A discussion about 2D inversion of magnetotelouric data using Bostick Transform

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Abstract

This study explores the use of the Bostick Transform as a tool for generating initial models in 2D Magnetotellurics inversion. The Bostick Transform is a method that estimates resistivity distribution based on apparent resistivity and phase data for 1D datasets. A comparison is made between the results obtained using the resulting distribution and those obtained using a homogeneous model with mean resistivity values. The findings indicate that the initial models generated using the proposed approach exhibit characteristics similar to the true model, although with some degree of smoothing. The Bostick Transform-based models demonstrate better performance in terms of lateral delimitation and capturing conductivity anomalies compared to the homogeneous model. Moreover, the inversion process shows improved convergence speed when initiated with the Bostick Transform-based models. Although the study utilizes synthetic data, the results suggest that incorporating the proposed prior knowledge can enhance the efficiency of 2D Magnetotellurics inversion.

Introduction

The 2D Magnetotellurics modelling is a highly non-linear problem, and the only way to recover the parameters of a data set is by an inversion procedure. The usage of this method in solving 2D MT is well-established and had been optimized for quite some time, to assure that the solutions are stable and unique, and to reduce the computational overhead of this process.

A common need among all proposed optimizations is that the first model fed to the inversion procedure has to be close as possible to the original/real parameters. For real surveys, the prior information is integrated by using parameters defined by other geophysical methods, if its scarce or non-existent, usually its assumed a homogeneous model with all parameters defined by the same value. The latter approach causes an increase in iteration numbers and more computational overhead.

Before the inversion become computable practical, it was common to employ approximated methods to recover conductivity on the subsurface, one of these methods is the Bostick transform, which maps a conductivity distribution on depth from a set of apparent conductivity and phase on frequency. This method can be resumed by two simple equations, calculated at no time. Geosystem (2011) and Neves (2021) uses the Bostick Transform to provide a preliminary guess model for 1D inversion.

We investigated the viability of building a first model by using the conductivity distribution of Bostick from our 2D data set, to initiate inversion closer to the solution in such a way as to reduce the number of iterations of the problem. We tested his approach in a synthetic data set and compared it with an initial model with the resistivity values set as mean of apparent resistivity.

Methods

Magnetotelluric Method

The Magnetotelluric method listens to the natural electromagnetic field on the surface, its independent polarization, to build an impedance matrix(Vozoff, 1991). For 2D evaluations, its gathered only the horizontal components of the field related by

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix} \tag{1}$$

In order to use the Bostick Transform, the data set has to be rearranged as four frequency-dependent sets: apparent resistivity and phase for two different Modes: Transversal Magnetic Mode,

$$\rho_{yx} = \frac{1}{\omega\mu_0}|Z_{yx}|^2, \quad \phi_{yx} = \frac{Im(\bar{Z_{yx}})}{Re(\bar{Z_{yx}})} \tag{2}$$

, and Transversal Electric Mode,

$$\rho_{xy} = \frac{1}{\omega\mu_0}|Z_{xy}|^2, \quad \phi_{xy} = \frac{Im(\bar{Z_{xy}})}{Re(\bar{Z_{xy}})} \tag{3}$$

A finite element method was used to model the 2D magnetotelluric data; it is a numerical technique that separates a physical domain into smaller subdomains called finite elements. The electric and magnetic field equations are approximated using basis functions for each element, therefore forming an algebraic equation system. Solving this system produces the electric and magnetic field for each element(Key & Weiss, 2006).

Bostick Transform

The Bostick transform is a method for generating a resistivity distribution by depth using magnetotelluric data. It was built by looking at the asymptotic behavior of the apparent resistivity curve and phase at low frequencies in models of a layer over an infinite basement. Bostick (1977) provides the correlations between resistivity in the frequency domain and resistivity in the space domain,

$$\rho(h_n) = \rho_a(T_n)\frac{\frac{\pi}{2} - \phi(T_n)}{\phi(T_n)} \tag{4}$$

Where h is an approximation of the skin depth for a semispace in which the resistivity is equal to the apparent resistivity on the frequency Tn,

$$h_n = \sqrt{\frac{\rho_{ap}(T_n)T_n}{2\pi\mu_0}} \tag{5}$$

Inversion

Given a set of observations y and a set of parameters p, there is a mathematical model f that describes data from parameters and variables independent, such as measurement positions or frequencies:

$$\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{p}), \tag{6}$$

the observations are the apparent resistivity and the phase for each frequency and the parameters are as layer resistivity of an interpretive model defined a priori as a discretization of the earth in the form of a stratified medium. To estimate the parameters p , it was implement the method of Gauss-Newton, with Levenberg-Marquardt iterations and the inclusion of regularization in the parameters (Pujol, 2007).

