



Study of anisotropy characterization in weakly anisotropic VTI media through diffraction traveltimes parameter clusters

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Abstract

Seismic processing uses a time-domain dataset to gather information about subsurface geological structures. However, lateral velocity variations and anisotropic contributions can cause inaccuracies in the inversion procedure. Determining whether a medium has intrinsic anisotropy and vertical heterogeneity is crucial. In order to achieve that, we explore the feasibility of diffraction traveltimes parameters that can be used as an intrinsic anisotropy signature indicator. By analyzing the behavior of diffraction velocity as a function of the traveltimes slopes estimated from the dataset, we can determine the average measurement velocity using a cluster of diffraction responses. This calculation helps determine whether the medium has lateral velocity variations based on the direction (anisotropy) or position (heterogeneity). The preliminary analysis shows that the proposed approach has the potential to be incorporated into seismic processing as an additional tool to identify anisotropic signatures distorting the estimated normal moveout velocities.

Introduction

Characterizing subsurface geologic structures to determine the location of hydrocarbon reservoirs can be achieved through a reliable seismic data processing framework. The presence of lateral velocity variation introduces an additional challenge to this task. Therefore, when such a phenomenon is caused by anisotropy, the kinematic parameters used for the inversion procedure are distorted with anisotropic contributions. The anisotropy may be intrinsically related to the preferred orientation of anisotropic mineral grains, induced by thin isotropic layers with thicknesses less than wavelength or related to fracture orientation. Based on these conditions, an analysis of how the lateral velocity variation changes wavefront kinematics is necessary.

The characterization of transversely isotropic (TI) symmetries by parameters given by Thomsen (1986) made it possible to incorporate the extraction of anisotropy parameters into seismic data. Alkhalifah (1997) presented a framework for a real field-data example that is based

on reflections, and parameters are extracted using an offset-dependent nonhyperbolic traveltimes. However, this procedure is only practical in anisotropic homogeneous media. When vertical heterogeneous media is considered, the *anelipticity* attribute η loses its kinematic interpretation and works as a best-fitting parameter. Consequently, the result is a better normal moveout velocity (V_{NMO}) estimation in this case. Besides, it is difficult to determine if an influence of the Thomsen parameter δ exists when the interval velocity is estimated. As for reflected waves, diffracted waves also have relevant information regarding the geological structure in the subsurface, with a notable advantage that the diffraction event as a whole is related to a single point in the subsurface. Therefore, determining the kinematic contributions for diffraction response is a topic of great interest, with applications varying from structural characterization to velocity analysis.

In this work, we explore the feasibility of diffraction traveltimes parameters as indicators of an anisotropy signature. By extracting the attributes of slope and curvature from the diffraction traveltimes, it is possible to determine a velocity given in function of diffraction slopes. Note that the normal moveout velocity depends on the slope, as shown in Alkhalifah and Tsvankin (1995). In our framework, two signatures are related to anisotropy and lateral heterogeneity, which can be obtained from the data.

Our synthetic experiments in vertical transversely isotropic (VTI) media show that the variation of the measured velocity is considerably tiny in the presence of pure vertical heterogeneity and has higher deviations in the presence of anisotropy. Based on this, we also determine the average measurement velocity using a cluster of diffraction responses, which we refer to calculate if the medium has lateral velocity variations based on the direction (anisotropy) or position (lateral heterogeneity).

Finally, this approach can be incorporated into seismic processing as an additional tool for users to make decisions regarding the anisotropic signatures distorting the estimated V_{NMO} .

Formulation

In order to perform our analysis, we will briefly describe the tool used to detect diffractions in a seismic-response dataset. Besides, we are considering this dataset modeled along a single horizontal line with the midpoint and half-offset defined as (m, h) from which one can derive source-receiver coordinates. A super gather of source-receiver pairs is assumed to be arbitrarily located

concerning a reference pair $(m_0, 0)$, called the zero-offset (ZO) coordinates. Considering no prior knowledge about the subsurface geological structures, the challenge is to make the extraction of kinematic parameters with precision from the seismic data. In order to perform such parameter extraction, the tools given by Facciopieri et al. (2016) and Coimbra et al. (2019) can be used, for example. The traveltime approximation that describes the kinematics behind these cited operators can be described as

