EVALUATION OF THE INFLUENCE OF THE DENSITY VALUE IN THE RESIDUAL TERRAIN MODEL REDUCTION

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Abstract

The terrain corrections are an important part of reducing gravimetric data, which are useful tools for smoothing the anomalies of gravity that are applied in calculations of geoid models and other studies that include the area of geophysics, oceanography, geological prospecting, etc. There are several methods of reducing gravimetric data and we can mention as an example the corrections Bouguer, Rudzki, Helmert and the Residual Terrain Modelling technique (RTM). One of the parameters that has an influence on the reductions in gravimetric data that involve the correction of the terrain refers to the geological constitution since the density of the rocks causes small variations in the gravitational field depending on the dimension and also the depth in which they occur. In this context, the objective of this study is to verify the influence of the density value in the calculation of the gravity anomaly using the RTM reduction. Thus, experiments were developed using three different models of densities in a test area of approximately 477 km² on the perimeter of the city of Porto Alegre. These results were compared with the standard density value adopted in a large part of scientific studies in the area of Geodesy, in this case, 2.67g / cm³. The results involving the three density models showed significant mean differences ranging from -0.21 mGal to 0.88 mGal.

Key words: Terrain correction, Residual Terrain Model, Density.

Introduction

Gravimetric reductions that relate the gravitational field to the topographic masses make an important role in geodetic applications such as, for example, in geoid models and in other types of studies that require knowledge of the gravitational field. The attraction of the topographic masses generates a strong signal that dominates the gravitational spectrum at shorter wavelengths and the topography can be used to smooth out that field before any modeling process. The complete or refined Bouguer reduction consists of adding the terrain correction to the simplified Bouguer reduction. There are several mathematical models in the literature to represent the terrain correction, are prisms, tesseroids, point mass, etc. (HECK and SEITZ, 2007).

The RTM (Residual Terrain Model) reduction technique is a terrain correction technique, described by Forsberg (1984), which is used for the purpose of estimating the gravity field and uses a high-resolution digital elevation model. This technique uses the Newton integral to calculate the values of the gravitational potential that are used in the Geodetic Boundary Value Problem (GBVP) in the determination of the Earth´s external gravity field for the determination of the geoid. The RTM reduction has some advantages over the Bouguer reduction, since density anomalies have oscillating positive and negative values, integrations for gravity field effects only need to be done at some suitable distance from the calculation point, eliminating the effects of the influence of the relief depending on distance. In addition, the effects of the terrain on height anomalies end up being small due to the adoption of a high resolution digital elevation model (TZIAVOS, 2013).

In the context of studies involving the gravitational field, one of the most important simplifications refers to the use of an average value for the density of the topographic masses. This value ends up being used in land reductions such as the Bouguer reduction and also in the RTM reduction. This simplification consists of adopting the density value of the surface rocks of the continental crust as 2.67 g / cm³. In calculations involving anomalies of gravity an approximate value of density is normally used in corrections involving topographic masses that end up being more appropriate for local geology than the value of 2.67 g / cm³. This fact becomes important in regions where the surface materials are low density unconsolidated sediments or sedimentary rocks and the local relief is sufficient to cause significant errors in gravity reductions due to an incorrect density assumption. The error in the severity anomaly can vary around 4.19 mGal for every 100 meters in the difference in elevation for each 1g / cm³ of error in the density model (HINZE, 2003).

Thus, in works where a higher level of precision in the study of the gravitational field is needed, such as in geoid models and geophysical studies, it is necessary to adopt density models with a higher level of detail. In this context, the main objective of this study is to verify the influence of the density value in the calculation of the gravity anomaly using the RTM reduction. The studies were developed in an area of approximately 477 km² in the perimeter of the city of Porto Alegre, capital of the state of Rio Grande do Sul (Brazil). Three density models were tested and the first model was extracted using geological data from the Porto Alegre Environmental Atlas developed by Menegat et al (1999) on a 1:185,000 scale. The second density model was extracted from vector data from the Geological Map of the State of Rio Grande do Sul prepared by the Companhia de Pesquisa de Recursos Minerais (CPRM). The third model was developed by Sheng et al (2019).

Materials and methods

The study area, Figure 1, covers the city of Porto Alegre, capital of the state of Rio Grande do Sul (Brazil), which has an area of 497 km² with an extension of approximately 36 km on the north south axis and 18 km on the axis East West.

