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Multiscale UFNO-FWI: A Hybrid Framework for Enhancing Seismic Inversion

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Abstract

We present a hybrid multiscale approach for full waveform inversion (FWI) that strategically integrates UNet Fourier Neural Operators (UFNO) with gradient-based optimization. In the initial frequency band, UFNO approximates the inverse of the Hessian matrix during early iterations, then shifts to conventional updates for later iterations. For other frequency bands, model updates are computed based solely on the gradient using the adjoint method. We conducted experiments using the Marmousi model to compare our hybrid UFNO-based approach with two others: a conventional multiscale FWI approach (without neural networks) and a neural network-enhanced approach where model updates for each frequency band incorporate a neural network-computed inverse Hessian applied to the gradient. Our results show that the hybrid UFNO-based approach achieves inversion accuracy competitive with the neural network-enhanced method while requiring less computation time.

Introduction

FWI is a powerful technique for estimating high-resolution subsurface velocity models, with success hinging on avoiding cycle skipping (Pratt et al., 1998; Tarantola, 1984). To mitigate cycle-skipping, FWI is usually performed in a multiscale approach, processing data in frequency bands from lowest to highest. When low-frequency data are available, this strategy effectively prevents cycle-skipping. Model updates in this approach tipically rely on the loss function's gradient to determine the update direction, neglecting the inverse of the Hessian's contribution. However, numerous studies highlight the Hessian's critical role in enhancing FWI's convergence rate.

Our work draws on ideas from Alfarhan et al. (2024) on Hessian approximation in FWI, proposing a hybrid multiscale framework combining UFNO (Li et al., 2021) with conventional gradient-based optimization. For low-frequency bands, UFNO approximates the inverse of the Hessian in early iterations to update the model, followed by gradient-based updates in later iterations. For high-frequency bands, model updates rely solely on gradients. By exploiting the inverse Hessian matrix at lower frequencies, the multiscale process generates an improved initial low-wavenumber velocity model, which is gradually refined with high-wavenumber components in subsequent bands.

Method and Theory

UFNO

The UFNO combines the hierarchical spatial feature extraction capabilities of UNet architectures with the spectral processing power of Fourier Neural Operators (FNO). This combination enables efficient learning of mappings between function spaces while capturing both local and global patterns in seismic data.

For a given function space mapping, $G : A(D) \to U(D)$, where A and U are Banach spaces of functions defined on domain D, the UFNO learns this mapping through a combination of spatial and spectral transformations. The core operation in FNO consists of:

$$h^{l+1} = \sigma\left(F^{-1}\left(R_{\theta,l}(F(h^l))\right) + W_l h^l\right), \tag{1}$$

where h^l represents the feature map at layer l, F and F^{-1} denote the Fourier transform and its inverse, $R_{\theta,l}$ represents learnable spectral convolution parameters, W_l are learnable weights for the spatial branch, and σ is a nonlinear activation function.

Born modelling and inverse Hessian approximation via deep learning

We consider the constant-density acoustic wave equation, given by:

$$m^{-2}(\mathbf{x})\partial_{tt}u(\mathbf{x},t) - \nabla^2 u(\mathbf{x},t) = f(\mathbf{x},t), \tag{2}$$

where $\mathbf{x} = (x, z)$, $m(\mathbf{x})$ is the medium velocity, $u(\mathbf{x},t)$ denotes the pressure wavefield, and $f(\mathbf{x},t)$ is the source term.