The whole process depends on the derivatives of the function f at to the parameters in vector p. This information is organized in the form of the so-called sensitivity matrix, defined as

$$S_{i,j} = \frac{\partial \boldsymbol{f}(y_i)}{\partial p_j}. \tag{7}$$

On the nth iteration, the parameter array is updated through the estimator

$$\boldsymbol{p}_{n+1} = \boldsymbol{p}_n - (\lambda_n \boldsymbol{I} + \boldsymbol{H}_d + \mu \boldsymbol{H}_r)^{-1}(\boldsymbol{G}_d + \mu \boldsymbol{G}_r) \tag{8}$$

where $\boldsymbol{H}_d$ is the matrix of the second derivatives of the function f, which in the Gauss-Newton method is approximated by

$$\boldsymbol{H}_d = \boldsymbol{S}^T \boldsymbol{S} \tag{9}$$

$\boldsymbol{g}_d$ is the gradient of the functional being adjusted in relation to the parameters; $\boldsymbol{H}_r$ and $\boldsymbol{g}_r$ are, respectively, the matrix of second-order derivatives and the gradient associated with the functional that describes the regularization; λ is the Marquardt parameter, which is adjusted in each iteration and which controls the behavior of the method, and $\boldsymbol{I}$ is the matrix identity.

The definitions of $\boldsymbol{H}_r$ and $\boldsymbol{g}_r$ depend on how you define the functional of regularization to the parameters. Regularization restrict the search for the problem solution to a subset with certain restrictions, in a certain configuration, or contain any a priori information. One way to define these restrictions is to create a new functional Φ, with a functional for the regularization parameter Φ_r, is a relative weight of the information of the links in relation to that of the data.

$$\Phi(\boldsymbol{p}) = \Phi_D(\boldsymbol{p}) + \mu\,\Phi_R(\boldsymbol{p}), \tag{10}$$

In this job, we implemented the Global Smoothness (GS), that establishes a relation of equality between the components of vector p, in the sense of least squares, to find a solution with the smallest variation between neighboring parameters (Constable et al., 1987). For N parameters, we have:

$$\Phi_R(\boldsymbol{p}) = \sum_{j=1}^{N-1} \|p_{j+1} - p_j\|^2 \tag{11}$$

To execute the inversion procedure, the inital marquadt parameter was set as $\lambda_0 = 0.001$ and global smoothness parameter $\mu = 0.02$.

Initial Model

The inversion grid was constructed within the boundaries of the first and last station, that are 6 kilometers appart. The maximum depth chosen was 3 kilometers. In terms of dimensions, the grid consisted of 51 nodes horizontally and 31 nodes vertically, resulting in a total of 1500 cells. Established a geometry model, the parameters should be atributed for each cell, two methods of filling were used. The first one, filled all cells with the same value of resistivity, taken from the geometric mean of apparent resistivity.

The second one, using the Bostick Transform for each station, it was found the resistivity-depth distribution, it was taken the log of this values, and were used for interpolated linearly the log resistivities for all the cells under each station. Once this is done, the remaining cells are fillied by a new interpolation, this time between each station. At the end, the values are coverted back to linear values of resistivity and phase.

Building the dataset

To build the dataset, it was simulated a sounding of a condutive block inside a two layers mean, the following layers were set as: And the block, 900 meters tall and 500

Resistivity ($\Omega.m$)	Depth (m)
100	1517
200	-

meters wide, 400 meters down the surface,

It were used 20 stations, 300 meters apart and get 20 impedance values for each one, in the frequency range from 1000 Hz to 0.001 Hz. All values for resistivity and phase were contaminated with a random noise of 1 % maximum.

