismic redatuming problems, the same procedure can then be applied a second time to common-receiver gathers at the datum, making use of source-receiver reciprocity, so as to achieve the redatuming of the sources as well. In this way, one can simulate a complete survey with sources and receivers at the datum.

Correlation-based redatuming

In this section, we present the basic formula for correlation-based redatuming as used for a comparison of the redatuming results. As before, we consider the medium as sketched in Figure 1. We start from the complex conjugate of the wavefield produced by a point source acting at $\mathbf{x}'$ in state A, given by $\hat{G}_-^{A*}(\mathbf{x},\omega;\mathbf{x}')$. We also consider that the wavefield recorded at the datum, produced by a point source at $\mathbf{x^B}$ in state B, is the scattered wavefield, $\hat{G}^S(\mathbf{x}',\omega;\mathbf{x^B})$. According to Vasconcelos et al. (2009) we can carry out a similar derivation like the one above using the reciprocity theorem of correlation type (Wapenaar and Fokkema, 2006) to represent the scattered wavefield at the datum by

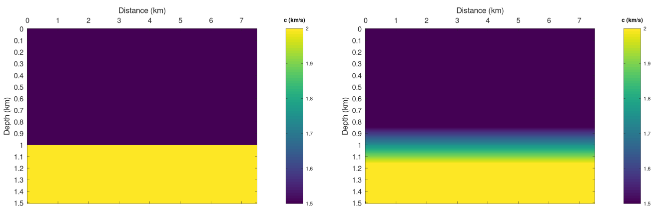
$$\hat{G}^S(\mathbf{x}',\omega;\mathbf{x^B}) \approx 2\iint_{S_0} \hat{G}_-^B(\mathbf{x},\omega;\mathbf{x^B})\partial_z \hat{G}_-^{A*}(\mathbf{x},\omega;\mathbf{x}')dS \quad (5)$$

On the right-hand side, $\hat{G}_-^B(\mathbf{x},\omega;\mathbf{x^B})$ is the seismic data, i.e. the wavefield recorded at receivers at $\mathbf{x}$ for a source at $\mathbf{x^B}$. Note that the integral is over the acquisition surface. Then, according to the equation (5), it is possible to redatum surface data by crosscorrelating them with the vertical derivative of the modeled transmitted wave from the datum to the acquisition surface at $\mathbf{x}$. By a single application of equation (5), only the receivers are redatumed from $\mathbf{x}$ to $\mathbf{x}'$. To complete the process, it has to be applied twice, using the source-receiver reciprocity, in the same way as explained above.

Results and applications

Single overburden interface model

The main purpose of this work is to provide a numerical comparison of the two redatuming procedures described by equations (4) and (5). After a consistency test with a single reflector below a homogeneous overburden (not



(a) (b)

Figure 2: (a) Exact 2D overburden velocity model (colors indicate propagation velocity in kilometers per second) and (b) its smoothed version.

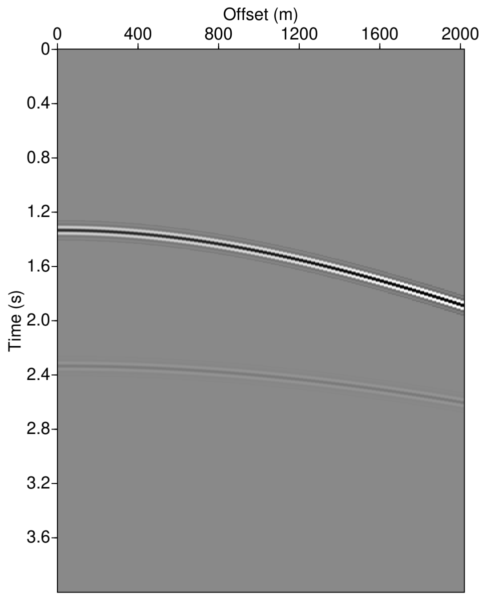


Figure 3: Reflection response for horizontal-layered velocity model with two layers.

shown here), we compared the behaviour for redatuming with a horizontal interface at 1 km depth, located between the acquisition and datum levels at 0 km and 1.5 km depth, respectively. Velocities are 1.5 km/s above and 2.0 km/s below the interface. This is in violation of the assumption for both tested methods that wave scattering in the overburden can be neglected. Figure 2a shows the true overburden model, and Figure 2b shows a smoothed version we used to simulate redatuming with the background velocity. The target event to be redatumed is a reflection at a second horizontal interface in 2 km depth, where the velocity increases to 2.5 km/s. We simulated a 2D seimsic acquisition with 41 sources spaced at 50 m and 100 receivers spaced at 20 m for each shot. One shot of the resulting data is shown in Figure 3.

