OHEX method: On-the-fly optimization for finite-difference 3D wave modeling.

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Abstract

Wave equation modeling is present in the solution of many geophysical problems, especially those based on seismic inversion techniques, where the modeling process must be performed several times. In this work, we present a computational optimization for calculating the finite difference method (FDM) applied to the 3D modeling of the wave equation. Based on the expanding-box method (EBM) that is used in the eikonal equation solution to calculate the wavefront, the On-the-fly Hexahedron (OHEX) method updates at each time step the vertices of the model to be applied to modeling, discarding unnecessary calculations in regions where the wavefront has not yet arrived. Results comparison for modeling using a spatial 8^{th}-order in a real velocity model show reductions over 44% in the total time and more than 57% in the total points calculated for the OHEX method. It was developed using C++ programming, GPU processing, and cPML absorbing boundaries.

Introduction

In geophysics, we commonly have to estimated properties using techniques like as seismic inversion. In its processing routine, the modeling of the wave equation is repeated in an iterative process until the solution converges. It takes a long time with a high computational cost. Currently, one of these main inversion methods is the Full Waveform Inversion (FWI) (Tarantola, 1984) and used in several works (Karsou et al., 2019) (da Costa et al., 2019).

The finite difference method (FDM) has been used for computational modeling of the acoustic wave equation for a long time (Virieux, 1986). However, this technique depends on the implementation of absorbing boundaries to simulate natural conditions, influencing an increase in computational cost with high processing time and memory usage.

The first absorbing boundary that appeared was the type damping zone (Cerjan et al., 1985) and later the type perfectly matched layer - PML (Berenger, 1994). Many other works have proposed improvements of these techniques, such as (Moreira et al., 2009), (Asvadurov et al., 2003), and (Rickard et al., 2003), emphasizing the high performance for the cPML boundary (Roden and Gedney, 2000).

Even with the efficiency of FDM, the seismic inversion of 3D models can be a high computational cost and time-consuming process, where optimization methods are necessary. The progress of computing in recent years allows for speeding up the FDM process, using GPUs - including multi-GPUs - instead of CPUs for mathematical processing. (Liu et al., 2021).

Regarding FDM optimization, Noack (2015) proposed an interesting optimization method for modeling based on creating subdomains and with multi-GPU processing. In his work, he separates model volumes that contain only the main wavefront, thus reducing the number of mesh points to be calculated. It is a process based on the expanding-box concept, first cited by Vidale (1990) in his work on the solution of the eikonal equation for heterogeneous media.

After Vidale, other works proposed other solutions to the eikonal equation obtaining the transit times for the first arrival of the wave. Among these, we can mention Podvin and Lecomte (1991), Zhang et al. (2006), and Noble et al. (2014), with iterative processes to reach the solution. However, as the FWI process uses information from reflected and refracted waves in a heterogeneous medium, we cannot perform the selection only of the voxels that contain the first arrival of the wave. Additionally, using an eikonal solution by the mentioned methods would depend on a previous calculation of the time model of transit for the entire domain, which implies an additional computational cost.

Thus, this work presents a solution for calculating the acoustic wave modeling only in voxels already reached by the wavefront, discarding other regions. The update of the vertices of the domain in which the processing is applied is performed at runtime. Results with information about the total time spent and the number of calculated mesh points are presented for scenarios with and without optimization.

Methods

The purpose of this work is to show the implementation of an optimized solution for the 3D modeling of the acoustic wave using an optimized FDM. For this, we will recalculate the hexahedron's limits that represent the velocity model's subdomain, to which the wave modeling will be applied. This calculation will occur at runtime, and this method will be called in ths work as On-the-fly Hexahedron (OHEX).

Theory

The acoustic wave equation used for FDM modeling in this work is defined in Equation 1.

$$\frac{\partial^2 p}{\partial t^2} - v^2 \nabla^2 p = \mathbf{f}, \quad (1)$$

where p is pressure, v is the median acoustic velocity, and $\mathbf{f}$ is the acoustic source. The ∇^2 is the Laplacian operator.

