The impact of pressure in parameters estimation of dry rock moduli calibration for carbonate rocks

Antonio Romero da C. Pinheiro[*1,2,3], *Fernando S. Moraes*[2,3], *Matheus Freire S. B. Guimarães*[2], *Viatcheslav I. Priimenko*[2,3], *Sérgio Adriano M. Oliveira*[2,3], *Igor Barbosa de Oliveira*[2], *Marco A. R. de Ceia*[2,3]

[1] *GFISA/UFAC – Laboratório de Geofísica Aplicada, Universidade Federal do Acre*
[2] *GIR/LENEP/UENF – Grupo de Inferência de Reservatório, Universidade Estadual do Norte Fluminense*
[3] *INCT-GP/CNPq – Instituto Nacional de Ciência e Tecnologia – Geofísica do Petróleo*

Abstract

We study the effect of pressure variations on the dry rock moduli model parameters estimated using lab data from outcrop carbonate samples. The analysis lab measurements, including chemical-mineralogical composition, porosity, permeability and elastic properties, guide the methodology. First, we use velocity measurements on dry samples to estimate dry frame model parameters for five well-known models from the literature. These results are used to analyze parameter sensitivities to variation in differential pressure. We then discuss strategies to incorporate pressure variation in the model and the validity of using models developed for siliciclastic rocks to model carbonate rocks. Results indicate that is possible to link pressure variations to the parameters of each model when porosity variations are not available as input data.

Introduction

In carbonate rocks, seismic velocities are determined mainly by sedimentary lithology and sedimentary processes such as cementation and dissolution. Although these rocks do not vary much in composition because they are usually composed of a single mineral species, seismic velocities vary with changes in composition and mineralogical density, porosity, pore space rigidity, changes in rock pressure, and pore aspect ratio (Misaghi et al., 2010).

Due to the micro- and macro-scale heterogeneity of carbonate rocks, there is no clear relationship between the pore structure and rock's physical properties, making it difficult to accurately estimate the elastic parameters of the carbonate reservoir by rock physics modeling. Therefore, to properly explore the correlation between the various physical properties of rocks, it is essential to develop a practical rock physics model for carbonate.

This work is concerned with analyzing and interpreting elastic properties obtained by different theoretical methods considering sedimentary and petrophysical properties to obtain a prior rock framework. Generally, moduli are measured in the laboratory, predicted by theory, or inferred under certain assumptions.

Several models are described in the literature that aim to determine the bulk modulus of dry rock from mineralogy and porosity, such as. Krief et al. (1990), Nur et al. (1998), and Pride et al. (2004). The most commonly used models in rock physics are the elastic limits, which define the average between the lower and upper limits for incompressibility and shear modulus (Hashin and Shtrikman, 1963; Hill, 1963). The theory of Kuster and Toksöz (1974) is used to approximate the compressional and shear wave velocities by including small fractions within the elastic limits. The most commonly used model for infinitesimal inclusions is the effective differential medium theory (DEM) (Berryman, 1992). Finally, an approach was developed by Keys and Xu (2002) that simplifies the application of the theoretical model of Xu and White (1995) and makes the method computationally more efficient, and is also related to the critical porosity approach.

The velocity ratio v_p/v_s is a useful parameter for evaluating the model characteristics of the elastic moduli of dry rock. The model of Krief et al. (1990) has shown that v_p/v_s has a constant behavior for dry rock, independent of porosity and pressure. In contrast, for grain contact theories based on the Hertz-Mindlin model, the v_p/v_s ratio decreases with increasing differential pressure and is independent of porosity. The model of Kuster and Toksöz (1974) predicts that v_p/v_s increases with increasing porosity. More recently, the theory of Pride et al. (2004) predicts that the velocity ratio decreases with increasing differential pressure and decreasing porosity. Thus, this parameter is not independent of porosity and pressure.