$$t_D(m, h) = \frac{1}{2} \left[\sqrt{(t_0 + A\Delta s)^2 + C(\Delta s)^2} + \sqrt{(t_0 + A\Delta r)^2 + C(\Delta r)^2} \right], \quad (1)$$

where $t_D(m, h)$ is the diffraction traveltime approximation, $\Delta s = \Delta m - h$, $\Delta r = \Delta m + h$, $\Delta m = m - m_0$, and t_0 is the two-way traveltime on the reference ray $(m_0, 0)$. Also, A is the slope of traveltime in the midpoint direction, and C is a species of the slowness squared, which can be specified in terms of derivatives of traveltime t as

$$A = \left. \frac{\partial t}{\partial m} \right|_{(m_0, 0)}, \quad \text{and} \quad C = t_0 \left. \frac{\partial^2 t}{\partial h^2} \right|_{(m_0, 0)}. \quad (2)$$

In practice, we can obtain the diffraction parameters of equation (1) through traveltime parameter search algorithms such as Ribeiro et al. (2023). Besides, as shown in Facciopieri et al. (2016), an optimal aperture in midpoints (based on the Fresnel zone) is necessary to achieve suitable results. Such parameters describe a time-migration velocity related to the points $(m_0, 0)$ in a ZO section, corresponding to the same point in depth.

If we set $A = 0$ and $m = m_0$ in equation 1, we find the common-midpoint (CMP) moveout, which is given by

$$t_D(m_0, h) = t_{\text{CMP}}(h) = \sqrt{t_0^2 + C_0 h^2}, \quad (3)$$

where we can interpret the parameter C_0 in terms of V_{NMO} as

$$C_0 = \frac{4}{[V_{\text{NMO}}(0)]^2}. \quad (4)$$

The parameter C_0 represents a wavefront curvature measured with respect to the imaging ray (Hubral, 1983). Additionally, by equation (2), we can define a time-migration (TM) velocity, V_{TM} , in the function of parameter A as (Coimbra et al., 2019)

$$V_{\text{TM}}(A) = \frac{2V_{\text{NMO}}(A)}{\sqrt{4 + A^2 V_{\text{NMO}}^2(A)}}. \quad (5)$$

Therefore, by measuring the TM velocity as a function of the kinematic parameter of the slope, we can extract meaningful information on the kinematic attributes of wavefront propagation. Furthermore, the equation (5) describes such behavior of this velocity on the kinematic response of a diffraction wavefront.

TM-velocity variation in homogeneous media

Let us start our analysis with the simplest case of an isotropic homogeneous medium with constant velocity V .

For this case, the physical interpretation of the kinematic parameter A is given by

$$A = \frac{2\sin(\beta)}{V}, \quad (6)$$

where β is the angle formed between the phase vector of the ray and the normal vector to the measurement surface at a midpoint m_0 . Therefore, in this media, $V_{\text{NMO}}(A)$ is given by

$$V_{\text{NMO}}(A) = \frac{2V}{\sqrt{4 - A^2 V^2}} = \frac{V}{\cos(\beta)}. \quad (7)$$

Replacing the equations (7) into (5), we have $V_{\text{TM}}(A) = V$. This implies that the TM velocity is independent of A , indicating no velocity variation. Now, let us examine the VTI Media (VTI) case for a homogeneous case considering the elliptic case, i.e., the Thomsen parameters $\varepsilon = \delta$. Following Alkhalifah and Tsvankin (1995) we have the exact expression for V_{NMO} , here in terms of A , given by

$$V_{\text{NMO}}(A) = \frac{2V_{\text{NMO}}(0)}{\sqrt{4 - A^2 V_{\text{NMO}}^2(0)}}, \quad (8)$$

with $V_{\text{NMO}}(0) = V_p \sqrt{1 + 2\delta}$ and where V_p is the vertical velocity of P-wave. Again, replacing the equation (8) into (5), we have $V_{\text{TM}}(A) = V_{\text{NMO}}(0)$, which implies that the TM velocity is constant along the diffraction response. For the general homogeneous case, consider true a weak approximation for V_{NMO} given by Alkhalifah and Tsvankin (1995) in terms of the slope A as

$$V_{\text{NMO}}(A) = \frac{2V_{\text{NMO}}(0)}{\sqrt{4 - A^2 V_{\text{NMO}}^2(0)}} [1 + \xi F(A)], \quad (9)$$

where $\xi = \varepsilon - \delta$ is the anisotropy correction, and

$$F(A) = \frac{y(4y^2 - 9y + 6)}{1 - y} \quad \text{with} \quad y = \frac{A^2}{C_0}. \quad (10)$$

