Bayesian inversion of laboratory complex resistivity measurements of carbonate rocks to estimate parameters of the Hybrid Dias/Cole-Cole model

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Abstract

We study the spectral electrical polarization (SIP) effect using the Dias and Hybrid models with lab measurements in carbonate rocks. The objective is to understand how this effect changes under more complex pore structures and surface properties caused by diagenetic alterations common in carbonates. Our experimental data of spectral electrical impedance are taken from 13 carbonate rocks samples from the Sergipe Sub-basin, using the frequency range from 1mHz to 100kHz. The Dias and Hybrid model parameters are first estimated by a trial and error (amplitude and phase) data fitting procedure used in our routine lab work. This method allows for fitting complex electrical resistivity for the entire frequency range but can not provide model parameter uncertainties and correlations. From SIP model parameters, it is possible to determine petrophysical properties of the rock samples such as permeability and pore radius. Another problem with this approach is that it can incorporate bias into the solution. Thus, we also use the Bayesian inference methodology based on Markov Chain Monte Carlo Method (MCMC) stochastic simulation method to improve the solution. We present and discuss the results for the Dias and Hybrid model parameter estimates obtained from 4 of the 13 studied samples. Analysis of the simulated chains and the quality of final amplitude and phase data fit shows that parameters have been reliably estimated for reservoir characterization applications.

Introduction

Inversion of induced polarization parameters is important in characterizing the electrical spectral response of porous rocks. In the spectral induced polarization (SIP) literature, three different approaches are often applied to interpret of SIP data. In the first approach, no fitting model is employed. Instead, the real and imaginary parts of the conductivity are interpreted in terms of its basic characteristics, like the peak value and frequency (Börner and Schoen, 1991; Kruschwitz et al., 2010). The second approach is to fit the SIP data using the generalized Cole-Cole model or any of its variants. In this case the optimization problem is overdetermined and the data fitting can be done using the non-linear least squares formulation. Lastly, the interpretation of the SIP data can be made by deconvolving the complex resistivity spectra in a linear superposition of relaxation models. A Warburg or Debye transfer function is typically used in the deconvolution scheme. In this approach the problem is underdetermined and requires optimal regularization.

Global optimization methods such as Markov-Chain Monte Carlo (MCMC) have advantages compared to least squares optimization techniques (Ghorbani et al., 2007), such a reduced the influence of initial values and the global coverage of the model parameter space. The latter allows for the computation of marginal probability distribution functions, from which we can make relevant inferences such as estimates and more realistic uncertainty bounds on the parameters. MCMC methods are effective for sampling complex, high-dimensional joint probability functions; have been increasingly used to invert geophysical data (Bosch, 1999; Buland and Omre, 2003; Gunning and Glinsky, 2003; Chen et al., 2004). Chen et al. (2008) developed an MCMC-based Bayesian model for inverting Cole-Cole parameters of SIP data. They compared its performance with the more traditional deterministic Gauss-Newton method for inverting synthetic and laboratory data. The Bayesian method estimates the joint marginal posterior distribution, defined by the likelihood functions of SIP data and prior distributions of unknown parameters, while the deterministic method searches for the optimal solution by minimizing the squared misfit of the model response with the SIP data.

Keery et al. (2012) describe several mathematical models that could represent the SIP response of porous media, emphasizing a simple approach that presents the overall SIP response as an accumulation of many simple relaxations. Their work describes and applies a deterministic method for estimating a distribution of relaxations (Nordsiek and Weller, 2008; Zisser et al., 2010), showing how this approach can be extended within a stochastic framework. Using a simple polynomial of such a distribution, the model is launched stochastically and is solved using an MCMC chain, thus allowing the calculation of uncertainties of the model parameters. The model is then applied to the synthetic data, demonstrating that the stochastic method can provide posterior distributions of the model parameters with tight bounds around the true values when little or no noise is added to the synthetic data. The widness of posterior distributions increases with increasing noise levels.