Based on the behavior of the ratio v_p/v_s, the model of Krief et al. (1990) obtains the dry rock bulk and shear moduli, a simple function of the Biot coefficient, similar to the bulk and shear modulus predicted by Biot theory. Thus, the elastic moduli of dry rock as a function of porosity ϕ, bulk modulus κ_m, and shear modulus μ_m of the mineral are thus given by:

$$\kappa_{dry} = \kappa_m \left(1-\phi\right)^{\frac{\mathbf{m}_\kappa}{(1-\phi)}} \quad (1)$$

$$\mu_{dry} = \mu_m \left(1-\phi\right)^{\frac{\mathbf{m}_\mu}{(1-\phi)}} \quad (2)$$

Similarly, Nur et al. (1998) proposes a linear relationship between elastic modulus and porosity, specifying a critical porosity ϕ_c. The critical porosity separates the mechanical and acoustic behavior, i.e., the porosity between two regions, the suspension domain ($\phi > \phi_c$) and the load domain ($\phi < \phi_c$). Thus, the elastic moduli of dry rock are determined by the equations:

$$\kappa_{dry} = \kappa_m \left(1 - \frac{\phi}{\phi_c}\right) \quad (3)$$

$$\mu_{dry} = \mu_m \left(1 - \frac{\phi}{\phi_c}\right) \quad (4)$$

The bulk and shear moduli for dry rock are given by the model of (Pride et al., 2004) as a function of porosity with a consolidation parameter c. The consolidation parameter depends on the differential pressure and the degree of consolidation of the grains.

$$\kappa_{dry} = \frac{\kappa_m (1 - \phi)}{(1 + c_\kappa \phi)} \quad (5)$$

$$\mu_{dry} = \frac{\mu_m (1 - \phi)}{(1 + c_\mu \phi)} \quad (6)$$

A useful simplification of the elastic properties of dry rock is to consider Poisson's ratio as constant. Since the bulk and shear moduli of the material inclusion in the dry rock are zero, it is possible to obtain approximations to the model of Xu and White (1995) for dry rock. Keys and Xu (2002) develop an approximation for dry rock, approximating very closely the velocities obtained using DEM. Keys and Xu improved the computational efficiency of the model of Xu and White by solving linear ordinary differential equations with solutions given by:

$$\kappa_{dry} = \kappa_m (1 - \phi)^p \quad (7)$$

$$\mu_{dry} = \mu_m (1 - \phi)^q \quad (8)$$

where the coefficients p and q are functions associated with the aspect ratio α of the pore given by:

$$p(\alpha) = \frac{1}{3} \sum_{l=s,c} f_l T_{iijj}(\alpha_l) \quad (9)$$

$$q(\alpha) = \frac{1}{5} \sum_{l=s,c} f_l F(\alpha_l) \quad (10)$$

where f_s and f_c are the volume fractions of sand and clay of the rock matrix, respectively, which implies that the pore space has different aspect ratios that can be divided into soft pores with small α and stiff pores with larger α. $T_{iijj}(\alpha)$ and $F(\alpha)$ are the functions related to the pore aspect ratio. Here, we propose a method that varies the aspect ratio as a function of pressure variation.

A proposed empirical formulation for predicting the effect of pressure on velocities in dry rock comes from studies by Eberhart-Phillips et al. (1989). Vernik and Hamman (2009) treat the Eberhart-Phillips equation as a basic theoretical model and reorganize it differently to allow physical interpretation of the parameters. They found that there are strong correlations between certain fitting parameters. Vernik and Kachanov (2010) also suggest that the modeling parameters p and q are very similar given by the theory of Kuster and Toksöz (1974).

A second category of methods deals with the dependence of the moduli of dry rocks on the effective pressure. MacBeth (2004) follows the formulation of Sayers and Kachanov (1995) and formulates a sigmoid function for the bulk and shear moduli as a function of pressure P as:

$$\kappa_{dry} = \frac{\kappa_\infty}{1 + E_\kappa e^{-P/P_\kappa}} \quad (11)$$

$$\mu_{dry} = \frac{\mu_\infty}{1 + E_\mu e^{-P/P_\mu}} \quad (12)$$

Here κ_∞ and μ_∞ are the bulk and shear moduli, respectively, in the high pressure asymptotes. P_κ and P_μ are characteristic pressures associated with pore geometry. They describe the rate at which the pressure increases and the rock structure reaches a state of relative insensitivity. The constants E_κ and E_μ are related to the change in relative pressure. Further pressure function models can be found in Mavko et al. (2020). We have limited our evaluation only to the model of MacBeth (2004) because it provides a good fit of the measured values and a consistent physical basis.

Therefore, these interpretive models are used to estimate the sensitivity of dry rock to the effects of varying mineralogy, porosity, pore shape function, and pressure. They form a series of simple models with excellent results for siliciclastics that can be applied to carbonate rocks.