The limits that will restrict the application of FDM to the 3D domain will be defined by two diametrically opposite vertices of a hexahedron, $\mathbf{v}_1$ and $\mathbf{v}_2$, containing the smallest and largest values of the coordinates, respectively (Equation 2). These vertices can be calculated prior to modeling using the traveltimes either generated by the solution of the eikonal equation or, more simply, using the maximum velocity value of the medium, ensuring that points of the domain where the wavefront has not yet reached are not used in the FDM. However, this can add extra processing of the same order of magnitude as the achieved economy - in the case of using eikonal - or minimize the optimization by oversize the hexahedron that is being expanded - in the case of considering the maximum speed of the medium.

$$v_1 = (x_{min}, y_{min}, z_{min}) \qquad v_2 = (x_{max}, y_{max}, z_{max}) \tag{2}$$

In order to make a solution for this problem and optimize the process, the hexahedron vertices will be calculated on-the-fly in the FDM processing. The method consists of creating a border around the actual subdomain (Figure 1). For the initial time, $t = 0$, this subdomain will be the source injection point. The border created has a width that is a function of the finite difference operator stencil used. The subdomain expansion will happen in this expanded region, and your rate depends on the values of medium velocity and the actual time step, according to a threshold value applied for the pressure.

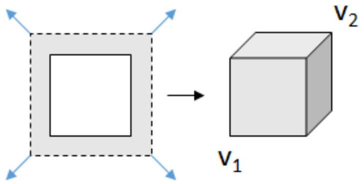


Figure 1: The hexahedron expansion of the OHEX method in a 2D projection view. The vertices v_1 and v_2 describe the subdomain used in each modeling process.

The first threshold value is calculated based on a fraction of the absolute maximum value of the source, which values below it (in modulus) will not be considered. As this threshold value is a function of the maximum value of the source and is valid only for that point of origin, it must be updated for the other points of the domain. In order to optimize the calculation process, this dynamic threshold will be calculated at each time step, equal to the original threshold with a decay that will be a function of the maximum speed of the medium. Equation 3 shows how the $threshold_i$, referring to a time step t_i is calculated.

$$threshold_i = \frac{threshold_0}{(v_{max} * t_i * \Delta t)^2} \tag{3}$$

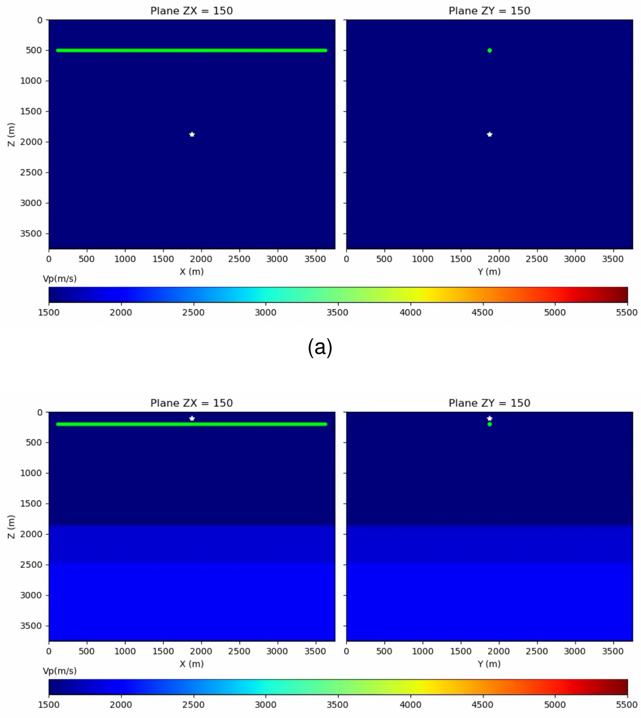
Very small values can be found in error measurement between the modeling with the OHEX method and the FDM standard. Thus, a metric for error calculation is used, and it is equal to relative value between the sum of the module of the difference of the pressure seismograms $(p_{OHEX} - p)$ and the sum of the values in the FDM seismogram (p) (Equation 4).

$$\varepsilon = \frac{\sum \|p_{OHEX} - p\|}{\sum \|p\|} \tag{4}$$

Tests

All tests were done using FDM with operators of 8^{th} order in space and 2^{nd} order in time, applied to 3D models. The absorbing boundary used in all tests was the cPML type with a width of 30 points. The code was written in C++ language and used the OpenACC library for GPU processing. The GPU used in the tests was a Quadro RTX 6000 from NVIDIA.

Four tests were done and used different velocity models, three of which are synthetic models, and one is a typical brazilian pre-salt model. For tests with synthetic models, we have Test01 with a homogeneous medium and central shot (Figure 2a). In Test02 was used a 3-layer model with close-to-surface shooting and a smooth transition between inner layers. Both Test01 and 02 models have dimensions of 301x301x301 (Figure 2b).

