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Robust Least-Squares Reverse Time Migration Using a Correntropy-Based Objective Function

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Abstract Summary

We propose a robust new approach to least-squares reverse time migration (LSRTM) based on non-Gaussian criterion that relaxes the traditional Gaussian assumptions of conventional LSRTM. In particular, we employ correntropy — a similarity measure that uses higher-order statistics to suppress the influence of outliers such as erratic seismic data or physically inconsistent forward modeling, as well as non-Gaussian noise. We derive a correntropy-based objective function and its gradient using the adjoint-state method. To validate our approach, we perform numerical experiments using an ocean-bottom node (OBN) acquisition geometry and evaluate two different noisy data scenarios: one with Gaussian noise and another with non-Gaussian noise. The results show that the proposed method provides better seismic images compared to conventional LSRTM without incurring additional computational costs.

Introduction

Least-squares reverse time migration (LSRTM) is a powerful method that aims to reconstruct reflectivity models by iteratively minimizing the difference between modeled and observed seismic data (residuals). Despite its superior resolution compared to conventional migration (Dai et al., 2011), LSRTM remains sensitive to noise, inaccurate velocity models, and the limitations of the Born approximation (Wu et al., 2024).

Conventional LSRTM is based on the least-squares objective function and assumes Gaussian statistics for the residuals (Tarantola, 2005). However, real seismic data often exhibit non-Gaussian properties, e.g., due to systematic noise and errors in forward modeling (Elboth et al., 2009). Thus, non-Gaussian-based objective functions, such as those based on cross-correlation (Zhang et al., 2015) or Laplacian statistics (da Silva et al., 2024), have shown improved robustness.

From this perspective, we propose an LSRTM approach based on a non-Gaussian criterion formulated using correntropy. Correntropy includes higher-order moments and provides natural resilience to non-Gaussian errors, specially to erratic data (outliers) (Santamaria et al., 2006). In particular, we introduce a new objective function that minimizes a correntropy-inspired cost and evaluate its performance under different noise conditions.

Correntropy-Based LSRTM

Correntropy quantifies the similarity between two random variables X and Y as: $V_\sigma(X,Y) = \mathbb{E}[\kappa_\sigma(X-Y)]$, in which $\kappa_\sigma(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\varepsilon^2}{2\sigma^2}\right)$ is the so-called Gaussian kernel, where ε denotes the residuals (or errors) and σ is the kernel size parameter. For a dataset $\{(x_i, y_i)\}_{i=1}^{N}$, the correntropy is given by $V_\sigma(X,Y) = \frac{1}{N\sqrt{2\pi}\sigma}\sum_{i=1}^{N}\exp\left(-\frac{(x_i-y_i)^2}{2\sigma^2}\right)$. In this work, instead of the usual approach of maximizing correntropy, we adopt a minimization framework based on the following objective function: $\phi_\sigma(m) = \sum_{i=1}^{N}\left[1-\exp\left(-\frac{\varepsilon_i^2(m)}{2\sigma^2}\right)\right]$, where $\varepsilon_i(m)$ denotes the residuals given a model m. As $\sigma \to +\infty$, $\phi_\sigma(m)$ reduces to the conventional least-squares objective function. For small σ, it emphasizes sparsity in the residuals and thus naturally attenuates large errors.

In this way, we formulate the correntropy-LSRTM optimization problem as follows:

$$\min_{m,\delta p}\sum_{s,r}\int_0^T\left[1-\exp\left(-\frac{(\Gamma_{s,r}\delta p_s - d_{s,r})^2}{2\sigma^2}\right)\right]dt \quad \text{subject to} \quad \frac{1}{c_0^2}\frac{\partial^2\delta p_s}{\partial t^2} - \nabla^2\delta p_s = -m\frac{\partial^2 p_{s_0}}{\partial t^2}, \quad (1)$$

where $m = m(\mathbf{x})$ is the reflectivity model, $c_0 = c_0(\mathbf{x})$ is the background (migration) velocity model; T is the acquisition time. $d_{s,r} = d_{s,r}(t)$ and $\delta p_s = \delta p_s(t)$ reprsent the observed data and the modeled scattered wavefields, where $\Gamma_{s,r}$ denotes the observation operator that extracts data at receiver positions. The background wavefield $p_{s_0} = p_{s_0}(\mathbf{x},t)$ fulfills the conditions: $\frac{1}{c_0^2}\frac{\partial^2 p_{s_0}}{\partial t^2} - \nabla^2 p_{s_0} = s(t)\delta(\mathbf{x}-\mathbf{x}_s)$; $s(t)$ is the source wavelet.

