Evaluation of the influence of porosity and saturation on seismic wave velocities in porous media

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This paper was prepared for presentation during the 18th International Congress of the Brazilian Geophysical Society held in Rio de Janeiro, Brazil, 16-19 October 2023.

Abstract

The integration of different geophysical methods used to study hydrocarbon reservoirs has been very successful since the early 1951s, the year of the publication of Gassmann's fluid replacement theory, which inaugurated a new area of study in Geophysics. From this theory, it was possible to distinguish the porous medium in grain of the rock matrix, dry rock and saturated rock, and from this distinction, he named three different incompressibility modules: of the grain of the rock matrix, dry rock and saturated rock, relating the last two to the porosity of the medium. Gassmann's studies were able to evaluate the influence of porosity and saturation on the velocities of seismic waves propagating inside hydrocarbon reservoirs, which in fact revolutionized the study of rock physics. In this paper, Gassmann's fluid replacement theory is defined and properly applied to a simple seismic reservoir consisting of flat, horizontal layers that have constant saturation and porosity and P- and S-wave seismic wave velocities that depend on these parameters with the goal of evaluating how the velocity curves behave with the variation of these parameters.

Introduction

The reflection seismic method is the geophysical method that uses the propagation of seismic waves in the subsurface to extract information from the rocks and their constituents inside the Earth, usually considering the subsurface as stratified and in parallel planes, whose differentiation between them is due to different physical parameters associated with each plane (YILMAZ, 2001; TELFORD, GELDART, SHERIFF, 1990; SHERIFF, GELDART, 1995).

According to Chapman (2004), when incident on different interfaces that separate media with different density values, seismic waves undergo the wave phenomena of reflection and transmission. In addition to layer densities, other physical properties that also influence the variation of seismic wave velocities are: the incompressibility modulus - which is the measure of the resistance of the medium to normal stresses due to the passage of the seismic wave - and the shear modulus of the medium - which is the measure of the resistance of the medium to shear stresses due to the passage of the seismic wave (LANDAU et al., 1986; NOVACKY, 1975).

The first studies using seismic wave propagation in subsurface, especially in porous and saturated ones, date back to the 1920s, such as those of Terzaghi and Peck (1948), for example, which used low frequency waves (0-100Hz), considering the solid and fluid phase as one. Work such as this, together with the classical theory of elasticity which, complemented by Darcy's Law for fluid flows in pores, led to the conclusion that fluids affect acoustic properties in mainly two ways: the first changes the elastic modulus of the rock and its seismic response; and the second introduces velocity dispersion, i.e., variation of velocity with frequency.

The distinction of rocks in the dry and saturated conditions using this geophysical method was made possible by the famous work of Gassmann (1951b), who developed his fluid substitution model. In this paper, the author related the incompressibility modulus of the saturated rock to the incompressibility modulus of the dry rock, the incompressibility modulus of the fluid, and the incompressibility modulus of the rock matrix grain.

According to Gassmann's fluid replacement model, the propagation velocities of primary seismic waves are greatly influenced by the incompressibility modulus of the saturated rock, which varies during the replacement process; on the other hand, the propagation velocities of secondary seismic waves are greatly influenced by the shear modulus of the rock, which remains unchanged during the fluid replacement process.

Besides the incompressibility and shear moduli of the rock, two other factors also influence the seismic wave velocities, being the porosity and saturation of the medium. In this sense, this paper studies the fluid substitution model described by Gassmann and evaluates the influence of rock porosity and saturation on seismic wave velocities to analyze its applications in petrophysics.

Materials and Methods

Gassmann Fluid Replacement Theory

According to AZEREDO (2017) and SMITH, SONDERGELD, and RAI (2003), the fluid replacement theory is a procedure that allows determining the physical properties of rocks in the presence of saturating fluids from measurements made in the laboratory, aiming to evaluate the influence of saturation variation on the incompressibility modulus of saturated rock, obtained through the Gassmann equation.

In GASSMANN's (1951a) fluid replacement theory, the incompressibility modulus of saturated rock is determined from prior knowledge of the incompressibility moduli of the dry rock, the rock matrix and the equivalent fluid, as well as the porosity of the medium.