Results

By forward modelling the parameters described in figure 1, it was gotten the following resistivity and phase data,

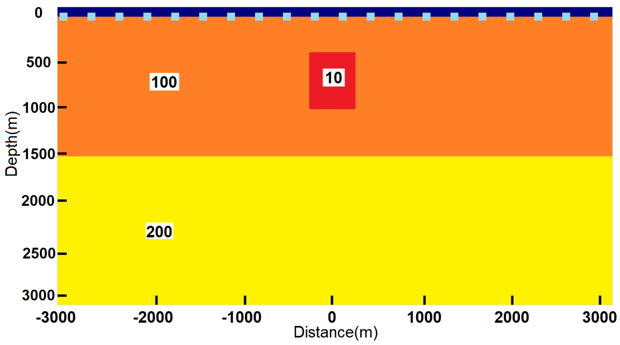

Figure 1 – Distribution of MT sounding stations across the 2d resistivity model, and the value for each layer and body.

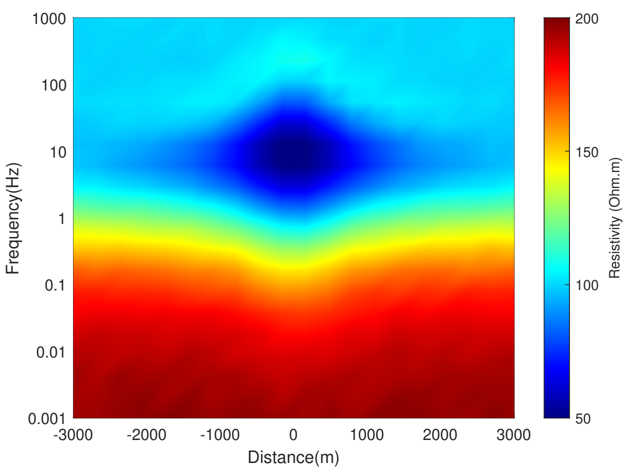

Figure 2 – Pseudosection of resistivty for TE mode

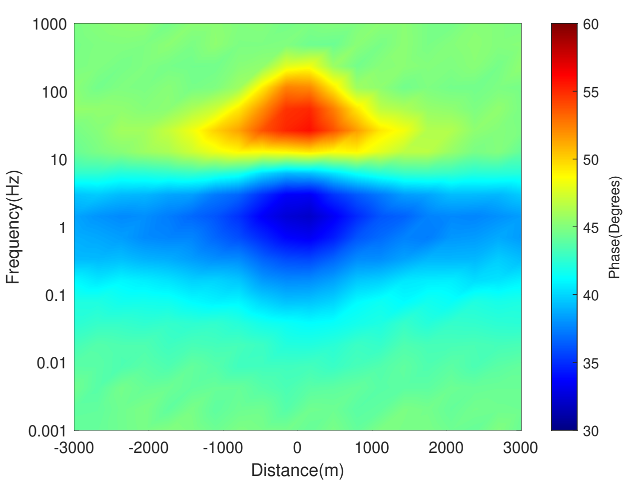

Figure 3 – Pseudosection of phase for TE mode

By inspecting figures 2 and 3, its clear that the conductive body causes a influence on the TE sounding higher frequencies that spreads for the majority of the stations, even the borders of the datatset. More pronounced on the resistity dataset than the phase.