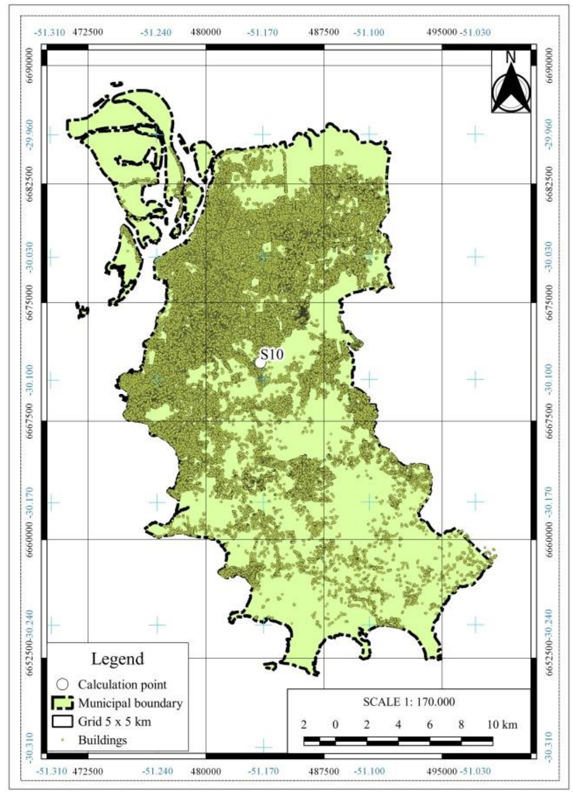

Figure 1 - Study area.

To calculate the value of gravity anomalies, using the RTM reduction (Δg_{RTM}), was used Eq. (1) (FORSBERG, 1984):

$$\Delta g_{RTM} = 2\pi G\rho(H - H_{ref}) - tc \qquad (1)$$

where G represents Newton's universal gravitation constant, ρ represents the value of the adopted density (generally 2670 kg / m³), H the origin of the calculation point, H_{ref} the height of the feature that will calculate the gravitational effect (FORSBERG, 1984; TZIAVOS; SIDERIS, 2013). In the second term of Eq. (1), in the case of terrain correction (tc), the geometric tesseroid element was used, whose mathematical development is found in Heck and Seitz (2007) and Wild-Pfeiffer (2008). The gravity anomaly that uses the RTM reduction (Δg_{RTM}) consists of Bouguer correction subtracted from the terrain correction. In this study, it was preferred to use the RTM reduction since it uses high resolution digital terrain models (MDT) in order to smooth out the shortest wavelengths that are related to topography. In this study, an MDT obtained from Lidar data (Light Detection and

Ranging) with a spatial resolution of two meters was used.

The data used to calculate the RTM reduction described in Eq. (1) refer to vector database of the study developed by Ferraz and Souza (2021) containing around 250 thousand centroids of buildings in the city of Porto Alegre. The altimetry data used refer to a digital terrain model (DTM), with spatial resolution of two meters, obtained from Lidar data measured in 2008 by the city hall of Porto Alegre. The estimate of positional errors of the point cloud generated by this system was around 7 cm, according to Zanardi et al (2013). The calculations were developed in Python programming language using the Numpy libraries for matrix operations, Pandas for database processing, and Matplotlib for graphic creation. To obtain the density values of each analyzed model, the software Qgis version 3.10.5 was used.

In the Table 01 contains an analysis of the maximum and minimum values for each density model analyzed in this study. Model 01 refers to the density model developed by Sheng (2019), Model 02 refers to the model developed by Menegat Et al (1999) and Model 03 the density model developed by the Companhia de Pesquisa de Recursos Minerais (CPRM).

VALUE	MODEL 01	MODEL 02	MODEL 03
Min	2.07	2.40	2.22
Max	2.79	3.20	3.20

Table 01 – Maximum and minimum values for each density model.

The spatial resolution of the Model 01 and Model 03 models was 30”. Model 02 was obtained from the vectorization of a map on a scale of 1: 185.000 and in this study it was resampled to 3”.

Results

Regarding the analysis of the results, the density values obtained from the Model 01, Model 02 and Model 03 density models were applied to the database and compared with the density value used in the vast majority of studies, such as, for example, Hinze (2003), in the case 2.67 g / cm³ or 2670 kg / m³. These values were used to calculate the value of the gravity anomaly in the RTM reduction.