Gassmann's Fluid Replacement Theory models and quantifies the incompressibility modulus of porous rock in situations where it is dry and fluid saturated (NOVACKY, 1975; SMITH, SONDERGELD, and RAI, 2003; HAN, BATZLE, 2004).

The type of fluid present in the rock pores causes variation in its seismic properties and through the fluid substitution technique the incompressibility modulus of the rock saturated by a fluid is obtained via values referring to its porosity, incompressibility modulus of the dry rock, saturation fluids and mineral matrix.

The assumptions made by Gassmann are:

i) the porous material is isotropic, elastic, homogeneous, and composed of one type of mineral. This assumption is violated if the rock is composed of multiple minerals;

ii) the porosity remains constant. This assumption ensures that the porosity does not change with different saturating fluids; in other words, no cementation or dissolution with changing geochemical conditions in the pores;

iii) the space between the pores must be well connected and in pressure equilibrium. This assumption is violated in sands of low porosity, such as in carbonate rocks, where the low connectivity between the pores would produce results that are unreliable to the results obtained by the Gassmann equation;

iv) the medium is a closed system, with no pore fluid movement across system boundaries. This assumption is to ensure that the pore-fluid system is sealed to prevent the fluid, from being moved to the sample surface in the laboratory experiments;

v) there are no chemical interactions between the fluids and the rock structure, which means that the shear modulus remains constant.

According to (GASSMANN, 1951a), the incompressibility modulus is given by the equation below:

$$K_{sat} = K_{rs} + \frac{\left(1 - \frac{K_{rs}}{K_0}\right)^2}{\frac{\phi}{K_{fl}} + \frac{(1-\phi)}{K_0} - \frac{K_{rs}}{K_0^2}} \quad (1)$$

where K_{sat} is the incompressibility modulus of the saturated rock; K_{rs} is the incompressibility modulus of the dry rock; Kfl is the incompressibility modulus of the fluid; K_0 is the incompressibility modulus of the rock matrix and ϕ is the porosity of the rock.

Compressional wave velocity in porous media

Considering the density of saturated rock, given by:

$$\rho_{sat} = (1-\phi)\cdot\rho_0 + \phi\cdot\rho_{fl} \quad (2)$$

where ϕ is the porosity of the rock, ρsat is the density of the rock saturated with the initial fluid, ρ_0 is the grain density of the rock matrix, and ρ_{fl} is density of the equivalent fluid, and its incompressibility modulus, shown in equation (1), one can rewrite the expression of the primary wave velocity as follows:

$$v_{p,sat} = \sqrt{\frac{k_{sat} + \frac{4}{3}\mu_{sat}}{\rho_{sat}}} \quad (3)$$

that takes the form:

$$v_{p,sat}(S,\phi) = \sqrt{\frac{1}{\rho_{sat}(S,\phi)}\cdot\left[k_{rs}(\phi) + \frac{\left(1-\frac{k_{rs}(\phi)}{k_0}\right)^2}{\frac{\phi}{k_{fl}(s)} + \frac{1-\phi}{k_0} - \frac{k_{rs}(\phi)}{k_0^2}} + \frac{4}{3}\mu_{sat}\right]} \quad (4)$$

where the shear modulus of the rock is assumed to remain constant during the fluid replacement process.

where, vp,sat and vs,sat are the P and S wave velocities for the saturated rock; K_{sat} is the incompressibility of the saturated rock; µsat is the shear modulus of the saturated rock and ρ_{sat} is the total density of the saturated rock.

Shear wave velocity in porous media

Considering the density of the saturated rock, present in equation (2), one can rewrite the expression for the secondary wave velocity as follows:

$$v_{s,sat} = \sqrt{\frac{\mu_{sat}}{\rho_{sat}}} \quad (5)$$

which takes the form:

$$v_{s,sat} = \sqrt{\frac{\mu_{sat}}{(1-\phi)\cdot\rho_0 + \phi\cdot\rho_{fl}}} \quad (6)$$

where $\mu_{sat} = \mu_{rs}$.