On the other hand, extensive work has been carried out in the Petrophysics laboratory of LENEP/UENF, in the search for optimization of the model by Dias (2000) for specific cases, carrying out spectral resistivity measurements in samples of siliciclastic rocks, aiming at the correct description of the IP effect and the better representation of laboratory measurements (Barreto and Dias, 2014; Marinho and Dias, 2017; Marinho, 2018; Carvalho, 2019). Meanwhile, a large volume of hydrocarbons in the world is accumulated in carbonate rocks, comprising the giant fields of the Middle East and the Brazilian pre-salt region (Santos, Campos and Espírito Santo Basins). The evaluation of carbonate reservoirs is a challenging task for petrophysicists and a good understanding of the transport properties in these porous media is still lacking. Relative to siliciclastics, carbonates may be simpler in terms of mineralogy, but they are incomparably more complex in terms of pore structure and surface properties (Fleury, 2002). In this sense, the use of the Dias model and Hybrid Dias/Cole-Cole model (for simplicity, we call it the Hybrid model) to describe the effect of spectral electrical polarization in carbonate rocks is proving to be a challenge, especially considering that these models were using siliciclastic rocks.

In this work, we performed MCMC inversion of stochastic simulation for Bayesian inference using the Dias model (Dias, 2000). According to (Bérubé et al., 2017), this approach offers significant advantages in terms of computation time when inverting SIP data with the decomposition approach, compared to the non-adaptive routine proposed by (Keery et al., 2012). The first test of the inversion procedure was performed to obtain the parameters of the original Dias model. Based on experimental data of spectral complex resistivity of siliciclastic rock available at the Petrophysics Laboratory of LENEP from samples I2401H, I2403H, I2446H from Campo D. João, Recôncavo Basin and from sample P0897H from Corvina field, Campos Basin, primitive parameters of the Dias model were estimated and amplitude and phase curves were constructed. Subsequently, the phase curve was correlated with the experimental data to find the best curve fit and thus obtain the parameters of the Hybrid model. It was concluded in this first stage of the research that the MCMC inversion could be considered in the future processing of experimental data of complex spectral resistivity, thus replacing the first steps of the procedure for obtaining parameters of the Hybrid model that is currently being carried out in the Laboratory of Petrophysics at the LENEP. Next, adjustments were made to the experimental data of 13 carbonate rocks samples from the Sergipe Sub-Basin (SE-Basin), through MCMC inversion using the Dias model (Dias, 2000).

Lastly, Bayesian inversion to inference Hybrid model parameters was performed with satisfactory results. This work presents the results in amplitude and phase curves of spectral electrical resistivity for 4 of the 13 studied samples.

Method

Dias model and Hybrid model

The Dias model used to describe the induced polarization effect is defined as a frequency function ($\omega = 2\pi f$) and is dependent on five primitive parameters (ρ_0,m,τ,η,δ):

$$\frac{\rho^* - \rho_\infty}{\rho_0} = \frac{1}{1 + i\omega\tau'(1 + \frac{1}{\mu})} \tag{1}$$

$i^2 = -1$, $\tau' = [\frac{1-\delta}{(1-m)\delta}]\tau$, $\mu = i\omega\tau[1 + \eta(i\omega)^{-1/2}]$, where $\rho_\infty = \mu = (1-m)\rho_0$. The terms ρ_0 and ρ_∞ are values of $\rho*$ when $f \to 0$ and $f \to \infty$, respectively; m is the chargeability (dimensionless); τ is the relaxation time (s) related to the Helmholtz's electrical double layer zone; η is the electrochemical parameter ($s^{-1/2}$); δ is the fraction of unit cell extension affected by polarization effect due to the solid-liquid interface (Dias, 2000).

The model has been introduced by Marinho and Dias (2017) and Marinho (2018). The model aims to best fit experimental data in the intermediate region of the frequency spectrum, where the Dias model generally does not get a good fit. The Hybrid model is defined as a frequency function given by:

$$\frac{\rho^* - \rho_\infty}{\rho_0} = \frac{m_{W1}}{1 + (i\omega\tau_{W1})^{\frac{1}{2}}} + \frac{m_D}{1 + i\omega\tau_D} + \frac{m_{W2}}{1 + (i\omega\tau_{W2})^c}, \tag{2}$$

with $0 \le c \le 1$.

where m_{W1}, m_{W2} and m_D are partial chargeability associated, respectively, to the low, intermediate and high frequencies of the observed spectrum. The partial chargeabilities relate to original Dias model chargeability (m) through the sum: $m = m_{W1} + m_{W2} + m_D$. The expression 2 is also dependent on three distinct main relaxation times. According to Barreto & Dias (2014), τ_W and τ_D and from the original Dias model are related, respectively, to the ionic diffusion process inside the electrical double layer (low frequencies) and to capacitive-resistive effect of this same formed structure (high frequencies). The term τ_{W2} is the main relaxation time corresponding to a peculiar diffusion process under influence of heterogeneities of each geological material. The c index, characteristic of Cole-Cole's functions ($0 \le c \le 1$), is related to the intensity of heterogeneities present in the rock-fluid system.