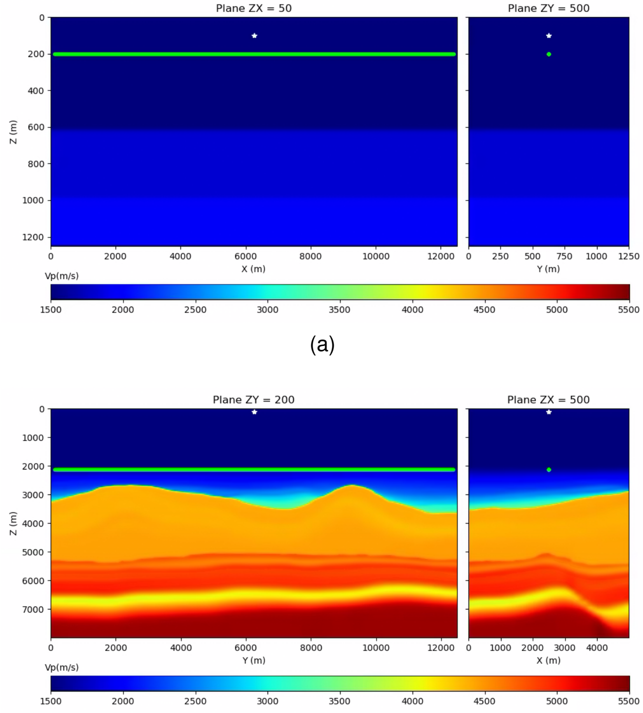


(a)

(b)

Figure 2: 3D Models: (a) is the homogeneous model used in Test01.(b) is the 3 layers model used in Test02.

Test03 purpose is to verify the optimization of the proposed method for a domain with a long offset. A 3-layer model was used with the same characteristics as Test02 but with dimensions of 1001x101x101 (Figure 3a). Finally, Test04 applies the OHEX method to FDM on a real velocity model, with high heterogeneity in its values and large distances. The Figure 3b shows a typical brazilian pre-salt velocity model. The template size is 640x400x1000. All models uses a spatial sampling of 12.5m for the FDM mesh in x, y, and z directions.



(a)

(b)

Figure 3: 3D Models: (a) is the model with long offset used in Test03. Attention for the X axis that is in a different scale and has a greater distance. (b) is a typical brazilian pre-salt model used in Test04.

In all tests, the white dot represent the acoustic source, and the light green dots represents the receivers. The acoustic source is the same for all tests: a Ricker wavelet with a maximum frequency of 15Hz and a delay of 300ms. Figure 4 shows the wavelet used.

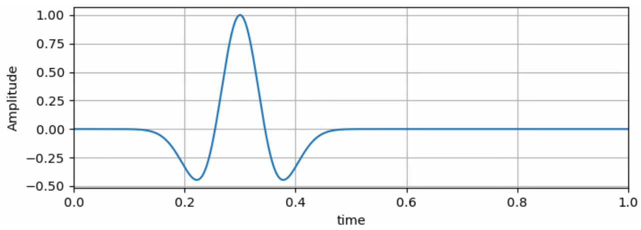


Figure 4: Ricker wavelet used in tests.

The FDM modeling will be executed with and without OHEX optimization. The results will be compared between the tests that use the pure FDM with the tests that applied the OHEX method. The values in the acquired histograms, the total execution time of the modeling, and the number of points calculated from the FDM will be compared.

Results

The seismograms with the results can be viewed in Figures 5 - 8. The left seismograms in the figures represent the default modeling using the traditional FDM. The right seismograms are the results of the modeling using the OHEX method. An amplitude gain was applied to turn visible lower values of reflected and refracted waves in all these seismograms.

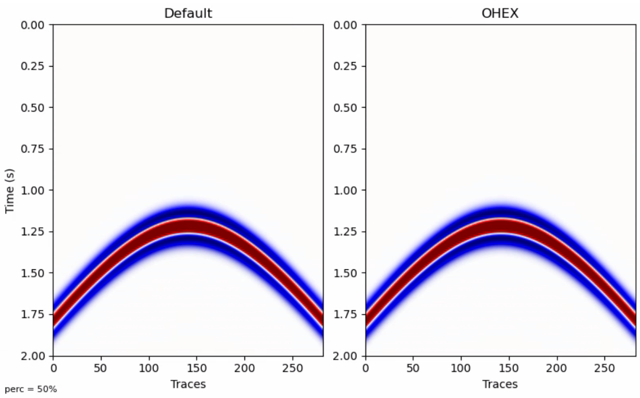


Figure 5: Test01 Seismogram.