We redatumed the data of Figure 3 with the exact overburden velocity model (Figure 2a) and a smoothed model (Figure 2b). Figure 4 compares the results using backpropagation (Figures 4a and 4b) to those obtained with crosscorrelation (Figures 4c and 4d) for a single common-shot section with 100 receivers. A dip filter was applied to reduce the artifacts caused by the limited aperture of the acquisition line. At first view, all four

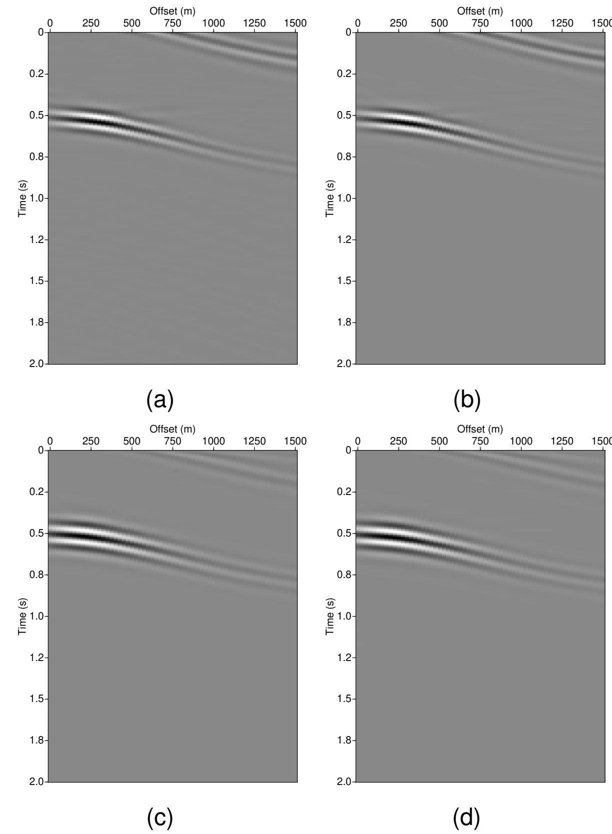


Figure 4: Redatuming using (a,b) backpropagation with the (a) exact and (b) smooth velocity model, and using (c,d) crosscorrelation with the (c) exact and (d) smooth velocity model.

sections of Figure 4 look very similar, indicating the comparable quality of the two redatuming procedures and their robustness regarding the use of a smoothed velocity model. In all cases, the event associated with the refletor in the overburden was not successfully removed for offsets greater than 0.5 km. Closer inspection reveals some slight differences between the results of backpropagation and crosscorrelation. The zero-offset trace seems to be slightly better recovered by the latter, such as the horizontal artifact at larger offsets at about 0.5 s which seems to be weaker in the crosscorrelation result. Regarding computation time, we observe that in this example, our implementation of crosscorrelation-based redatuming consumed about 1.5 times the CPU time of backpropagation-based redatuming.

For a more detailed evaluation of the redatuming results, Figure 5 compares the four redatumed zero-offset traces from Figure 4 to the modeled trace at the datum. We recognize high similarity of all traces, indicating the good quality of all four redatuming results.

SEG/EAGE Overthrust model

For an evaluation of the redatuming methods in a more realistic scenario, we also tested our implementation on 2D synthetic acoustic finite-difference data modeled in a slice of the 2D SEG/EAGE Overthrust model (Aminzadeh et al., 1997). The dimensions used were approximately

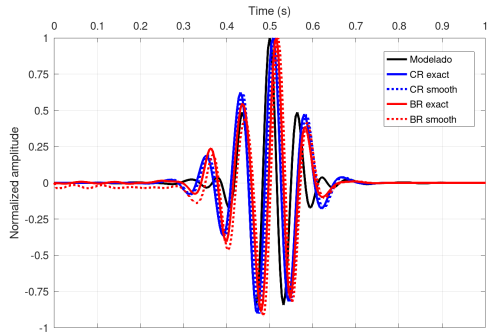
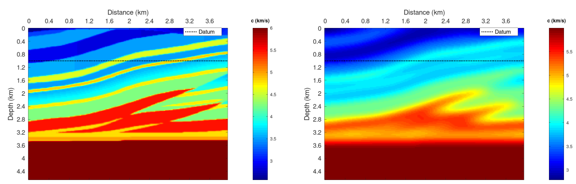


Figure 5: Comparison of the trace at zero offset obtained from redatuming using backpropagation and crosscorrelation in the exact and smooth overbuden.