Results and Discussions

In this topic the expected results for the present work are presented, about the knowledge obtained through the information generated to introduce the concepts of physical rock properties and fluid replacement theory in seismic, in order to estimate the velocities of P and S waves in porous media, Considering to be a sedimentary environment formed by fluid-saturated rocks, in which the proposed model for analysis is of a monomineralogical rock matrix and formed by quartz grains.

Therefore, we consider two different situations for fluid filling of the pores. The considered saturations simulate water and oil reservoirs. The values of the physical properties of the saturating fluids, as well as, of the constituent grain of the dry rock matrix required for the proposed analysis are presented in tables 1, 2 and 3, respectively.

In this regard, it is necessary to understand the behavior of P-wave velocity, S-wave velocity, and the density to recognize changes in the fluid or properties, as well as predict the effect of these changes on seismic amplitude. The velocity equation can be written in terms of the porous part, the fluid part, the dry rock and the saturated rock, where it is possible to observe the different seismic velocities for different saturations; the higher the porosity and saturation, the higher the velocity.

Tables

The tables presented below refer in the rock physics parameters. Where the values of the physical properties of the saturating fluids, as well as of the grain constituent of the dry rock matrix required for the proposed analysis, are presented in tables 1, 2 and 3, respectively. Simultaneously, a discussion regarding the exposed results is raised.

The sealing layer of the phlegm characterized by its density and P-wave velocities, as shown in Table 1.

Table 1: *Sealant layer parameters: shingle*

Parameter	Sealing Foil	Unit	Denomination
P	2250	Kg/m³	Density
v_p	2600	m/s	Wave speed P

Source: Authors (2023).

A reservoir layer composed of relatively clean sandstones with a mineral matrix was considered, as shown in Tables 2 and 3.

Table 2: *Reservoir parameters: mineral matrix (Eocene sandstone)*

Parameter	Ocean Sandstone	Unit	Denomination
ρ_0	2450	Kg/m³	Density
K_0	$32 \cdot 10^9$	Pascal	Modulus of incompressibility of the rock matrix grain
μ_0	$38 \cdot 109$	Pascal	Shear modulus
Φ	5% a 39%	dimensionless	Porosity

Source: Authors (2023).

The reservoir contained in the model was characterized based on information from an Eocene sandstone from the Campos Basin Romanelli (2010) considering its pores saturated by only two phases: water and oil, as shown in table 3.

Table 3: *Reservoir parameters: water and oil*

Parameter	Water and Oil	Unit	Denomination
ρ_{water}	1049	Kg/m³	Density of water
K_{water}	$2.90 \cdot 10$	Pascal	Incompressibility modulus of water
ρ_{oil}	700	Kg/m³	Oil's density
K_{oil}	$1.02 \cdot 10$	Pascal	Oil's incompressibility module

Source: Authors (2023).

Discussion of Results

The Gassmann model related to equation (1), was used to determine the incompressibility of saturated rock from data obtained from dry rock. The incompressibility of the dry rock was calculated using equation (i) (Appendix A), for the fluid incompressibility was defined by equation (ii) (Appendix B), the values listed in table (3) were used.

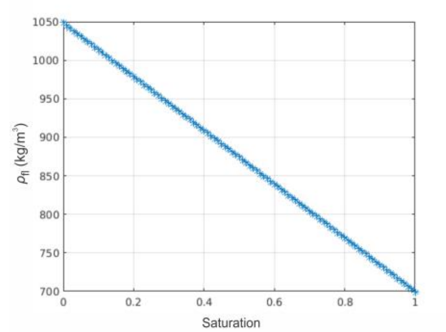
Since Gassmann's model assumes that the rock has only one mineral in its composition, it was assumed that the model was composed of quartz mineral. With the incompressibility of the saturated rock calculated using the Gassmann model, equations (3) and (4) were used to calculate the Gassmann wave propagation velocities.

The discussions of the results obtained by applying Gassmann theory in porous media will be discussed below.