Bayesian inference using MCMC

Monte Carlo techniques using Markov chain are based on stochastic simulations. The motivation for these techniques is mainly given to the Bayesian statistical framework, where the inference is made about by a posterior function, represented by $\pi(\vec{m}|\vec{d})$. On many occasions, Bayesian inference needs to integrate very high dimensional distributions (e.g.,hundreds of parameters). The application of MCMC techniques consists of two steps: (i) generate a sample $\vec{m}_1$,..., $\vec{m}_n$ using a Markov chain whose stationary distribution is the desired one and (ii) take parameter samples by Monte Carlo integration and make inferences about the parameters.

This general structure allows for solving many problems. Under a Bayesian approach, all the parameters and data of the model are random quantities. Denoting the observed data by $\vec{d}$ and the set of parameters of the model by $\vec{m}$. In Bayesian statistics, the posterior function can be expressed as follows, applying Bayes' theorem:

$$\pi(\vec{m}|\vec{d}) = \frac{p(\vec{d}|\vec{m})q(\vec{m})}{\int p(\vec{d}|\vec{m})q(\vec{d})d\vec{m}}, \tag{3}$$

where $\pi(\vec{m}|\vec{d})$ is the information about $\vec{m}$ after observing $\vec{d}$, also known as the *posteriori* distribution and can be written as:

$$\pi(\vec{m}|\vec{d}) = \frac{p(\vec{d}|\vec{m})q(\vec{m})}{h(\vec{d})}, \tag{4}$$

where $h(\vec{d}) = \int p(\vec{d}|\vec{m})q(\vec{d})d\vec{m}$ is the normalization constant. The posterior distribution is then proportional to the numerator of the equation 3. For a fixed value of $\vec{d}$ the function $\mathcal{L}(\vec{m};\vec{d}) \approxeq p(\vec{d}|\vec{m})$ is defined as likelihood function, which provides the plausibility of $\vec{m}$ values, given the observations made $\vec{d}$. Following these definitions the Bayes' theorem can be represented in the no-normalized form: $\pi(\vec{m}|\vec{d}) = \mathcal{L}(\vec{m};\vec{d})q(\vec{m})$. The solution of the inverse problem can be obtained through the technique of Metropolis-Hasting algorithm that allows to randomly sample the distribution of probabilities $\pi(\vec{m}|\vec{d})$. To get random samples of $\pi(\vec{m}|\vec{d})$, an appropriate Markov chain is defined in the space of unobservable quantities. This chain proposes random samples of $\vec{m}$ that are or are not accepted by the algorithm. The collection of accepted samples has a frequency distribution proportional to $\pi(\vec{m}|\vec{d})$, which represents the complete solution of the inverse problem.

Implementing MCMC for SIP inversion

In this work, MCMC techniques were implemented to estimate parameters of the Dias and Hybrid models by using the likelihood function

$$\mathcal{L}(\vec{m}|\vec{d},\epsilon) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\epsilon_i} \exp\left[-\frac{1}{2\epsilon_i^2}(\vec{d}_{obs_i} - \vec{d}_{calc_i})^2\right], \quad (5)$$

where $\vec{d}_{obs_i}$ represent the measurements (observed data) and ϵ its uncertainties in the inverse problem, and $\vec{d}_{calc_i}$ is defined as the modeled complex resistivity as a function of SIP model parameters $\vec{f}(\vec{m})$, defined here as $\vec{f}_D(\vec{m}_D)$, $\vec{f}_P(\vec{m}_P)$ and $\vec{f}_H(\vec{m}_H)$ for the Dias, partial fraction Dias and Hybrid models, respectively. The Prior distributions are uniform on a bounded interval.

Results

Modeling the spectral electrical data of carbonate samples rocks of SE-Basin

In this section, the amplitude (Ωm) and phase $(mRad)$ curves of the experimental spectral electrical resistivity data in the frequency range from 1mHz to 100kHz and the discussion of their respective modeling using the Dias and Hybrid models for 4 samples of carbonate rocks from the Sergipe sub-basin are presented. The data fitting by the traditional method uses trial and error, and the agreement between measured and modeled data is checked individually using Normalized Root Mean Square Error (NRMSE) of amplitude and phase values. The results are shown in Figure 1. Table 1 presents the final parameters estimates obtained from modeling experimental data.