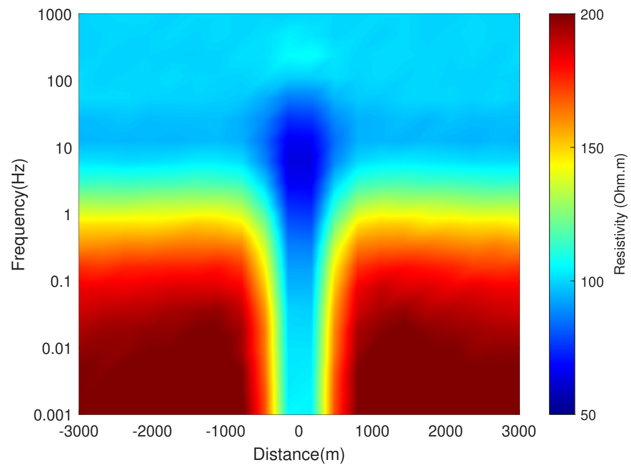

Figure 4 – Pseudosection of resistivty for TM mode

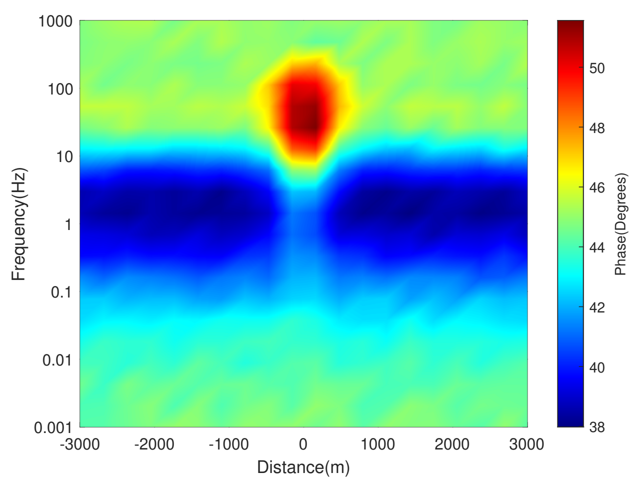

Figure 5 – Pseudosection of phase for TM mode

Figures 4 and 5 shows that the influence of the body is more restricted, to the stations immediate near to it, in which it provokes a phenomenon called static shift (Jones, 1988), in this sounding, that distorts all the signal for frequencies less than $100\Omega.m$, for stations not over the conducive body, the signal for both phase and resistivity seems almost 1D.

The initial model are set by Bostick transform from TE data,

By inspecting the model described in figure 6, we can see that it has the main characteristics of the true model, with resistivity values exaggerated smoothened.

Figure 7 shows a model with a static shift observed in data, near to the conductive body, it seems less verticality accurate, but it gives a good lateral delimitation between

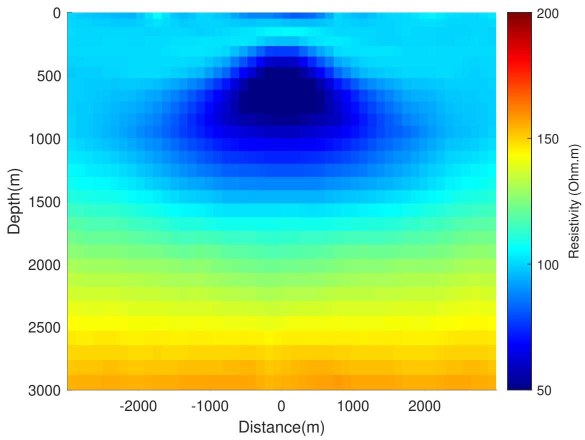

Figure 6 – Initial model from TE impedance

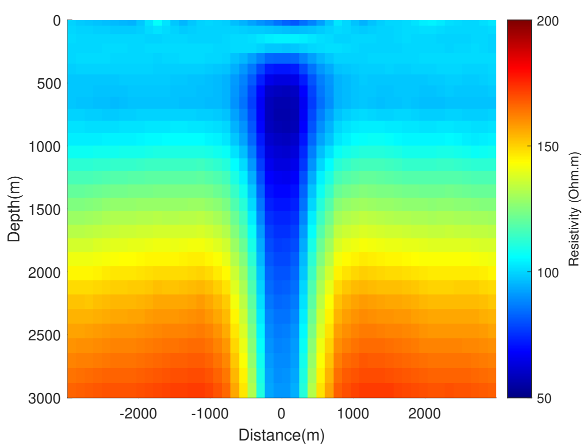

Figure 7 – Initial model from TM impedance

the surrounding mean and the body, it shows more strong values of resistivity, but still somewhat smooth.

If the grid were deep enough, the Bostick transform would eventually recover the resistivity value from the substract. The magnetotelluric method suffers from parameter resolution, and that problem is potentialized by the approximation.