Equivalent fluid density variation with saturation

According to equation (iii) (Appendix C), the density of the fluid resulting from the water-oil mixture, called equivalent fluid, varies with saturation, with a decrease in its value as more oil is added and more water is removed from the mixture. This addition of oil accompanied by the reduction of water in the mixture causes the density of the equivalent fluid to decrease, as illustrated in figure 1.

Figure 1: *In the initial condition of the fluid, its initial density is 1049 Kg/m³, which corresponds to the density of water, and as saturation increases, the equivalent fluid density decreases, reaching the minimum value of 700 Kg/m³, which corresponds to the density of oil.*

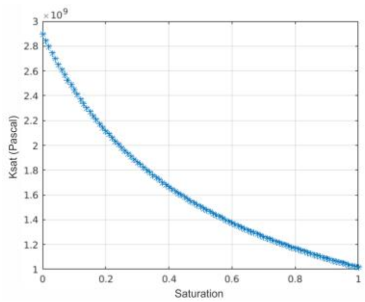


Fonte: Autores (2023).

Variation of the incompressibility modulus of the equivalent fluid with saturation

According to equation (ii) (Appendix B), the incompressibility modulus of the fluid resulting from the water-oil mixture, called equivalent fluid, varies with saturation, with a decrease in its value as more oil is added and more water is removed from the mixture. This addition of oil accompanied by the reduction of water in the mixture causes a decrease in the incompressibility modulus of the equivalent fluid, as illustrated in figure 2.

Figure 2: *In the fluid's initial condition, its initial incompressibility modulus is 2.90 -10^{9} Pascal, which corresponds to the incompressibility modulus of water, and as saturation increases, the equivalent fluid's incompressibility modulus decreases, reaching a minimum value of 1.02 -10^{9} Pascal, which corresponds to the incompressibility modulus of oil.*



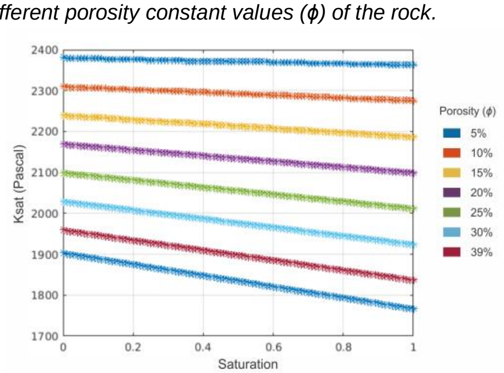
Source: Authors (2023).

Variation of saturated rock density with saturation

Considering the homogeneous and isotropic medium with saturation equal to zero (0) - for rock completely filled with water - and saturation equal to one (or 100%) - for rock completely filled with oil, it is observed that the densities of saturated rock suffer less or more influence from saturation variation, depending on the porosity value that the rock has.

This fact occurs for all porosity values evaluated, however, the variations of saturated rock densities with lower porosity values are lower than the variations of saturated rock densities that have higher porosity values, i.e., the higher the rock porosity, the greater will be the variation of its density with saturation, as illustrated in figure 3.

Figure 3: *Variation of saturated rock density (ρ_{sat}) with saturation for different porosity constant values (ϕ) of the rock.*



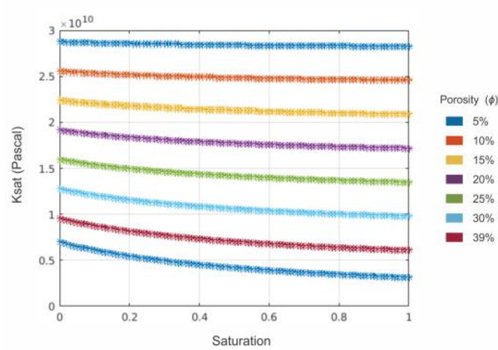
Source: Authors (2023).

Variation of the incompressibility modulus of saturated rock with saturation

Considering the homogeneous and isotropic medium with saturation equal to zero (0) - for rock completely filled with water - and saturation equal to one (or 100%) - for rock completely filled with oil, it is observed that the incompressibility modulus of saturated rock suffers less or more influence of the saturation variation, depending on the porosity value that the rock has.