The quality of experimental electrical measurements was considered satisfactory, with spectral complex resistivity curves without relevant noise signals throughout the analyzed frequency spectrum. The phase curves of the Dias model showed a good fit covering the two maxima observed on the phase spectrum. The first at low frequencies located around 1 Hz which is associated with the process of anion-selective diffusion of the N_a^+ ion from the electrolyte solution at the clay/electrolyte interface inside the Helmholtz electrical double layer, and the second at high frequencies, associated with the capacitive-inductive effect inside the electrical double layer as a whole (Barreto and Dias, 2014). In all samples the presence of an extensive and high phase plateau in the intermediate frequency region (10^1 Hz to 10^4 Hz) is noted, which is associated with the differentiated diffusion process caused by the presence of heterogeneities such as microfractures and fissures, roughness on the surface of the grains and on the pore wall, added to the complex geometry of the clay minerals present in the samples.

It was observed that despite the good fit of the two phase maxima made with the Dias model (with mean errors of 0.042 for amplitude and 0.417 for phase), as theoretically predicted by this model, it did not obtain adequate fitting of the extensive and high phase threshold in the intermediate frequency region requiring the use of the Hybrid model. The Hybrid model provided an excellent fit of the phase curve in the intermediate frequency region with mean errors of 0.009 for amplitude and 0.145 for phase.

Table 1: Parameters of the Dias and Hybrid models, obtained by a trial and error data fitting procedure for samples 4A-SE, 4A1-SE, 4B-SE and 4B1-SE.

	Dias model original parameters
Amostra 4A-SE	$\rho_0 = 94.5\Omega m$ $m = 0.29; \tau = 1.6s$ $\delta = 0{,}68; \eta = 2.2s^{-1/2}$
	Hybrid model approximate parameters
	$\rho_0 = 94.5\Omega m; m = 0{,}35;$ $m_{w1} = 0.078\ \tau_{w1} = 0.613s;$ $m_D = 0.123; \tau_D = 6.5e^{-7}s$ $m_{w2} = 0.15; \tau_{w2} = 9.4e^{-4}s$; $c = 0.26$
	Dias model original parameters
Amostra 4A1-SE	$\rho_0 = 94.5\Omega m$ $m = 0.28; \tau = 1.6e^{-6}s$ $\delta = 0.68; \eta = 2.8s^{-1/2}$
	Hybrid model approximate parameters
	$\rho_0 = 94.5\Omega m; m = 0.323;$ $m_{w1} = 0.089; \tau_{w1} = 0.379s;$ $m_D = 0{,}134; \tau_D = 6.4e^{-7}s;$ $m_{w2} = 0.1; \tau_{w2} = 5.3e^{-4}s;$ $c = 0.33$
	Dias model original parameters
Amostra 4B-SE	$\rho_0 = 89\Omega m$ $m = 0.31; \tau = 1.65e^{-6}s$ $\delta = 0.73; \eta = 2.9s^{-1/2}$
	Hybrid model approximate parameters
	$\rho_0 = 89\Omega m; m = 0.34;$ $m_{w1} = 0.08; \tau_{w1} = 0.317s;$ $m_D = 0.139; \tau_D = 6.8e^{-7}s;$ $m_{w2} = 0.12; \tau_{w2} = 4e^{-4}s;$ $c = 0.28$
	Dias model original parameters
Amostra 4B1-SE	$\rho_0 = 90.5\Omega m$ $m = 0.31; \tau = 1.65e - 6s$ $\delta = 0.73; \eta = 2.8e^{-2}s^{-1/2}$
	Hybrid model approximate parameters
	$\rho_0 = 90.5\Omega m; m = 0.35;$ $m_{w1} = 0.08; \tau_{w1} = 0.341s;$ $m_D = 0.139; \tau_D = 6.8e^{-7}s$ $m_{w2} = 0.13; \tau_{w2} = 4e^{-4}s;$ $c = 0.28$

MCMC inversion with Dias model

The MCMC simulation with Dias model was made considering errors of 2% for the amplitude, and 2° for the phase data for 4 carbonate samples rocks of the Sergipe Sub-Basin. In the inversion process, the space of parameters was explored by fitting the number of walkers to explore the