Method

The theoretical and experimental petrophysical studies of the elastic properties of carbonate rocks are developed here by velocity measurements (v_p and v_s) in six outcrops of carbonate rocks of the Maruim Member of the Riachuelo Formation (Sergipe Group) of the Sergipe Subbasin and six outcrops of carbonate rocks (coquinas) of the Morro do Chaves Formation (Coruripe Group), Alagoas Subbasin.

The measurements of the velocities of the P and S waves (S_1 and S_2) are carried out for the samples under dry conditions in the regime of different differential pressures ($5MPa$–$25MPa$), using experiments with the System of Physics and Rock Mechanics (ErgoTech) equipment located in the Laboratory of Physics and Rock Deformation of the Laboratory of Petroleum Engineering and Exploration – LENEP/UENF.

The petrophysical properties of the sample set can be found in Table 1, which also lists the mineral phases identified by the X-ray diffraction (XRD) method. In the samples of the Sergipe sub-basin, the carbonates show a mineralogical diversity composed mainly of calcite and dolomite and, in secondary form, of quartz and clay minerals, including illite. In the samples of the Alagoas sub-basin, coquina consists mainly of calcite.

The theoretical methodology is based on the calculation of the parameters of the presented interpretive models from the inversion of the functions 1 to 8 of the total porosity models and the estimation of the parameters from the minimized residual function between the observed data and those calculated for the function by the equations 11 and 12. The corresponding fitting parameters are given for the bulk modulus and shear modulus depending on the sensitivity factor, be it mineralogy, porosity, geometric aspect ratio function, or pressure. Since the seismic velocities were measured, the dry bulk and shear moduli were estimated from the velocities using the following equations:

Table 1: Basic petrophysical measurements performed on the plugs. ρ (density), ϕ (porosity in Ultrapore300), k (permeability in Coreval700), mineralogical composition by XRD, dominant pore type and predominant texture classification in thin section.

Sample	ϕ (%)	ρ (gm/cc)	k (mD)	Calcite (%)	Dolomite (%)	Quartz (%)	Illite (%)	Pyrite (%)	Pore type	Lithology texture
4A1-SE	18.23	2.81	16.23	0	92.74	6.72	0.54	0	intercrystalline	dolomite
4A4-SE	18.54	2.8	27.16	0	97.45	2.28	0.27	0	intercrystalline	dolomite
4A5-SE	19.14	2.8	25.11	0	95.71	0.79	0.5	0	intercrystalline	dolomite
4A6-SE	19.53	2.8	21.65	0	92.41	4.73	0.07	0	intercrystalline	dolomite
4B-SE	18.93	2.81	16.79	1.88	80.25	14.41	3.45	0	intercrystalline	dolomite
4B6-SE	19.87	2.75	35.65	2.45	83.57	4.48	9.49	0	intercrystalline	dolomite
7B1-AL	12.37	2.69	42.6	96.57	0	3.42	0	0.01	moldic	grainstone
7C-AL	18.77	2.68	635.17	71.77	0	18.28	0	9.72	vug	rudston
9E-AL	21.01	2.67	229.83	88.9	0	11.06	0	0.04	intercrystalline	grainstone
10A-AL	19.7	2.68	140.1	82.97	0	13.76	0	0.07	intercrystalline	packstone
10B-AL	15.15	2.68	35.1	87.59	0	10.56	0	0.01	vug	rudstone
10D-AL	23.93	2.64	29.24	81.45	0	13.28	0	0.17	intercrystalline	packstone

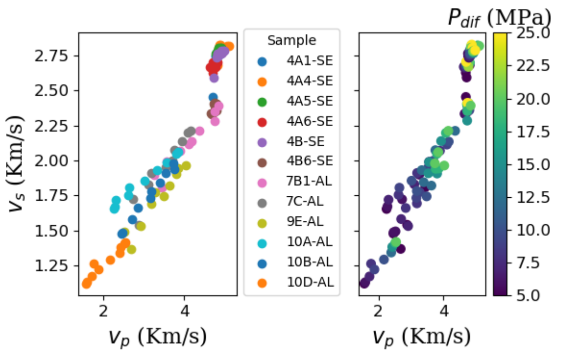

Figure 1: (Left) Plot v_s versus v_p. This sample color pattern is used in all analyzes. (Right) Plot v_s versus v_p of the dry samples in relation to pressure sensitivity. The Alagoas samples were subjected to pressures up to 20 MPa due to their high degree of disaggregation.

$$\kappa_{dry} = \rho_b \left(v_p^2 - \frac{4}{3} v_s^2 \right) \quad (13)$$

$$\mu_{dry} = \rho_b v_s^2 \quad (14)$$

and assumed a constant density value with a pressure variation.