Replacing equation (10) into (5), we have an expression for the TM-velocity counterpart (valid when $|\varepsilon| \ll 1$ and $|\delta| \ll 1$) given by

$$V_{\text{TM}}(A) = \frac{V_{\text{NMO}}(0)}{\sqrt{1 + 2y\xi F(A) + y\xi^2 F(A)^2}} [1 + \xi F(A)]. \quad (11)$$

Observe that $\xi = 0$ implies the medium is isotropic or elliptic. Indeed, the above expression works well even for moderate anisotropy in the small- A limit, making it possible to use it to obtain the best fit concerning parameter ξ , considering the velocity V_{TM} obtained from a dataset.

Results in a homogeneous VTI media

In this section, we analyze two homogeneous anisotropic examples. First, we take a VTI medium with $\varepsilon = 0.195$, $\delta = -0.220$, and vertical P and S velocities given by $V_p = 3292$ m/s and $V_s = 1768$ m/s, respectively. Using the diffraction traveltime approximation, given by equation (1), to extract A and $V_{\text{TM}}(A)$ from a diffraction response located at a depth of 1000 m. Figure 1 shows the TM velocity variation (blue line) as a function of slope A obtained by fitting diffraction traveltime approximation with the diffraction response. Also, in Figure 1, we have the

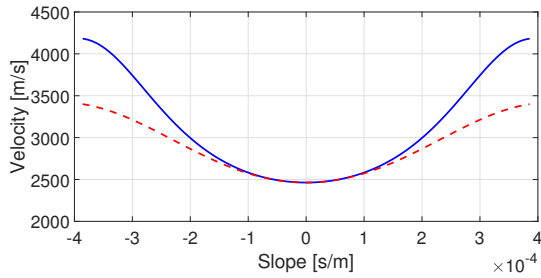


Figure 1: TM velocity measured (blue line) of a diffraction embedded in a homogeneous VTI medium with $\varepsilon = 0.195$, $\delta = -0.220$ and velocities $V_p = 3292$ m/s and $V_s = 1768$ m/s. The red dashed line represents the best fitting using equation (11).

best fit of the curve described by the equation (11) (red dashed line). From that, the extracted anisotropy correction parameter is $\xi_E = 0.5$. However, its original value is $\xi = 0.4180$, resulting in an absolute error $|E_A| = 0.082$.

Second, we take a VTI medium with $\varepsilon = 0.110$, $\delta = -0.035$ considering P and S velocities given by $V_p = 3368$ m/s and $V_s = 1829$ m/s, respectively. Again, Figure 2 shows the TM velocity variation (blue line) as a function of slope A obtained by fitting the curves between diffraction traveltimes approximation and diffraction response. Moreover, Figure 2 also shows the best fit of the curve described by the equation (11) (red dashed line). In this second experiment, we see a better fit with an extracted anisotropic correction parameter of $\xi_E = 0.1735$. The modeled parameter is given by $\xi = 0.1450$, resulting in an absolute error given by $|E_A| = 0.0285$.

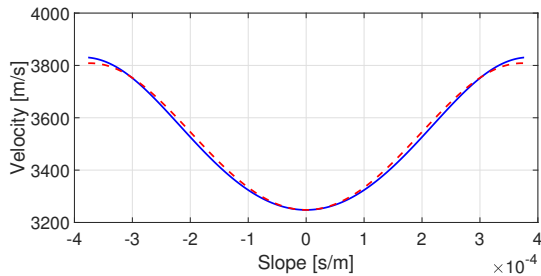


Figure 2: TM velocity measured (blue line) in a homogeneous VTI medium with $\varepsilon = 0.110$, $\delta = -0.035$ and velocities $V_p = 3368$ m/s and $V_s = 1829$ m/s. The red dashed line represents the best fitting using equation (11).