In Born modeling, the model $m(\mathbf{x})$ is split into a background velocity $m_0(\mathbf{x})$ and a perturbation $\delta m(\mathbf{x})$ with $m(\mathbf{x}) = m_0(\mathbf{x}) + \delta m(\mathbf{x})$. When using the Born approximation, the linear Born modeling can be expressed as $\delta u = L\delta m$, where $\delta m(\mathbf{x}) = \delta m(\mathbf{x})/m_0(\mathbf{x})$, and L is the Born modeling operator. By using a sampling operator P to extract the scattered wavefield at receiver positions and produce data, we have $d = PL\delta m = \tilde{L}\delta m$. Born modelling and its adjoint are used to approximate the inverse Hessian operator. The gradient of FWI can be summarized as $g = \tilde{L}^T\Delta d$, where $\tilde{L}^T$ is the adjoint of the Born modeling operator, and Δd is the data residual. Considering the true perturbation δm, $\Delta d = \tilde{L}\delta m$, the gradient can be rewritten as $g = \tilde{L}^T\tilde{L}\delta m$, which suggest that g and δm are linked via demigration/migration operations. A blurred gradient can be obtained by:

$$\delta m_1 = \tilde{L}^T\tilde{L}g. \tag{3}$$

Given availability of both g and δm_1 an approximation of the inverse of the Hessian $(\tilde{L}^T\tilde{L})^{-1}$ can be attained. In this work, we followed Alfarhan et al. (2024) ideias and use UFNO to learn the mapping from δm_1 to g, that is, the action of the inverse Hessian operator.

Multiscale approach

To validate the accuracy and feasibility of our proposed hybrid UFNO-based approach, we conducted a multiscale FWI experiment using the Marmousi model. We selected three frequency bands with dominant frequencies of $2\,\mathrm{Hz}$, $4\,\mathrm{Hz}$, and $10\,\mathrm{Hz}$, performing 20 iterations per band. Ricker wavelets with these dominant frequencies served as source signals to generate observed seismic data for each band. We implemented FWI using Deepwave (Richardson, 2023) for forward and adjoint wave propagation and the Barzilai-Borwein (BB) method to compute step sizes. In the initial iterations of the lowest frequency band (2 Hz), we applied Born modeling and its adjoint to the FWI gradient to obtain the perturbation δm_1. The UFNO network was then trained to map δm_1 to the gradient, producing the inverse of the Hessian operator which is used to update the model when applied to the FWI gradient. After several iterations, updates shifted to conventional gradient-based methods with *short* BB step sizes. The UFNO architecture featured 20 Fourier modes in both dimensions and a width of 64, and was trained using the L_2 norm loss function ($\mathcal{L} = \|g_{\text{pred}} - g_{\text{true}}\|_2^2$) with the AdamW optimizer. For higher frequency bands (4 Hz and 10 Hz), we used only gradient-based updates with *short* BB step sizes. We compared our approach to a conventional multiscale FWI and a neural network-enhanced approach that applies UFNO-based/UNet-based Hessian correction in every iteration across all frequency bands, retraining the UFNO/UNet network for each new band.

Results

Figure 1 shows the true Marmousi velocity model and its smoothed version, which was used as starting model for experiments. Table 1 presents a comprehensive comparison of different inversion strategies in terms of computational time and final loss value.

Table 1: Performance comparison of FWI approaches. The test setup used one NVIDIA Tesla V100 GPU.

Method	Time (min)	Final Loss
Conventional multi-scale FWI	9:48	1.59×10^{-15}
UFNO hybrid (first band + conv.)	8:21	8.54×10^{-14}
UNet hybrid (first band + conv.)	14:04	1.20×10^{-14}
First band with UFNO	10:10	9.39×10^{-14}
First band with UNet	19:08	4.16×10^{-14}
UFNO multi-scale	19:45	9.39×10^{-14}
UNet multi-scale	45:00	1.39×10^{-13}

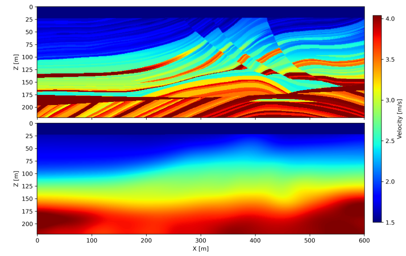

Figure 1: Marmousi velocity model and its corresponding smoothed version used in all FWI experiments.