With the proposed approach for the initial model, the best fit model took 26 minutes to be achieved. In contrast, using the usual homogenous initial model, the best fit was achieved in 31 minutes. Meaning a reduction of computational overhead of approximated 20 %.

The path until convergence for the models are as followed,

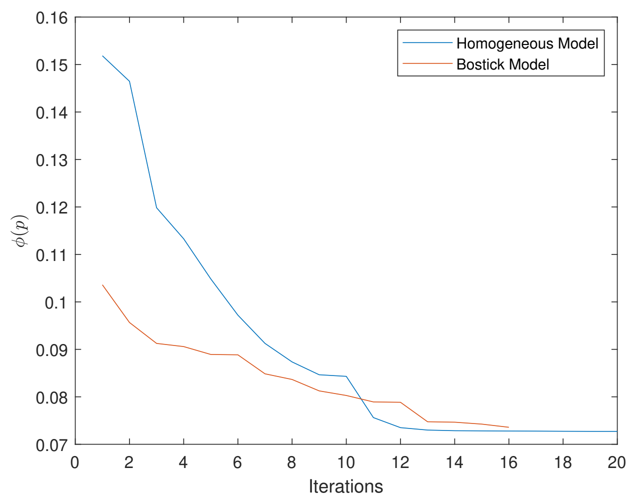

Figure 8 – Convergence path for the problem: Proposed Approach for initial model vs Homogenous Approach

The initial misfit for Bostick model in figure 8 indicates that this initial model is near to the real parameters than the Homogenous model. But, Bostick model shows a slower drop on the misfit functional than the one using the homogenous model, indicating that using the same Marquadt parameter λ and Smoothness Parameter μ for both models may not be adequate.

The model resulted of the inversion shows that both models were successful to recover the main features of the original model, showing two layers and a conductive body at the center of the model.

Conclusion

Despite approximations and simplifications in magnetotellurics modelling and inversion, the Bostick Transform method proves advantageous for initializing models. It saves computational time compared to homogeneous models by reducing iterations required for a solution.

Additionally, initial models utilizing this method display key characteristics of the true model, albeit not perfectly

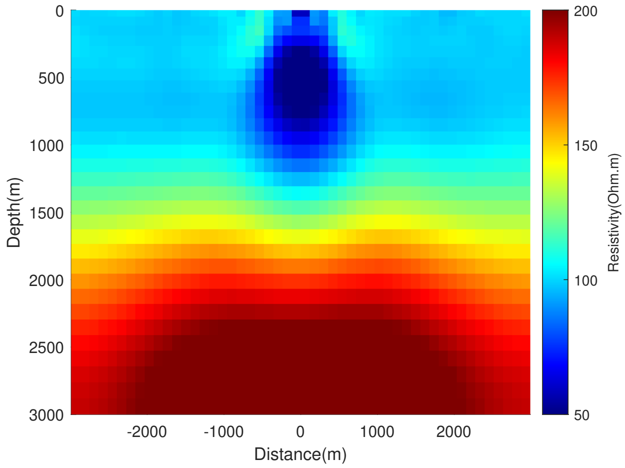

Figure 9 – Best fit model generated by the inversion

accurate. This suggests that it reasonably approximates the subsurface resistivity distribution, bringing the inversion process closer to the true solution.

Furthermore, comparing initial models generated using the Bostick Transform and homogeneous models with mean resistivity values reveals the former's superiority in lateral delimitation and capturing conductivity anomalies.

While these results are based on synthetic data and require validation with real-world data, they offer promising insights into the advantages of utilizing the Bostick Transform for initializing 2D Magnetotellurics inversion. Incorporating prior knowledge derived from the Bostick Transform can potentially enhance inversion accuracy and efficiency, leading to more reliable interpretations of subsurface conductivity structures.

In conclusion, the Bostick Transform demonstrates promise for generating initial models in 2D Magnetotellurics inversion. It reduces computational overhead, expedites convergence, and reasonably approximates the true resistivity distribution, capturing essential subsurface features. Further research and real-world applications are necessary to fully assess its effectiveness and benefits in the field of Magnetotellurics.

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