The group velocity (ray velocity) for P- and SV- waves in TI media has the direction dependence as presented in Tsvankin (2001). As a consequence, the group velocity V_g can be written in terms of phase velocity V , taking into account the direction correction as

$$V_g(\theta) = V \sqrt{1 + \left(\frac{1}{V} \frac{dV}{d\theta} \right)^2}, \quad (12)$$

where θ represents the phase direction angle. Also, one way to track the symmetry axis is to evaluate those directions where $V_g = V$. For such cases, if ϕ represents

the angle with the vertical representing the symmetry orientation, then the derivative of phase velocity, when applied to $\theta = \phi$ is zero. For the specific case of VTI media, the equation (12) is still valid regarding the vertical symmetry axis, which implies $\phi = 0$. Meanwhile, for that same media, by Coimbra et al. (2023), the TM velocity can be written in terms of θ as

$$V_{TM}(\theta) = V \sqrt{1 + \frac{1}{V} \frac{d^2V}{d\theta^2}}. \quad (13)$$

Therefore, it is clear the influence direction of the media, in anisotropic cases, on the wave propagation has implications on the obtained datasets for purposes of seismic processing, but it is important to note that, unlike the isotropic medium, we cannot associate the TM velocity with the group velocity despite these changes concerning the phase angle. Next section, we examine examples that provide clues to determine some signature on seismic data to get the information of pure heterogeneity or the possible existence of anisotropy contributions.

Diffraction Cluster - with vertical heterogeneity

Until now, we considered a straightforward diffraction response, which provides an indication of the TM velocity variation and anisotropic influence on the wavefront path. However, a single diffraction response cannot take relevant information about the phenomenon, being necessary to consider more diffraction responses. For that, we considered a cluster of diffraction responses near the target region to achieve suitable redundancy of information. Figure 3 shows their distribution and specifies the velocity model. The diffraction traveltimes approximation, equation 1, is applied to recover all these diffractions from the dataset, obtaining an individual TM velocity related to each extracted diffraction response. Here, we use the global optimizer differential evolution (DE) for the traveltimes fitness (Storn and Price, 1997). After this, an average TM velocity is created.

Figure 4 shows the TM velocity average represented by the solid blue line. Here, it is possible to see a pattern of strong high-velocity variation and the individual diffraction case. Besides, when we apply the equation (11) to this average velocity curve, we obtain $\xi_E = 0.4525$, which represents an approximation for the anisotropic deviation of the medium where the exact anisotropy has $\xi = 0.4180$. Also, such a cluster can be used in the isotropic case, illustrated by Figure 5 where verifying a small average velocity variation is possible. It was obtained $\xi_E = 0.007$, where $\xi = 0$ is the correct, emphasizing the probable prevalence of heterogeneity since the velocity is not constant. For the example with velocity $v_p = 3368 + 0.7z$ m/s and $v_s = 1829$ m/s where $\varepsilon = 0.110$ and $\delta = -0.035$ the result is illustrated in Figure 6. The extracted parameter was $\xi_E = 0.1510$ in contra-position to exact one $\xi = 0.1450$ resulting in the absolute error $|E_A| = 0.006$. The case $\varepsilon < \delta$ with $\varepsilon = 0.110$ and $\delta = 0.150$, illustrated in Figure 7 where $\xi_E = -0.0555$ and the exact one is $\xi = -0.04$. From the concavity orientation of the velocity, the diffraction signature can be used to set if ε is greater than δ and vice versa.

Diffraction Cluster - with lateral heterogeneity

Let us move on to the case with lateral heterogeneity where the velocity is described by $V_p(x, z)$ as illustrated

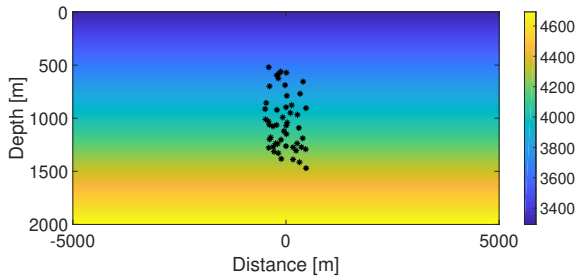


Figure 3: Cluster of diffractions varying in depth and lateral position in a heterogeneous medium with velocities $V_p = 3292 + 0.7z$ m/s and $V_s = 1768$ m/s. A similar cluster configuration was used for other velocities.