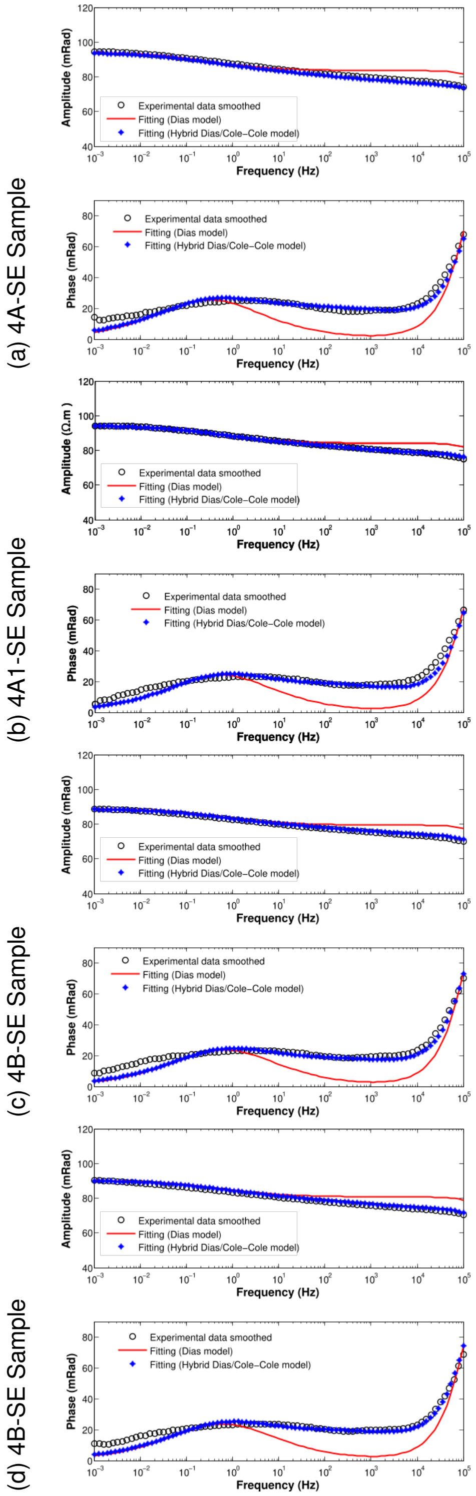


Figure 1: Experimental data and modeled amplitude and phase curves for samples (a) 4A1-SE, (b) 4A1-SE, (c) 4B-SE and (d) 4B1-SE.

parameter space and the number of steps to perform in the MCMC simulation. The amplitude and phase curves obtained from the model were not able to fit the data curves, implying that the quality of the inversion results was not satisfactory and the parameter model (ρ_0, m, τ, δ, η) were not correctly recovered for this type of samples as shown in Figures 4 to 7

MCMC inversion with the Hybrid model

In this work, the MCMC inversion with the Hybrid model was made by adaptation of the BISIP code. The first step was to include the partial fractions Dias model defined by Barreto and Dias (2014) to recover the model parameters (ρ_0, m, τ, δ, η, m_{W1}, τ_{W1}, m_D, τ_D, c), which were used to delimit the fit interval of the Hybrid model parameters and subsequently obtain the recovered parameters (ρ_0, m, m_{W1}, τ_{W1}, m_D, τ_D, m_{W1}, τ_{W2}, c) shown in Figures 2. The amplitude and phase curves of the electrical complex resistivity (Figures 3 to 7) have been constructed from experimental and modeled data using parameters of the Dias and Hybrid models recovered from the inversion, as presented in Table 2. The fit of the experimental data using the Hybrid model (green curves) was satisfactory, with some discrepancies in the high frequencies. Figure 3 compares the fit made by the traditional method and the MCMC inversion for the 4ASE sample. In this case, the parameters recovered from the inversion better fit the experimental data curve in the lower and intermediate frequency region.

Discussion

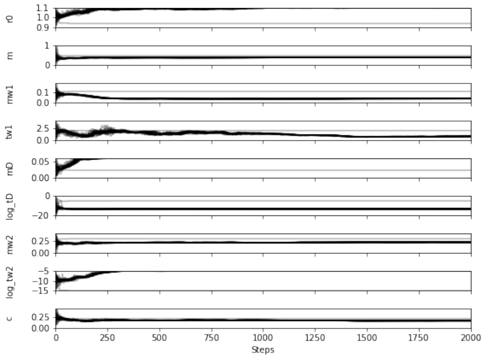
The traditional manual way of fitting the parameters of the Dias model currently used in the Petrophysics Laboratory consists of a curve fitting methodology by trial and error, aiming to preserve the uniqueness of the solution. This procedure is done in the modeling stage using the Dias and Hybrid models correlated to the curves of the experimental data, with some peculiarities arising from the fact that the Dias Model has five parameters, whereas the Hybrid Model has eight parameters. In the latter, there is an ambiguity in the solution, which is avoided by solving the two models in sequence.