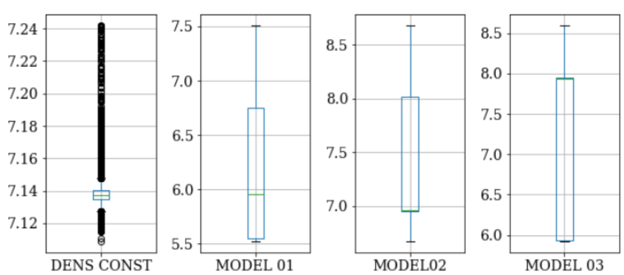

Figure 2 – Analysis of experiments.

In the Figure 2 contains an analysis of the results obtained in each experiment. The values in the graphs are in mGal. In the first experiment (DENS_CONST) in Figure 2, the density value of 2.67 g / cm³ was adopted in the calculation of the gravimetric anomaly. In the other experiments, the density values of each model analyzed in Table 01 were used. It was found that using a constant density value for a large database, in this case a vector base with 250 thousand building centroids, many outliers occurred. In comparison, using the models described in Table 01, no outlier was verified. Table 2 contains a statistical summary of the results obtained using the models described in Table 01 (results in mGal).

ANALYSIS	DENS CONST	MODEL 01	MODEL 02	MODEL 03
Mean	7.137	6.252	7.352	7.433
Std	0.006	0.657	0.635	1.014
Max	7.242	7.509	8.679	8.597
Min	7.109	5.521	6.670	5.920

Table 02 – Statistical analysis of the results obtained in each experiment.

In the Figure 3, there is a graphical analysis of the mean of the gravity anomaly (blue circle) and the value of the standard deviation (black lines) in each experiment.

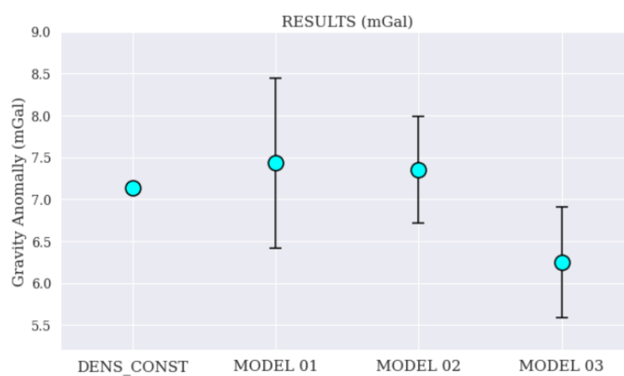

Figure 3 – Average and standard deviation for each experiment.

Although the experiment using the constant density value (DENS_CONST, in Figure 3) has shown the largest discrepancies, as shown in Figure 2, it is noted that the value of the standard deviation was almost null. In the other experiments, the value of the standard deviation and the mean oscillated a lot, but it was found that this oscillation was greater in the density models whose spatial resolution was lower, in this case the models with a spatial resolution of 30 ”(Model 01 and Model 03). Thus, it was found that in this study the spatial resolution of the density models generated a significant variation in the mean and in the value of the standard deviation of the value of the severity anomaly. Making an analysis of variance, these differences were significant with a significance level of 5%, with a p-value of 0.035.

Another analysis that was carried out in this study, was in relation to each of the terms referring to Eq.

(1). How the value of density enters into the calculation of the Bouguer reduction, which is the first term of Eq. (1), and it is also part of the calculation of the terrain correction (tc), which is the second term of Eq. (1), a correlation analysis was performed on each of these terms in isolation. This procedure was carried out with the purpose of verifying if there was a linearity relationship between the density value of each model and the Bouguer reduction along with the correction of the terrain.

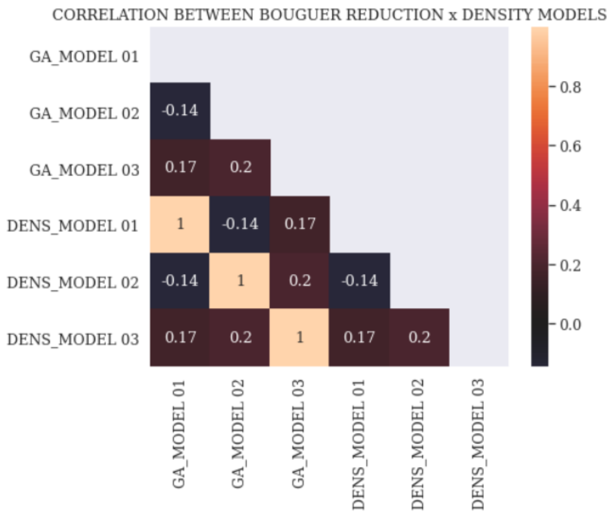

Figure 4 – Correlation between Bouguer reduction versus density models.