Results

The measured velocities v_p and v_s of all selected samples were performed under dry conditions using the rock physics system presented in the previous section. The left plot of Figure 1 shows the v_s versus v_p of the samples, and in each pressure regime the plot on the right.

From the v_p and v_s data, the dynamic bulk and shear moduli of the samples are estimated, checking the sensitivity to different pressures, mineral moduli, total porosity, and functional geometries in terms of pore aspect ratio according to the selected rock physics model. With the exception of the MacBeth (2004) model, the mineral modulus is the limiting parameter of rock stiffness. The theoretical estimate of the matrix modulus is obtained from the average

Table 2: Elastic properties of the minerals contained in the composition of the selected samples (Prasad et al., 2002; Wang et al., 1998; Katahara, 1996; Mavko et al., 2020).

Mineral	κ_m (GPa)	μ_m (GPa)	ρ (g/cc)
Quartz	37,0	44,0	2,65
Illite	60,1	25,3	2,71
Calcite	77,0	32,0	2,71
Dolomite	95,0	45,0	2,87
Pyrite	147	132	4,93

of Voigt-Reuss-Hill (Hill, 1963), using the values of the table 2 from the literature (Prasad et al., 2002; Wang et al., 1998; Katahara, 1996; Mavko et al., 2020).

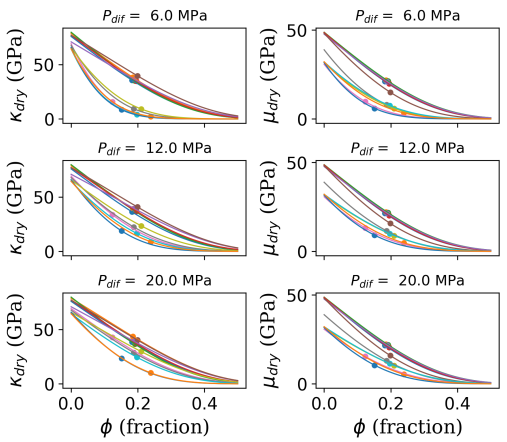

Figure 2: Modulus of elasticity measured (points) and calculated (curves) with the model by Krief et al. (1990). The samples from Alagoas are friable and could not resist high pressure. For this reason, we made a cut of 20 MPa in the analysis of elastic moduli.

Figures 2, 3, 4, 5, and 7 show the elastic moduli calculated according to the theoretical model compared to the measured values. The behavior of the parameters was obtained by inverted functions $\mathfrak{m}(\kappa)$ and $\mathfrak{m}(\mu)$, $\phi_c(\kappa)$ and $\phi_c(\mu)$, $c(\kappa)$ and $c(\mu)$, $p(\kappa)$ and $q(\mu)$ as a function of pressure, as shown in Figure 6, except for the model of MacBeth (2004), which is a pressure model. Ideally, the effect of pressure on porosity should be observed as in Silva et al. (2019), but since this experimental measurement is

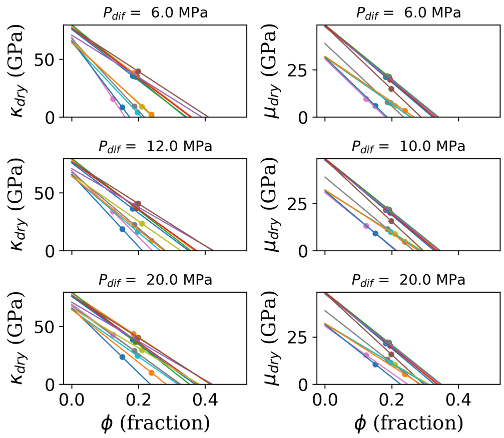

Figure 3: Modulus of elasticity measured (points) and calculated (curves) with the model of Nur et al. (1998)