In this sense, the implementation of Bayesian inversion using MCMC algorithms offers significant advantages compared to the fit by the traditional manual method, since the MCMC techniques use Markov chains that propose an iterative simulation scheme where each iteration of the algorithm depends only on the previous iteration that converges to a stationary distribution providing a good solution and estimation of the Hybrid model parameters.

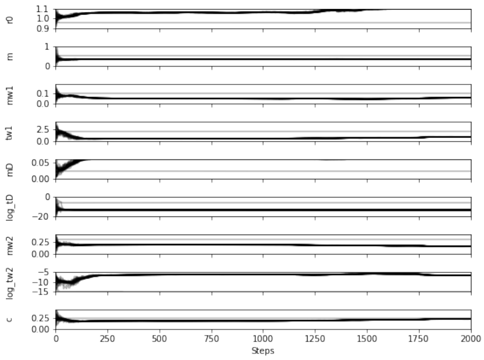
Conclusions

The spectral electrical resistivity measurements of the samples from the Sergipe sub-basin expressed through amplitude and phase curves were of good quality without the presence of significant noise. The Dias and Hybrid models satisfactorily fit the experimental data across the considered frequency spectrum using the traditional method by manual fit, showing mean errors with values of 0.412 for amplitude and 0.458 for phase (Dias model) and with values of 0.009 for the amplitude and 0.136 for the phase (Hybrid model). However, the model parameters are adjusted and estimated through user intervention, which can cause a weak correlation between the data set depending on the interpreter's decisions.

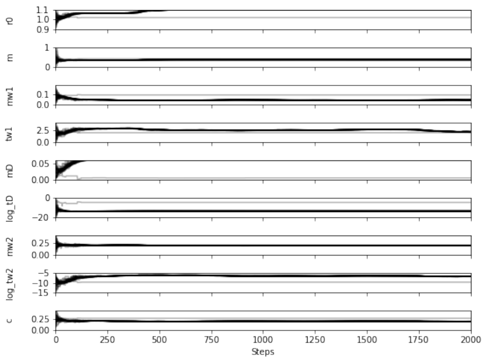
The Hybrid model parameters were estimated from MCMC inversion. The amplitude and phase curves constructed



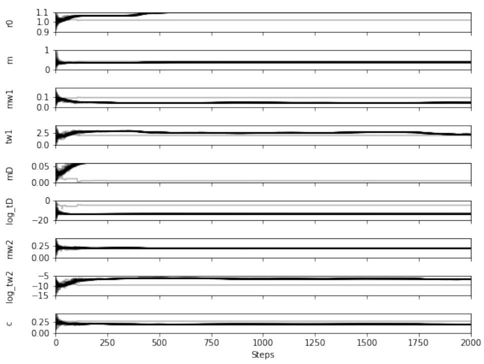
(a) Sample 4A-SE



(b) Sample 4A1-SE



(c) Sample 4B-SE



(d) Sample 4B1-SE

Figure 2: Curves of parameters recovered from the inversion to fit the Hybrid model of samples: (a) 4A-SE, (b) 4A1-SE, (c) 4B-SE e (d) 4B1-SE.

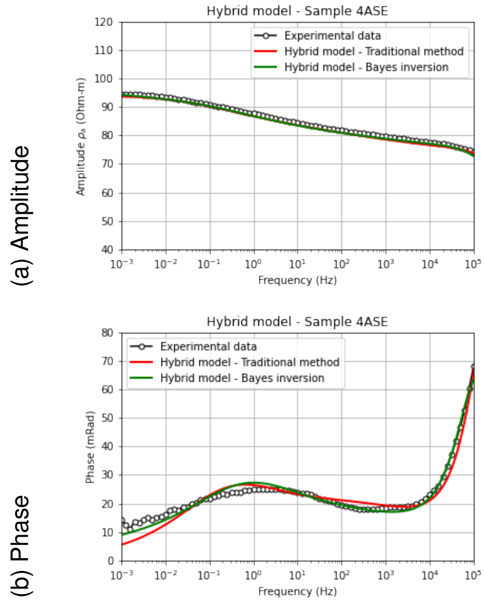


Figure 3: Curves of (a) amplitude and (b) phase of the spectral electrical resistivity using parameters estimated from the traditional method and parameters recovered from the inversion to fit the Hybrid model – Sample 4A-SE.