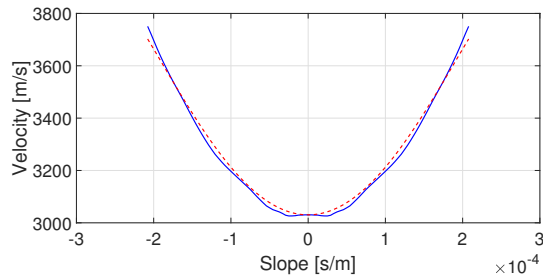


Figure 4: Average TM velocity response (blue line) of a diffraction cluster embedded in the VTI medium with $\varepsilon = 0.195$, $\delta = -0.220$ and velocity described in Figure 3. The red dashed line represents the best fitting using equation (11).

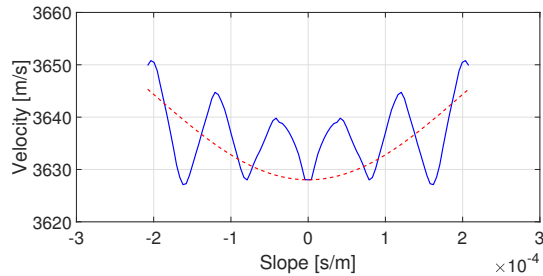


Figure 5: Average TM velocity (blue line) in the heterogeneous medium with velocity described in 3. The red dashed line represents the best fitting using equation (11).

in Figure 8. As before, Figure 9 shows the TM velocity response represented by the solid blue line. The red dashed line is the best fit obtained using equation (11), presenting a poor performance. To consider such cases, it is necessary to modify that equation to consider the lateral heterogeneity. We propose a correction for considering lateral heterogeneity in the form

$$V_{TM}(A) = \frac{V_{NMO}(0)(1 + \alpha A)}{\sqrt{1 + 2y\xi F(A) + y\xi^2 F(A)^2}} [1 + \xi F(A)], \quad (14)$$

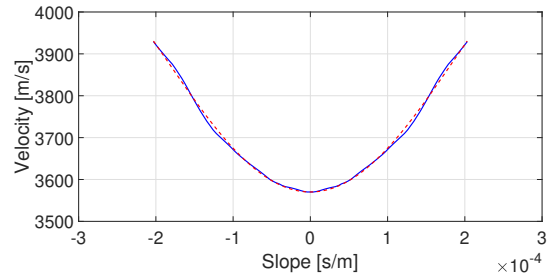


Figure 6: Average TM velocity response of a diffraction cluster (blue line) embedded in the VTI medium with $\varepsilon = 0.110$, $\delta = -0.035$ with velocity $v_p = 3368 + 0.7z$ m/s and $v_s = 1829$ m/s. The cluster is the same described in Figure 3. The red dashed line represents the best fitting using equation (11).

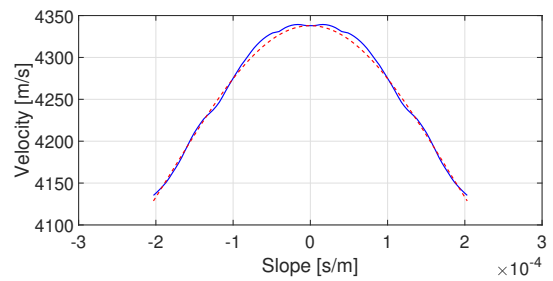


Figure 7: Average TM velocity response of a diffraction cluster (blue line) embedded in the VTI medium with $\varepsilon = 0.110$, $\delta = 0.150$ with velocity $v_p = 3368 + 0.7z$ m/s and $v_s = 1768$ m/s. The cluster is the same described in Figure 3. The red dashed line represents the best fitting using equation (11).

where $F(A)$ preserves its defined structure, see equation (10), and

$$y = \frac{A^2}{C_0} (1 + \alpha A)^2. \quad (15)$$

The parameter α has the dimension of velocity (m/s) here named lateral velocity factor. Considering this correction, Figure 10 shows the performance of fitting where the extracted parameter was $\xi_E = -0.03$ and the lateral velocity factor is $\alpha = 453$ (m/s). Applying this equation to other examples makes clear the correction's effectiveness. Figure 11 illustrates other example where the fitting parameter obtained is $\xi_E = 0.1745$ and $\alpha = 528$ (m/s). In this case, we can see a good fit performance. Figure 12 shows the performance of our approach when considering a strong anisotropy where $\xi_E = 0.4980$ and $\alpha = 500$ (m/s). The last example shows the performance for the pure homogeneous case where the parameter $\xi = 0$. The signature for this case is well illustrated in Figure 13, and for this case, $\xi_E = 0.002$ and $\alpha = 412$ (m/s).