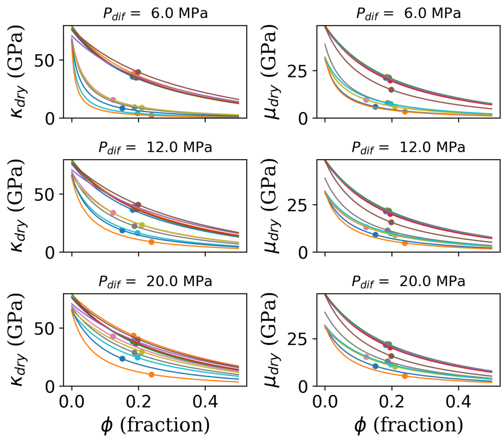

Figure 4: Modulus of elasticity measured (points) and calculated (curves) with the model of Pride et al. (2004)

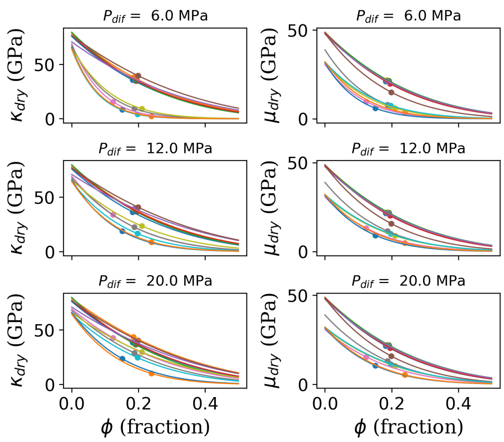

Figure 5: Modulus of elasticity measured (points) and calculated (curves) with the model of Keys and Xu (2002)

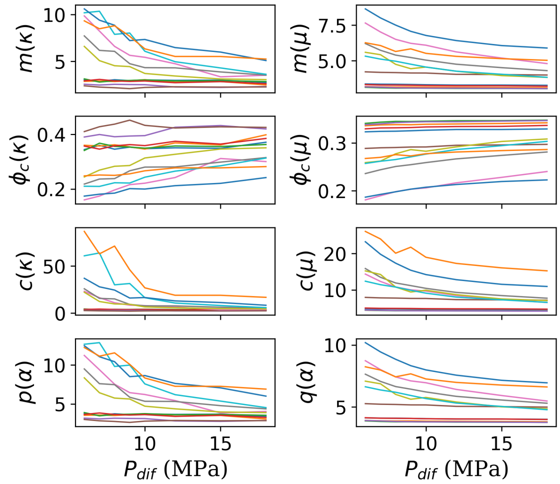

Figure 6: Effect of pressure on the estimation of the calibration parameters of the dry rock moduli for the selected samples. The colors of the curves represent the identification of each sample, as shown in Figure 1.

not available, the effect of pressure on the parameters was analyzed. It can be assumed that the effect of pressure on density is also small. The statistics of these calculated parameters are available in Table 3 (left), which were estimated by inversion to fit the models Krief et al. (1990); Nur et al. (1998); Pride et al. (2004); Keys and Xu (2002). Table 3 (right) shows the parameters of the model MacBeth (2004) estimated by the nonlinear least squares method with fitting to the observed data.

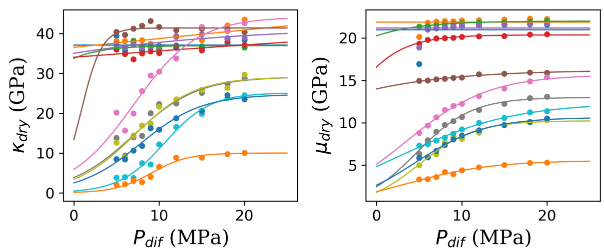

Figure 7: Modulus of elasticity measured (points) and calculated (curves) with the model of MacBeth (2004)

Discussions

Figure 1 shows that the compression and shear velocities of the samples from Sergipe range from 4.634 to 5.112 km/s and from 2.332 to 2.822 km/s, respectively. The Alagoas samples range from 1.570 to 4.159 km/s and from 1.121 to 2.212 km/s for v_p and v_s, respectively. The samples from Sergipe are mainly dolomites with a relatively consolidated structure, showing higher velocities and lower dispersion. In contrast, the samples from Alagoas have lower velocities, which could be related to the high calcite content and a strong degradation process, and they show a greater compliance to the pressure increase.