This fact occurs for all porosity values evaluated, however, the variations of the incompressibility modulus of saturated rock with lower porosity values are lower than the variations of the incompressibility modulus of saturated rock with higher porosity values, i.e., the higher the porosity of the rock, the greater the variation of its incompressibility modulus with saturation, as illustrated in figure 4.

Figure 4: *Variation of the saturated rock incompressibility modulus (K_{sat}) with saturation for different porosity constant values (ϕ) of the rock.*



Source: Authors (2023).

Variation of P-wave velocity of saturated rock with saturation

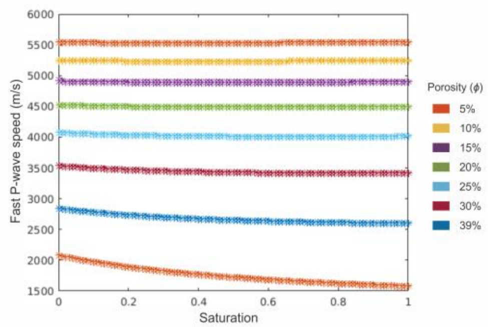
Considering the homogeneous and isotropic medium with saturation equal to zero (0) -for the rock completely filled by water- and saturation equal to one (or 100%) -for the rock completely filled by oil, it is observed that S-wave velocities suffer less influence than P-wave velocities during the fluid replacement process, because the S-wave velocity depends on two parameters (μ and ρ), while the P-wave velocity depends on three parameters (μ, K and ρ).

Similar results were verified in the experimental work of Sayers, Chopra (2009); Baechle et al., (2009); Galvin, Gurevich and Sayers, (2007); Gurevich et al., (2009) and Verwer et al., (2010).

For all cases, the P-wave velocity decreases as saturation increases, indicating that the filling of the rock pore space causes the P-wave velocity in the rock to decrease, or in other words, the dry rock condition provides higher P-wave propagation velocity than the saturated rock condition.

This is explained by the decrease in density of the saturated rock as it is filled with oil and the decrease in incompressibility modulus of the saturated rock, as illustrated in figure 5.

Figure 5: *Variation of the P-wave velocity ($v_{p,sat}$) with saturation for different porosity constant values (ϕ) of the rock.*



Source: Authors (2023).

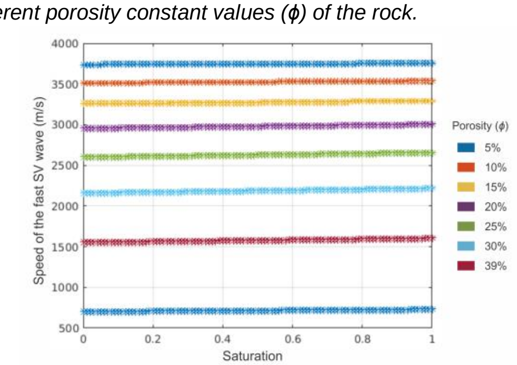
Variation of the S-wave velocity of saturated rock with saturation considering porosity constant at 25%.

Similar analysis is done in relation to the S-wave velocity, which increases as saturation increases, indicating that the filling of the rock pore space causes an increase in the S-wave velocity in the rock, or, in other words, the dry rock condition provides higher S-wave propagation velocity than the saturated rock condition.

This result is explained by the fact that, in Gassmann's theory, the shear modulus remains constant during the fluid replacement process, causing the S-wave velocity to vary only as a result of the variation in density of the saturated rock, which decreases as the rock is filled with oil, which replaces water, i.e., saturated.

Thus, filling the rock with oil decreases its density and therefore increases the velocity of propagation of the shear wave inside it, as illustrated in figure 6.

Figure 6: *Variation of S-wave velocity ($v_{s,sat}$) with saturation for different porosity constant values (ϕ) of the rock.*



Source: Authors (2023).

figures 5 and 6 present the compressional seismic wave velocities vp and shear vs estimated from equations (iv) and (v), both in Appendix D and E. The range of porosity values between 5% and 39% are considered. It can be seen that the variation in porosity values causes variation in P and S wave velocities, however, this variation is more intense in the P wave velocities.