Discussion

We start the discussion by considering the relationship between heterogeneity and velocity spreading in contrast to anisotropic influence. Cameron et al. (2007) presented a well-established relation between migration

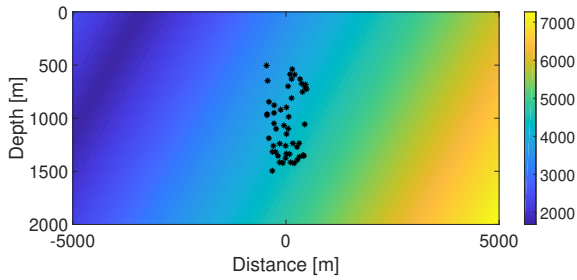


Figure 8: Cluster of diffractions varying in depth and lateral position in a heterogeneous medium with velocities $V_p = 3368 + 0.5x + 0.7z$ m/s and $V_s = 1829$ m/s.

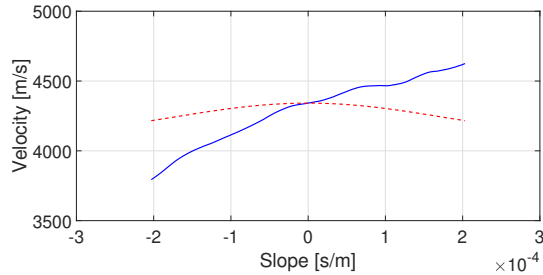


Figure 9: Average TM velocity response (blue line) of a diffraction cluster embedded in the VTI medium with $\varepsilon = 0.110$, $\delta = 0.150$ with velocity $v_p = 3368 + 0.5x + 0.7z$ m/s and $v_s = 1768$ m/s. The cluster is the same described in Figure 8. The red dashed line represents the best fitting using equation (11), which does not follow the velocity signature.

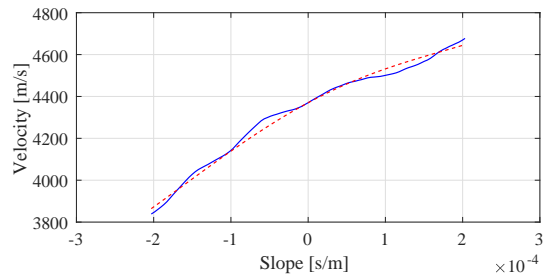


Figure 10: Average TM velocity response of a diffraction cluster embedded in the VTI medium with $\varepsilon = 0.110$, $\delta = 0.150$ with velocity $v_p = 3368 + 0.5x + 0.7z$ m/s and $v_s = 1768$ m/s. The cluster is the same described in Figure 8. The red dashed line represents the best fitting using equation (14).

velocity and wave propagation velocity (Dix velocity) for stratified isotropic media. In that work, they have shown that

$$V_{\text{Dix}} = \frac{V_g}{|Q|}, \quad (16)$$

where $|Q|$ represents the geometric spreading of wavefront if we consider a line source on measurement surface and telescopic ray, taking into account that, for stratified media with no lateral velocity variation, we have $|Q| = 1$, then the conclusion is $V_{\text{Dix}} = V_g$. However, for complex media, the spreading factor $|Q| \neq 1$, the relation makes clear the

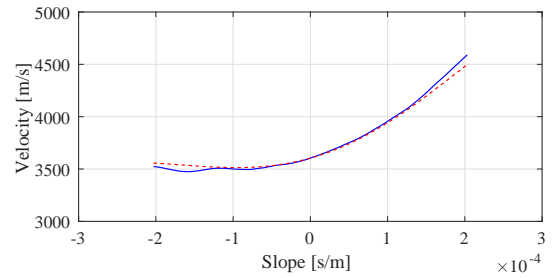


Figure 11: Average TM velocity response of a diffraction cluster (blue line) embedded in the VTI medium with $\varepsilon = 0.110$, $\delta = -0.035$ with velocity $v_p = 3368 + 0.5x + 0.7z$ m/s and $v_s = 1829$ m/s. The cluster is the same described in Figure 8. The red dashed line represents the best fitting using equation (14).