Table 3: (Left table) Statistics with mean and standard deviation of the calculated parameters of the respective dry rock models of total porosity ranging from 6 MPa to 20 MPa. (Right table) Estimated parameters for the model of MacBeth (2004)

Model	(Krief et al., 1990)		Nur et al. (1998)		Pride et al. (2004)		Keys and Xu (2002)	
Sample	$\overline{\mathfrak{m}_\kappa}$	$\sigma_{\mathfrak{m}_\kappa}$	$\overline{\phi_c}$	σ_{ϕ_c}	$\overline{c_\kappa}$	σ_{c_κ}	$\overline{p(\alpha)}$	$\sigma_{p(\alpha)}$
4A1-SE	2.96	0.099	0.35	0.008	3.81	0.223	3.62	0.121
4A4-SE	2.84	0.154	0.36	0.015	3.60	0.339	3.49	0.188
4A5-SE	2.95	0.096	0.36	0.008	3.94	0.232	3.64	0.118
4A6-SE	2.85	0.127	0.36	0.011	3.79	0.312	3.55	0.158
4B-SE	2.43	0.129	0.41	0.016	2.76	0.267	3.00	0.160
4B6-SE	2.25	0.081	0.43	0.011	2.48	0.168	2.81	0.101
7B1-AL	5.93	2.100	0.23	0.051	10.15	6.242	6.76	2.397
7C-AL	5.10	1.334	0.27	0.031	11.66	6.467	6.28	1.642
9E-AL	4.25	1.116	0.30	0.036	9.45	5.461	5.38	1.413
10A-AL	6.93	2.420	0.25	0.036	28.44	21.50	8.63	3.013
10B-AL	7.62	1.738	0.20	0.021	19.36	9.059	8.98	2.049
10D-AL	7.12	1.544	0.26	0.013	43.59	25.53	9.36	2.030
Sample	$\overline{\mathfrak{m}_\mu}$	$\sigma_{\mathfrak{m}_\mu}$	$\overline{\phi_c}$	σ_{ϕ_c}	$\overline{c_\mu}$	σ_{c_μ}	$\overline{q(\alpha)}$	$\sigma_{q(\alpha)}$
4A1-SE	3.33	0.032	0.33	0.002	4.69	0.080	4.07	0.039
4A4-SE	3.15	0.025	0.34	0.002	4.31	0.062	3.86	0.031
4A5-SE	3.08	0.027	0.34	0.002	4.27	0.068	3.81	0.034
4A6-SE	3.28	0.055	0.33	0.003	4.88	0.090	4.08	0.069
4B-SE	3.10	0.038	0.34	0.003	4.27	0.093	3.82	0.046
4B6-SE	4.12	0.076	0.29	0.003	7.56	0.263	5.14	0.095
7B1-AL	6.14	0.876	0.21	0.018	9.94	2.403	7.01	1.000
7C-AL	5.15	0.592	0.26	0.014	11.04	2.533	6.34	0.729
9E-AL	4.61	0.594	0.28	0.016	10.36	2.782	5.83	0.753
10A-AL	4.61	0.493	0.28	0.015	9.43	1.929	5.74	0.613
10B-AL	7.05	0.896	0.21	0.012	15.67	3.987	8.31	1.056
10D-AL	5.61	0.409	0.28	0.006	19.94	3.584	7.37	0.538

Model	(MacBeth, 2004)		
Sample	P_κ	κ_∞	E_κ
4A1-SE	0.03	37.08	0.02
4A4-SE	13.32	43.52	0.26
4A5-SE	3.22	36.97	0.09
4A6-SE	10.98	37.96	0.16
4B-SE	8.64	39.83	0.16
4B6-SE	1.20	41.40	2.08
7B1-AL	3.41	42.68	7.23
7C-AL	3.63	28.82	6.80
9E-AL	3.53	29.10	7.48
10A-AL	3.27	24.56	24.56
10B-AL	3.53	24.57	8.99
10D-AL	3.83	10.05	10.05
Sample	P_μ	μ_∞	E_μ
4A1-SE	1.09	21.64	21.64
4A4-SE	0.91	22.17	22.30
4A5-SE	4.77	21.99	0.09
4A6-SE	3.18	20.36	0.23
4B-SE	0.98	21.58	21.73
4B6-SE	5.64	15.93	0.18
7B1-AL	4.48	15.39	2.20
7C-AL	3.25	13.00	4.29
9E-AL	3.25	10.24	4.69
10A-AL	4.79	11.37	1.80
10B-AL	3.94	10.51	3.16
10D-AL	4.19	5.32	2.24