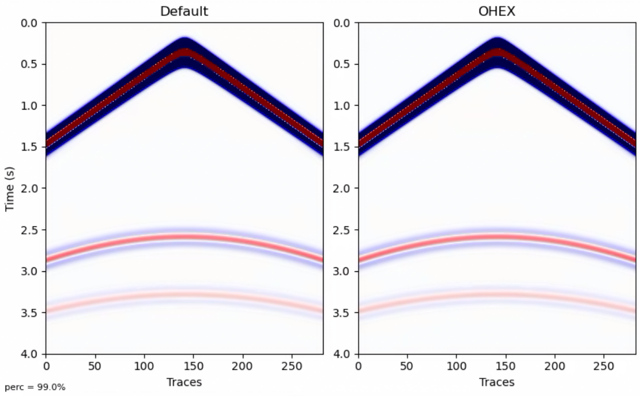


Figure 6: Test02 Seismogram.

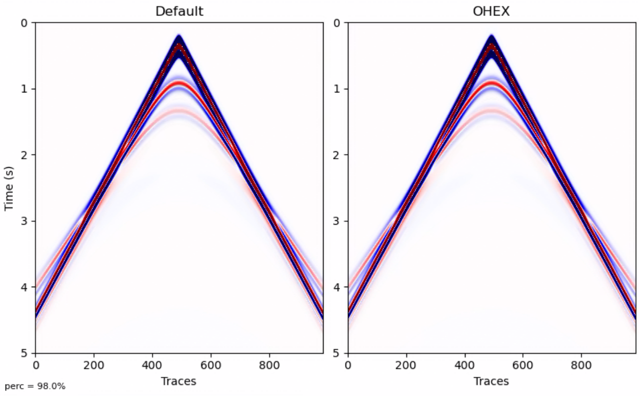


Figure 7: Test03 Seismogram.

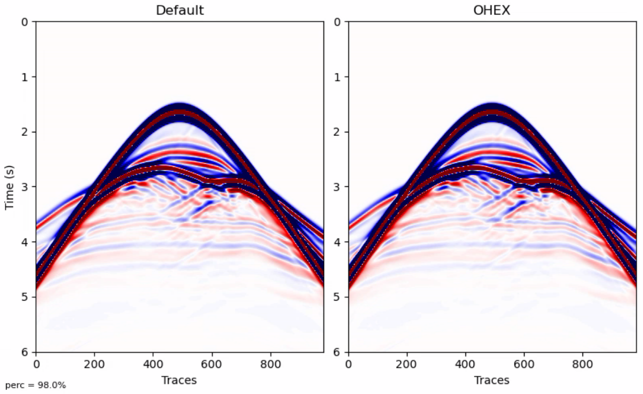


Figure 8: Test04 Seismogram.

No difference could be seen between the seismograms, and the simple subtraction between the images returned lower values, around 10^{-9}. Thus, the relative error is calculated using the Equation 4 and shown in Table 2. The results for the optimization in the total processing time for the modeling are shown in Table 1. The relative total time saved in the process is calculated and represents the process efficiency. As the OHEX method calculates a smaller domain with fewer points in a specific step time, a total point calculated for the whole modeling process is a good metric. Table 3 shows the total points for a default FDM modeling, including a cPML boundary, and the total reduction in mesh points processing. The unit is giga points (GP).

Table 1: Runtime for the tests with the default FDM modeling and with the OHEX modeling.

Test	Modeling	Run time (s)	Processing Time Saved
01	default	24.000	48.46%
	OHEX	12.370	
02	default	45.954	28.60%
	OHEX	32.813	
03	default	43.013	21.65%
	OHEX	33.700	
04	default	427.91	44.30%
	OHEX	238.35	

Table 2: Relative errors for the OHEX tests.

Test	Error
01	6.36 x 10^{-6}
02	8.23 x 10^{-6}
03	8.21 x 10^{-6}
04	1.52 x 10^{-5}

Table 3: Total mesh points calculated in each test with default modeling and with the OHEX method.

Test	FDM GPoints (GP)	OHEX Reduction
01	88.017	55.44%
02	175.99	35.10%
03	123.27	39.21%
04	1974.6	57.09%

Conclusion

The results showed that the OHEX method greatly improved the FDM modeling in the applied tests. A saving of 44.30% in processing time was achieved using the velocity model of brazilian pre-salt field, which uses real data. The reduction of processed points reached higher savings values (57.09%). As this reduction contains points both inside the velocity model and those on the absorbing boundary - which has a much higher computational cost - it was expected that this value would actually be higher. It was verified that the model's size and, mainly, the total modeling time directly influence the optimization gain. Excessively long times will imply more modeling steps using the whole domain. Thus, a correct dimensioning of the total modeling time will considerably impact the performance gain of the OHEX method.

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