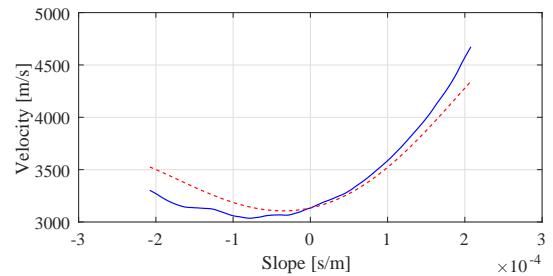


Figure 12: Average TM velocity response of a diffraction cluster (blue line) embedded in the VTI medium with $\varepsilon = 0.195$, $\delta = -0.220$ with velocity $v_p = 3292 + 0.5x + 0.7z$ m/s and $v_s = 1768$ m/s. The cluster is the same described in Figure 8. The red dashed line represents the best fitting using equation (14).

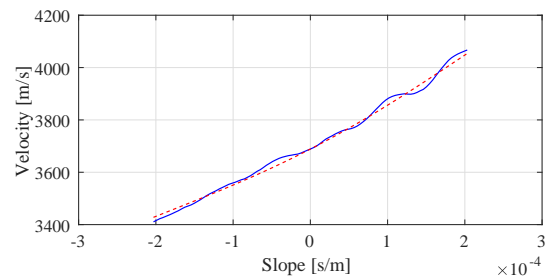


Figure 13: Average TM velocity response of a diffraction cluster (blue line) embedded in the heterogeneous medium with velocity $v_p = 3368 + 0.5x + 0.7z$ m/s and $v_s = 1829$ m/s. The red dashed line represents the best fitting using equation (14).

impossibility of an exact correspondence between the Dix velocity and the real group velocity, which implies an error when one performs the inversion procedure for such cases of heterogeneous isotropic media. However, for anisotropic general media, Coimbra et al. (2023) have shown that

$$V_{\text{Dix}} = \frac{V}{|Q|} \sqrt{1 + \frac{1}{V} \frac{d^2 V}{d\theta^2}}. \quad (17)$$

The influence of anisotropy will be noticed even in the symmetry axis in the VTI medium. Consequently, the Dix and Group velocity differs even in homogeneous and vertical heterogeneous media, where $|Q| = 1$, according to the analysis of Thomsen (1986). How to determine $|Q|$ for practical examples is unclear. In our approach, the proposed diffraction cluster may indicate possible velocity distortions during the inversion procedure.

The introduction of this additional parameter, named lateral velocity factor, α combined with the anisotropy signature ξ_E , can help in corrections during the inversion task. Note that equation (14) can be used in both cases since α is estimated null for $V_p(z)$ allowing us to obtain a general framework based on this approach. On the other side, for the almost elliptic ($\varepsilon \approx \delta$) case, there is a bottleneck to the procedure, which is to differentiate pure heterogeneity from anisotropy. The signature must be carefully examined for those cases to find curvatures that will happen only in the VTI case –comparing the figures 10 and 13, it is clear the difference. The fitting curve is a straight line for the pure heterogeneous case, while there is a small curvature for the VTI case. Therefore, the proposed approach is a tool to help to figure out distortions during the inversion. An additional procedure to estimate the group velocity from the diffraction is necessary to determine the anisotropy parameters. Coimbra et al. (2023) have shown how to do this for complex anisotropic media using an Eikonal-type equation of the wave-equation time migration for the generalized case. Accordingly, in that work, the authors generalize the result proposed by Fomel and Kaur (2021). That technique can be combined with the proposed approach to characterize the anisotropy of the medium entirely.

Conclusions

Diffraction signatures are of great interest for velocity model building which is essential to depth conversion procedures. In this work, an additional feature about diffraction events is presented. Using the TM velocity, it is possible to track variations of the medium velocity concerning direction, which characterizes anisotropy. Such a procedure can be done directly from any diffraction dataset. Besides, the numerical experiments for vertical heterogeneity (even thin vertical layers) show that the TM velocity variation presents a distinct signature compared to heterogeneous VTI media. Even more, a TM-velocity approximation is proposed considering the most challenging case of lateral heterogeneity. The result is a framework that can be used to track signatures of anisotropy in the medium and pronounced lateral velocity originated by pure heterogeneity. Further investigations may suggest introducing an additional step in seismic data processing to explore such features. Finally, in ongoing work, we will study if this pattern has the same behavior for TTI and HTI media and anisotropies with fewer symmetries.

Acknowledgments

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