(a) (b)

Figure 6: Slice of the SEG/EAGE Overthrust velocity model (a) and (b) its smoothed version.

4 km (horizontal) and 4.5 km (vertical), as shown in Figure 6a. We also redatumed using a smoothed version of the Overthrust model, as shown in Figure 6b. For the data generation, we placed 159 sources and 159 receivers with a spacing of 25 m along the surface. We chose the datum level to be at 1 km, with the same source and receiver configuration as at the acquisition surface.

In Figures 7a and 7b, we show a resulting common-shot gather for the complete redatuming process by using backpropagation with the exact and the smooth velocity model, respectively. For comparison, we also show in Figure 7c and 7d the results obtained by the crosscorrelation with exact and smooth model, respectively. The reference reponse in the medium, modeled with source and receivers at the datum, is shown in Figure 7e. We can observe that both tecniques retrieve all expected events, in accordance with the modeled data (see Figure 7e). We also note that the result obtained by backpropagation is very similar to the correlation technique, both with respect to the recovered reflection events and to the amount of artifacts present in the redatumed data, even when the smooth model was used. As in the previous example, redatuming by backpropagation took about a factor of 1.5 less computation time to complete the process than redatuming by crosscorrelation.

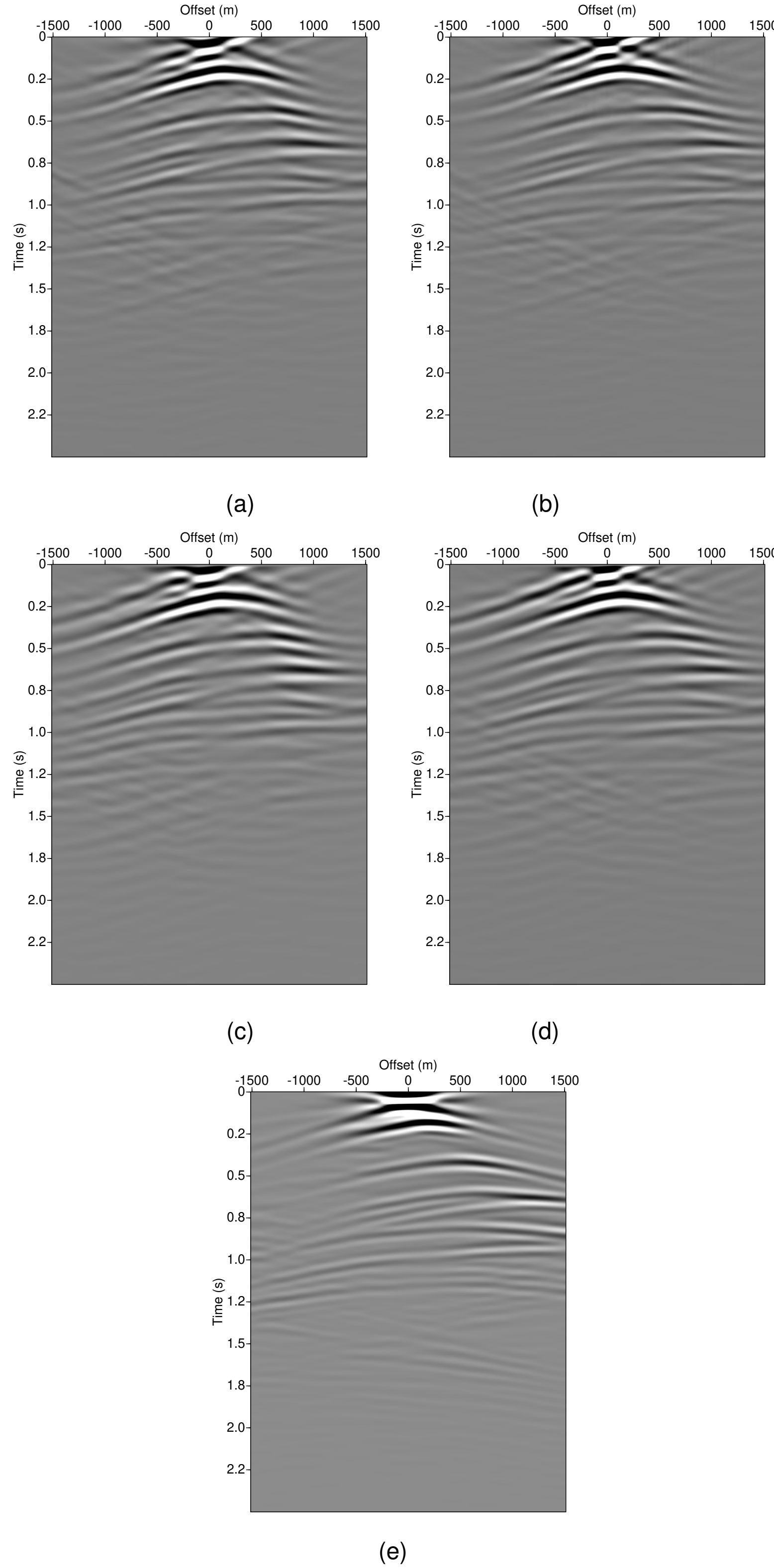


Figure 7: Redatuming using (a,b) backpropagation with the (a) exact and (b) smooth velocity model, and using (c,d) crosscorrelation with the (c) exact and (d) smooth velocity model. (e) Modeled data for sources and receivers at the datum

Conclusions

We have developed a new version of redatuming by backpropagation, based on an interferometric equation derived from the reciprocity theorem of the convolution type. This method performs redatuming by numerically backpropagating the seismic data from the acquisition surface to the datum in depth. For this purpose, it requires a velocity model in the overburden. As usual in redatuming, the sources are redatumed in a second step by backpropagating common-receiver gathers, making use of source-receiver reciprocity, in this way creating data for virtual sources and receivers at a desired depth level.