As shown in Table 1, the conventional multi-scale FWI achieves the best final loss value, demonstrating its effectiveness in finding optimal solutions. Our proposed hybrid UFNO approach, using UFNO for the first 12 iterations of the initial frequency band (e.g., $2\,\mathrm{Hz}$), then gradient-based updates for the remaining 8 iterations and higher frequency bands, yields a similar loss value with reduced computational time. In contrast, using UNet instead of UFNO in the hybrid approach increases computation time while maintaining a comparable loss value. Additionally, Table 1 presents results for two other approaches: one applying UNet/UFNO-based Hessian-corrected updates for all iterations across all bands, and another using UNet/UFNO-based Hessian-corrected updates only in the first band, followed by gradient-based updates in subsequent frequency bands.

Our experiments revealed that applying UNet/UFNO-based Hessian-corrected updates at each iteration for all bands yielded suboptimal results, with low loss values but no model improvement, notably in the $4\,\mathrm{Hz}$ and $10\,\mathrm{Hz}$ bands. Moreover, computational times were significantly longer, especially for UNet (see last row in Table 1). This observation motivated our hybrid strategy, which limits neural network use on the initial frequency band for maximum benefit. Figure 2, shows the results of conventional multiscale FWI (top row), and our hybrid approach using UNet (middle row) and UFNO (bottom row) in the first band. Although the hybrid approach yields a slightly higher final loss value than conventional FWI, visual inspection of the inverted models (Figure 2) indicates comparable quality, though UNet introduces some stripe-like noise.

Conclusions

We propose a hybrid multi-scale FWI method that use UFNO to approximate the inverse of the Hessian, enhancing inversion robustness. The hybrid approach combines Hessian-corrected updates (applying UFNO to FWI gradients) with gradient-only updates in all or just the first low-frequency band. Applied only in the initial band, it improves low-wavenumber components, while conventional FWI refines high-wavenumber components in higher frequency bands. Experiments show this achieves equivalent model quality in less time than neural network-enhanced methods using UNet/UFNO-based Hessian-corrected updates for all bands. Furthermore, additional research will be required to gain a deeper comprehension of these networks for seismic inverse issues.

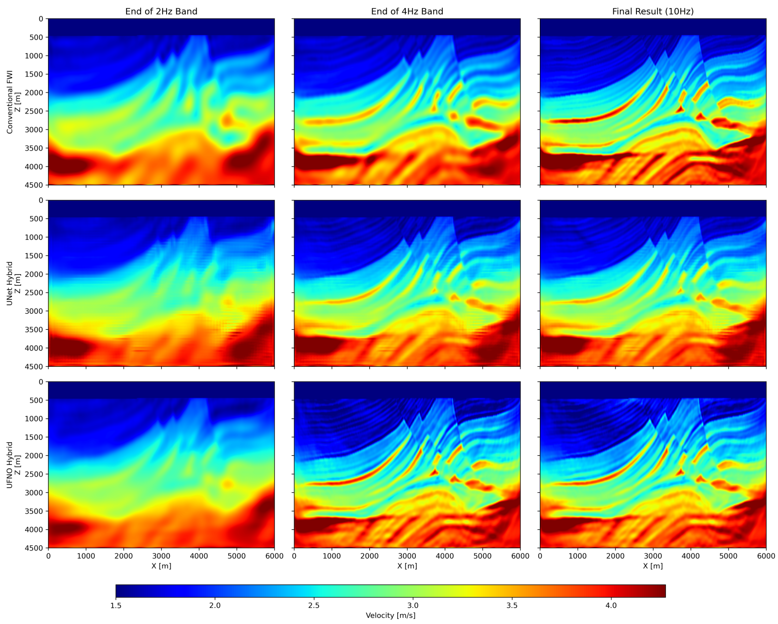

Figure 2: Comparison of velocity models obtained through different FWI approaches across frequency bands ($2\,\mathrm{Hz}$, $4\,\mathrm{Hz}$, $10\,\mathrm{Hz}$).

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