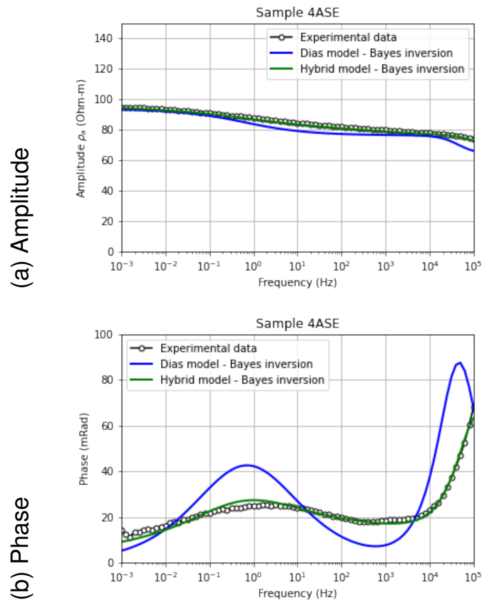


Figure 4: Curves of (a) amplitude and (b) phase of the spectral electrical resistivity using parameters recovered from the inversion to fit the original Dias model and the Hybrid model – Sample 4A-SE.

from the recovered parameters showed a satisfactory fit to the experimental data curves for over the entire frequency range presenting small discrepancies in the high frequencies possibly caused by the δ parameter fitting, which is directly related to the intensity of this last maximum of the spectrum. The ambiguity in determining the parameters of the Hybrid model is reduced by the Bayesian inference method based on a probabilistic mechanism that considers the a priori distribution of probabilities of the parameter conditional on the data obtained.

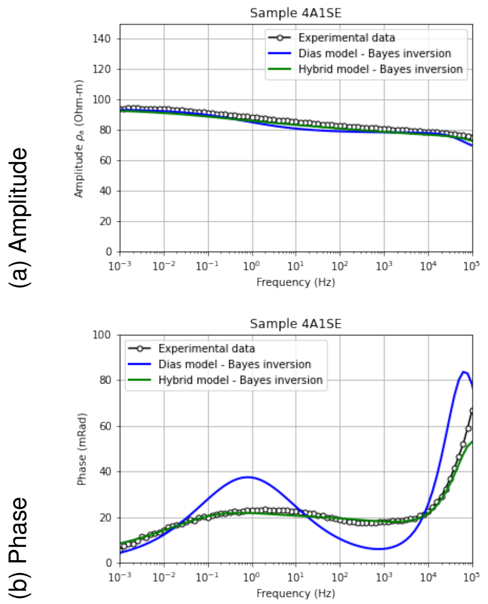


Figure 5: Curves of (a) amplitude and (b) phase of the spectral electrical resistivity using parameters recovered from the inversion to fit the original Dias model and the Hybrid model – Sample 4A1-SE.

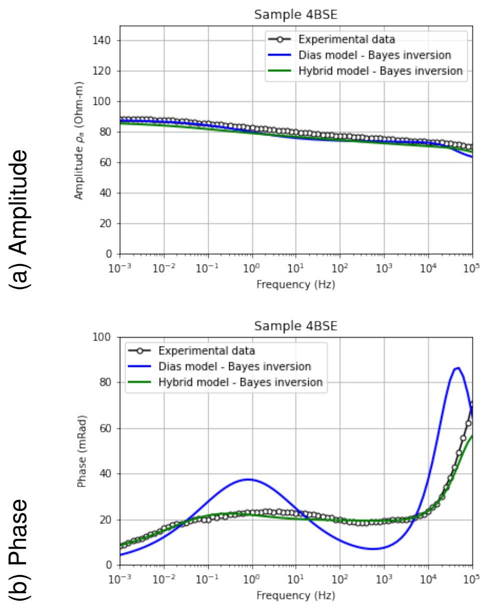


Figure 6: Curves of (a) amplitude and (b) phase of the spectral electrical resistivity using parameters recovered from the inversion to fit the original Dias model and the Hybrid model– Sample 4B-SE.