In our numerical tests, redatuming by backpropagation produced similar results to those obtained by correlation-based redatuming, as demonstrated with a simple and a more complex synthetic-data example. Our tests indicate that a smooth background velocity model can be sufficient to successfully redatum the data. As the major advantange of the backpropagation technique, the process requires less computational effort as compared to the correlation-based technique.

Acknowledgments

We are grateful to Petrobras, which funded part of this research by means of a research project, to CAPES, which provided a scholarship to develop this work, and to the INCT Program of CNPq for additional funds.

References

Aminzadeh, F., J. Brac, and T. Kunz, 1997, 3-d salt and overthrust models: SEG/EAGE 3-D Modeling Series No.1.

Barrera, D. F., J. Schleicher, and J. Brackenhoff, 2021, Interferometric redatuming by deconvolution and correlation-based focusing: Geophysics, **86**, no. 1, Q1–Q13.

Berryhill, J. R., 1979, Wave-equation datuming: Geophysics, **44**, 1329–1344.

——, 1984, Wave-equation datuming before stack: Geophysics, **49**, 2064–2066.

Bleistein, N., J. Cohen, and J. Stockwell, 2001, Mathematics of multidimensional seismic imaging, migration, and inversion: Springer, New York, USA.

der Neut, J. V., I. Vasconcelos, and K. Wapenaar, 2015, On Green's function retrieval by iterative substitution of the coupled Marchenko equations: Geophysical Journal International, **203**, 792–813.

Vasconcelos, I., R. Snieder, and H. Douma, 2009, Representation theorems and green's function retrieval for scattering in acoustic media: Phys. Rev. E, **80**, no. 3, 036605.

Wapenaar, C. P. A., H. L. H. Cox, and A. J. Berkhout, 1992, Elastic redatuming of multicomponent seismic data: Geophysical Prospecting, **40**, 465–482.

Wapenaar, K., and J. Fokkema, 2006, Green's function representations for seismic interferometry: Geophysics, **71**, SI33–SI46.

Wapenaar, K., J. Thorbecke, J. V. der Neut, F. Broggini, E. Slob, and R. Snieder, 2014, Green's function retrieval from reflection data, in absence of receiver at the virtual source position: The Journal of the Acoustical Society of America, **135**, 2847–2861.