In the Figure 4 shows the correlation analysis between the Bouguer reduction as a function of the density value of each model analyzed. On the horizontal axis, the legend GA refers to the term gravity anomaly in relation to each model (MODEL 01, MODEL 02 and MODEL 03). On the vertical axis, the label DENS_MODEL refers to the density value of the MODEL 01, MODEL 02 and MODEL 03 models (models shown in Table 01). It was verified by the results of the correlation analysis in Figure 4 that the density value has a positive correlation, of a value equal to 1, between the Bouguer reduction and the analyzed density models.

In the Figure 5 contains an analysis similar to that developed in Figure 4, but in this case, the analysis was carried out by analyzing the correlation between the value of the terrain correction and the density of each model analyzed. In this case, it was found that the correlation value was very close to zero with values of -0.15, 0.12 and -0.19. This fact indicates that the linear relationship between the terrain correction and the density value is very low.

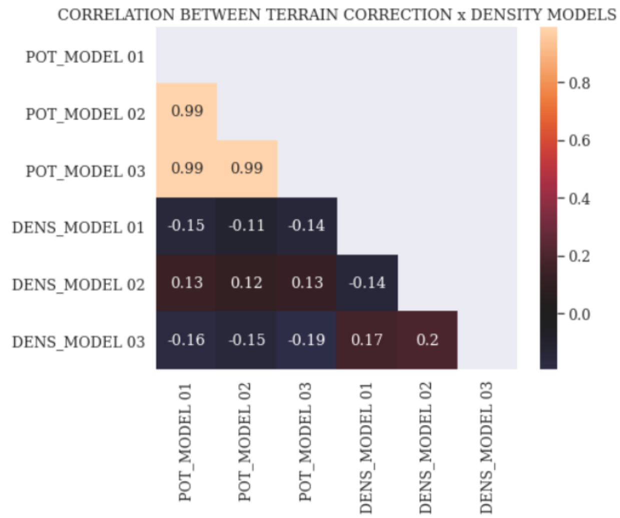

Figure 5 – Correlation between terrain correction (tc) versus density models.

From the correlation analyzes performed in Figure 4 and Figure 5, it was found that the density value has a positive correlation in the first term of Eq. (1), which term refers to Bouguer's reduction. It was also found that the density has practically no linear relationship with the terrain correction using the geometric tesseroid element described by Heck and Seitz (2007) and Wild-Pfeiffer (2008).

Conclusions

The main objective of this study was to verify the influence of the density value in the calculation of the gravity anomaly using the RTM reduction. The study was carried out in the city of Porto Alegre using three different density models. One of the models used was developed by Sheng et al (2019) with a spatial resolution of 30”. Other model used in this study was developed from the Environmental Atlas of Porto Alegre on a scale of 1: 185.000 (Menegat et al, 1999), where this map was vectorized and then transformed into a raster format with a spatial resolution of 3”.

The third model was obtained from the geological map of the Estado do Rio Grande do Sul obtained on the website of the Companhia de Pesquisa de Recursos Minerais (CPRM), where it was transformed into a matrix format with spatial resolution of 30”. The results of the calculation of the gravity anomaly using the RTM reduction obtained from these models were compared with a density value that are commonly used in scientific studies, in this case, the value of 2.67 g / cm³.

From these results it was found that using density models with different spatial resolutions, the standard deviation values showed less variation in the model with higher spatial resolution. It was also found that using a constant density value the results present the greatest discrepancies in comparison to models that have the variable density value. From these results it was found that using density models with different spatial resolutions, the standard deviation values showed less variation in the model with higher spatial resolution. It was also found that using a constant density value the results present the greatest discrepancies in comparison to models that have the variable density value. Another factor that was observed in the calculation of the gravity anomaly using the RTM reduction is that the density value has a greater correlation in the term corresponding to the Bouguer reduction in comparison to the term corresponding to terrain correction.

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