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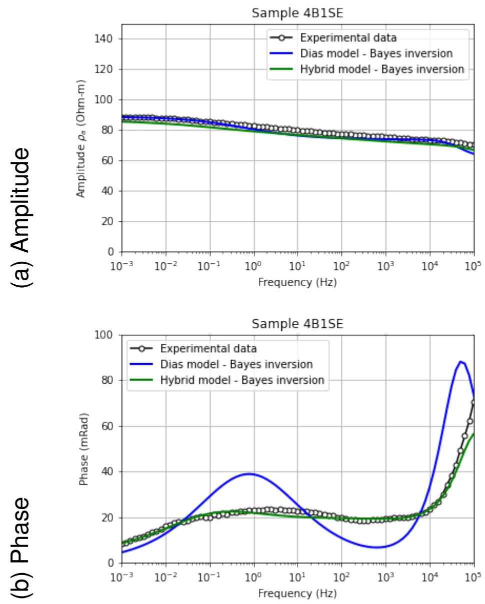

Figure 7: Curves of (a) amplitude and (b) phase of the spectral electrical resistivity using parameters recovered from the inversion to fit the original Dias model and the Hybrid model– Sample 4B1-SE.

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Table 2: Parameters of the Dias model and Hybrid model, obtained from the inversion procedure for samples 4A-SE, 4A1-SE, 4B-SE and 4B1-SE.

Amostra 4A-SE	**Dias model recovered parameters**
	$\rho_0 = 93.6182\Omega m$ $m = 0.316 \pm 0.101; log\tau = (-11.980 \pm 2.976)s$ $\delta = 0.507 \pm 0.144; \eta = (4.638 \pm 1.132)s^{-1/2}$
	Hybrid model Recovered parameters
	$\rho_0 = 103.970\Omega m; m = 0.383 \pm 0.012;$ $m_{w1} = 0.055 \pm 0.006; \tau_{w1} = (0.280 \pm 0.003)s;$ $m_D = 0.073 \pm 0.018; log\tau_D = (-13.536 \pm 0.700)s$ $m_{w2} = 0.193 \pm 0.002; log\tau_{w2} = (-5.042 \pm 0.212)s;$ $c = 0.165 \pm 0.005$
Amostra 4A1-SE	**Dias model recovered parameters**
	$\rho_0 = 93.521\Omega m$ $m = 0.294 \pm 0.077; log\tau = (-12.316 \pm 2.548)s$ $\delta = 0.524 \pm 0.131; \eta = (0.524 \pm 0.131)s^{-1/2}$
	Hybrid model Recovered parameters
	$\rho_0 = 101.95\Omega m; m = 0.356 \pm 0.031;$ $m_{w1} = 0.053 \pm 0.010; \tau_{w1} = (0.785 \pm 0.267)s;$ $m_D = 0.061 \pm 0.007; log\tau_D = (-13.205 \pm 1.277)s$ $m_{w2} = 0.177 \pm 0.024; log\tau_{w2} = (-6.616 \pm 1.525)s;$ $c = 0.213 \pm 0.016$
Amostra 4B-SE	**Dias model recovered parameters**
	$\rho_0 = 87.689\Omega m$ $m = 0.297 \pm 0.047; log\tau = (-11.910 \pm 2.656)s$ $\delta = 0.533 \pm 0.133; \eta = (4.693 \pm 1.133)s^{-1/2}$
	Hybrid model Recovered parameters
	$\rho_0 = 97.223\Omega m; m = 0.376 \pm 0.007;$ $m_{w1} = 0.047 \pm 0.009; \tau_{w1} = (2.560 \pm 0.171)s;$ $m_D = 0.060 \pm 0.010; log\tau_D = (-13.292 \pm 1.519)s$ $m_{w2} = 0.192 \pm 0.002; log\tau_{w2} = (-6.684 \pm 0.551)s;$ $c = 0.207 \pm 0.012$
Amostra 4B1-SE	**Dias model recovered parameters**
	$\rho_0 = 88.769\Omega m$ $\rho_0 = 88.769\Omega m$ $m = 0.305 \pm 0.063; log\tau = (-12.073 \pm 3.402)s$ $\delta = 0.530 \pm 0.141; \eta = (4.637 \pm 1.208)s^{-1/2}$
	Hybrid model Recovered parameters
	$\rho_0 = 98.895\Omega m; m = 0.372 \pm 0.008;$ $m_{w1} = 0.039 \pm 0.011; \tau_{w1} = (1.940 \pm 0.273)s;$ $m_D = 0.060 \pm 0.010; log\tau_D = (-13.189 \pm 1.552)s$ $m_{w2} = 0.194 \pm 0.004; log\tau_{w2} = (-5.364 \pm 0.793)s;$ $c = 0.211 \pm 0.012$