Figure 2 shows that the samples from Sergipe have lower $\mathfrak{m}_\kappa$ and $\mathfrak{m}_\mu$ Krief coefficients than those from Alagoas. The $\mathfrak{m}_\kappa$ values for Sergipe range from 2.30 to 3.13, while those for Alagoas range from 3.20 to 10.60. The $\mathfrak{m}_\mu$ coefficients range from 3.03 to 4.22 for the samples from Sergipe. The coefficients for the samples from Alagoas range from 3.76 to 8.64.

According to the plots of the calculated parameters, ϕ_c of the model Nur et al. (1998), figure 6, the values of the critical porosity tend to a constant behavior, which is an expected result, but the theoretical-experimental method, which analyzes the performed measurements and calculations made, establishes uncertainty on the process.

According to the calculations of the model of Pride et al. (2004), the consolidation parameter, c_κ and c_μ, behaves inversely proportional to the increase in pressure. The values range from 2.40 to 4.40, while for Alagoas the values range from 3.40 to 86.78. The high pressure sensitivity is due to the high dispersion and low elastic modulus values.

The calculation results of the model Keys and Xu (2002), both of the 5 and 6 Figures, are similar to those of the model Krief et al. (1990), because the calculation was of the geometric function, p and q, rather than the pore aspect ratio. The inversion of the model for aspect ratio is very complex because the expressions of T_{iijj} and F are mathematically very large. In this case, the nonlinear regression of the parameters is indicated by the need for porosity data that vary with pressure.

In the case of the pressure model, it was possible to use it in the characterization of the carbonate rock in a more complete modeling to describe the effect of the effective pressure of the samples using the parameter estimates. The results show two groups of elastic responses where dolomites are less sensitive to pressure variations than coquinas.

More realistic modeling considers the effects of key petrophysical and mechanical parameters. Lee (2005) proposes a model with a pressure effect on the rock framework by coupling the model of Pride et al. (2004) with the model of MacBeth (2004), which is based on the constant Poisson's ratio and is limited to integration only, without considering the behavior and physical significance of the parameters, and does not consider the pore aspect ratio. The Mur and Vernik (2020) model also focuses on analyzing the effect of mineralogy, porosity, pore shapes, and effective stresses on elastic properties, but is based on rock complacency, i.e., it places pores and cracks at the average effective stress in the solid matrix of the material. This model sets fixed and non-arbitrary aspect ratios. Another proposal to consider porosity and mineralogy along with compressive sensitivity comes from Grana (2016), who modifies the model of MacBeth (2004), but unfortunately, the dependence on mineralogy is not explicit, nor is pore geometry considered.

Conclusions

The estimated parameters of the dry rock moduli models tested, using lab measurements at differents pressures conditions, show these models to be appropriate for the selected carbonate rocks used in this study. However, more complete data, such as the response of porosity to pressure changes, are required for a more comprehensive analysis in addition to knowledge of the complex pore structure of carbonates from image analysis.

All models confirm a high discrepancy between textures, both in the total porosity models and in the pressure model; that is, there is a separation of dolomite and coquina groups in the outcrops rock of the Sergipe-Alagoas Basin. Among the models, the critical porosity model shows greater stability with respect to pressure variations, since it has a parameter with a lower standard deviation. Dolomites are less sensitive to pressure than coquina. This is seen from the constant behavior of the parameters associated with this sample group and the lower variability of the calculated dry rock moduli on all models. This is contrasted with MacBeth pressure model, which shows the asymptote curves for dolimite. The calculation results of the Keys and Xu model are similar to those of the Krief model due to the inverse function method used. Regardless of any consideration about parameter significance, a models can adequately describe the dry moduli in carbonates for approximately constant pressure conditions.

Therefore, this summary represents a first overview of a more comprehensive theoretical-experimental study dry rock moduli modelign for pressure and fluid sensitivities. The modeling used here begins with simpler models and statistical features, the best possible strategy to model elastic carbonate responses.