Concluding Remarks

The theory of fluid replacement proposed by Gassmann is the most widely used theory to verify the influence of petrophysical properties of rocks, such as porosity, saturation, incompressibility modulus and shear modulus, when subjected to the propagation of seismic waves in their interiors.

This theory has great importance in the study of Rock Physics and Petrology, due to its application in hydrocarbon reservoirs, especially in the study of 4D seismic, where the fluid substitution theory can be applied based on the saturation variation of the oil contained inside the reservoir as another fluid is injected to replace it.

This saturation variation of the phase mixture of fluids present in the seismic reservoir causes variation in the densities and incompressibility moduli of the equivalent fluid, and this quantification of the saturation variation is evaluated as written in equations (ii) and (iii), both in Appendix B and C, from which it can be seen that the density of the equivalent fluid decreases with increasing saturation, as illustrated in figure 1 with the same occurring for the incompressibility modulus of the equivalent fluid, as illustrated in figure 2.

In addition to observing the variation of the equivalent fluid behavior during the process of fluid replacement, Gassmann's theory was successful in evaluating the influence of saturation and porosity on the incompressibility modulus of the saturated rock, with the major contribution of his work being the fact that it allows to calculate the incompressibility modulus of the saturated rock from the previous knowledge of the incompressibility modulus of the dry rock, the grain of the rock matrix and the incompressibility modulus of the equivalent fluid.

This fact allows Gassmann's theory to be able to evaluate the velocities of compressional seismic waves propagating in porous media and allows the classical equation for compressional seismic wave velocities, written in equation (iv) (Appendix D), to be rewritten using the equation for the incompressibility modulus of the saturated rock, written in equation (1) and the equation for the density of the saturated rock, written in equation (2).

The result of the saturated rock density evaluation, shown in figure 3, shows that increasing saturation causes the saturated rock density to decrease, and the same occurs when porosity increases.

The result of the evaluation of the incompressibility modulus of the saturated rock, shown in figure 4, shows that increasing saturation causes a decrease in the incompressibility of the saturated rock, the same occurring when increasing porosity.

The result of the compressional seismic wave velocity evaluation, shown in figure 5, shows that increasing saturation causes a decrease in compressional seismic wave velocity, and the same occurs when porosity increases.

The result of the evaluation of the seismic shear wave velocity, shown in figure 6, shows that increasing saturation causes an increase in the velocity of this wave, the same occurring when increasing porosity.

From the evaluation of the results shown in figures 1, 2, 3, 4, 5 and 6, it can be concluded that variations in reservoir porosity and fluid saturation have a decisive influence on seismic wave velocities in porous media, justifying the importance of the study and application of Gassmann's theory in porous media.

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APPENDIX - A

$$K_{rs}(\phi) = K_0\left(1-\frac{\phi}{\phi_c}\right) = K_0 \cdot (1-2.5\phi) \qquad \text{(i)}$$

where K_0 is the incompressibility modulus of the mineral; ϕ is the porosity and ϕ_C is the critical porosity.

APPENDIX - B

$$K_{fl} = \left[\frac{(1-S)}{K_{água}} + \frac{S}{K_{óleo}}\right]^{-1} \qquad \text{(ii)}$$

where K_{water} is the modulus of incompressibility of water; K_{oil} is the modulus of incompressibility of oil, and S is the saturation of oil in the reservoir rock, which ranges from 0 (100% water) to 1 (100% oil).

APPENDIX - C

$$\rho_{fl} = S\rho_{óleo} + (1-S)\rho_{água} \qquad \text{(iii)}$$

where S is the oil saturation in the reservoir; ρ_{oil} is the oil density and ρ_{water} is the water density.

APPENDIX - D

$$v_p = \sqrt{\frac{K + \frac{4}{3}}{\rho}} \qquad \text{(iv)}$$

where K is the incompressibility modulus; μ is the shear modulus and ρ is the rock density.