Acknowledgments

We thank PETROBRAS for the financial support of this research. The authors also acknowledge the technical and financial support of the National Institute of Science and Technology: Petroleum Geophysics (INCT–GP).

References

Berryman, J. G., 1992, Exact effective-stress rules in rock mechanics: Physical Review A, **46**, no. 6, 3307–3311.

Eberhart-Phillips, D., D.-H. Han, and M. D. Zoback, 1989, Empirical relationships among seismic velocity, effective pressure, porosity, and clay content in sandstone: **54**, 82–89.

Grana, D., 2016, Pressure-âvelocity relations in reservoir rocks: Modified macbeth's equation: Journal of Applied Geophysics, **132**, 234–241.

Hashin, Z., and S. Shtrikman, 1963, A variational approach to the theory of the elastic behaviour of multiphase materials: Journal of the Mechanics and Physics of Solids, **11**, no. 2, 127–140.

Hill, R., 1963, Elastic properties of reinforced solids: Some theoretical principles: Journal of the Mechanics and Physics of Solids, **11**, no. 5, 357–372.

Katahara, K. W., 1996, Clay mineral elastic properties, *in* SEG technical program expanded abstracts 1996: Society of Exploration Geophysicists, 1691–1694.

Keys, R. G., and S. Xu, 2002, An approximation for the xu-white velocity model: Geophysics, **67**, no. 5, 1406–1414.

Krief, M., J. Garat, J. Stellingwerff, and J. Ventre, 1990, A petrophysical interpretation using the velocities of p and s waves (full-waveform sonic): **31**.

Kuster, G. T., and M. N. Toksöz, 1974, Velocity and attenuation of seismic waves in two-phase media: Part i. theoretical formulations: **39**, 587–606.

Lee, M. W., 2005, Proposed moduli of dry rock and their application to predicting elastic velocities of sandstones: US Department of the Interior, US Geological Survey.

MacBeth, C., 2004, A classification for the pressure-sensitivity properties of a sandstone rock framesandstone pressure sensitivity: Geophysics, **69**, no. 2, 497–510.

Mavko, G., T. Mukerji, and J. Dvorkin, 2020, The rock physics handbook: Cambridge University Press.

Misaghi, A., S. Negahban, M. Landrø, and A. Javaherian, 2010, A comparison of rock physics models for fluid substitution in carbonate rocks: Exploration Geophysics, **41**, no. 2, 146–154.

Mur, A., and L. Vernik, 2020, Rock physics modeling of carbonates: Society of Exploration Geophysicists. (doi:10.1190/segam2020-3427703.1).

Nur, A., G. Mavko, J. Dvorkin, and D. Galmudi, 1998, Critical porosity: A key to relating physical properties to porosity in rocks: **17**, 357–362.

Prasad, M., M. Kopycinska, U. Rabe, and W. Arnold, 2002, Measurement of young's modulus of clay minerals using atomic force acoustic microscopy: Geophysical research letters, **29**, 13–1.

Pride, S. R., J. G. Berryman, and J. M. Harris, 2004, Seismic attenuation due to wave-induced flow: Journal of Geophysical Research: Solid Earth, **109**, no. B1.

Sayers, C. M., and M. Kachanov, 1995, Microcrack-induced elastic wave anisotropy of brittle rocks: **100**, 4149–4156.

Silva, R. A., M. Ceia, R. Missagia, and I. A. Lima Neto, 2019, Analysis of elastic velocities behavior and porosity in carbonates submitted to external pressure variation: Journal of Applied Geophysics, **166**, 10–21.

Smith, T. M., C. M. Sayers, and C. H. Sondergeld, 2009, Rock properties in low-porosity/low-permeability sandstones: **28**, 48–59.

Vernik, L., and J. Hamman, 2009, Stress sensitivity of sandstones and 4d applications: **28**, 90–93.

Vernik, L., and M. Kachanov, 2010, Modeling elastic properties of siliciclastic rocks: **75**, E171–E182.

Wang, Z., H. Wang, and M. E. Cates, 1998, Elastic properties of solid clays, *in* SEG Technical Program Expanded Abstracts 1998: Society of Exploration Geophysicists, 1045–1048.

Xu, S., and R. E. White, 1995, A physical model for shear-wave velocity prediction: Geophysical Prospecting, **44**, no. 4, 687–717.