We compute the gradient of the correntropy-based objective function using the adjoint-state method for computational efficiency (Plessix, 2006). The associated adjoint wave equation is given by:

$$\frac{1}{c_0^2}\frac{\partial^2\lambda_s(\mathbf{x},t)}{\partial t^2} - \nabla^2\lambda_s(\mathbf{x},t) = -\frac{1}{\sigma^2}\sum_r \Gamma_{s,r}^{\dagger}\varepsilon_{s,r}(T-t)\exp\left(-\frac{\varepsilon_{s,r}^2(T-t)}{2\,\sigma^2}\right), \quad (2)$$

where $\varepsilon_{s,r}(T-t)$ represents the residuals in reverse time and λ_s the backpropagated wavefield.

For a finite and fixed value of σ, the source term on the right-hand side of the adjoint equation (2) vanishes as $\varepsilon_{s,r} \to \infty$, due to the exponential decay. This decay effectively suppresses the influence of very large residuals in the adjoint source and thus increases the robustness against outliers. As $\sigma \to \infty$, the objective function defined on the left-hand side of Eq. (1) converges to the conventional least-squares formulation. In this limit, the adjoint source becomes a simple backpropagation of the residuals, without any weighting or suppression. Consequently, conventional LSRTM treats all residuals equally, regardless of their magnitude, resulting in large residuals dominating and thereby reducing robustness.

Results

In this section, we present numerical experiments based on the Marmousi model, depicted in Fig. 1(a), together with the corresponding reflectivity model (Fig. 1(b)). The acquisition geometry follows an ocean-bottom-node (OBN) configuration consisting of 17 nodes spaced 400 m apart (red triangles in Fig. 1) and 681 sources spaced 10 m apart (green lines in Fig. 1). As the seismic source, we consider a Ricker wavelet with a peak frequency of 15 Hz and a recording time of 5 seconds. For the inversion, we employ the conjugate gradient method with the stopping criterion set to 30 iterations. Figure 1(c) shows the background P-wave velocity model utilized in this work.

We consider two noise scenarios in our numerical experiments. In the first case, the observed data are contaminated with additive Gaussian noise at a signal-to-noise ratio (SNR) of 10 dB. In the second case, we add non-Gaussian noise to the same data already contaminated with 10 dB Gaussian noise. Specifically, we introduce outliers by randomly corrupting 0.5% of the seismic traces as described by Brossier et al. (2010). Figure 2 shows the LSRTM results for the first noise scenario. Panel (a) shows the model estimates obtained using conventional LSRTM, which exhibit wavepath footprints and reduced resolution, especially in the shallow region where strong artifacts are observed at the OBN positions. Panels (b)–(d) show the results obtained using the proposed correntropy-based

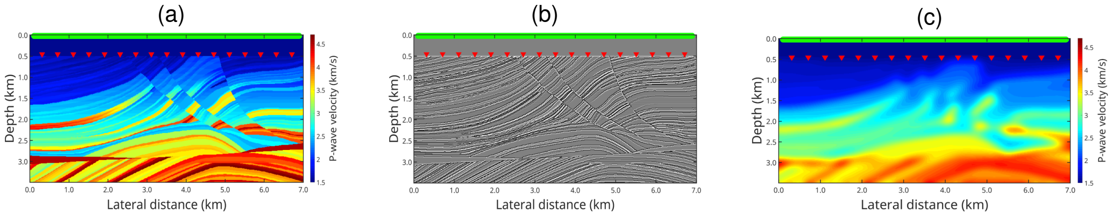

Figure 1: (a) Marmousi P-wave velocity model; (b) Benchmark reflectivity; (c) Background P-wave velocity model. Red triangles: OBN positions; green lines: seismic source locations.

LSRTM for different values of the kernel size σ. As σ decreases, the method becomes increasingly robust to noise, resulting in improved image continuity and artifact suppression. In particular, the resulting model in panel (d) obtained with $\sigma = 1$ has the clearest structural features and the lowest noise. Panel (e) illustrates the convergence of the normalized objective function for each method. The correntropy-based approach accelerates the convergence compared to the conventional LSRTM as σ decreases.

Given the superior performance previously observed for $\sigma = 1$, we consider only this kernel size in the second noise scenario, where non-Gaussian noise is introduced. In this case, conventional LSRTM completely fails to produce reasonable estimates, as depicted in Fig. 3(a). In contrast, the correntropy-based LSRTM provides much cleaner results (Fig. 3(b)) with fewer artifacts; the geologic interfaces appear sharper and more continuous, comparable even to those obtained under purely Gaussian noise conditions. Moreover, the objective function exhibits a faster decay rate than that of the conventional approach, as can be seen in Fig. 3(c).

Conclusions

In this work, we have proposed a correntropy-based LSRTM approach that effectively mitigates the effects of non-Gaussian errors and reduces wavepath footprints in the resulting seismic images. Our results proved that conventional LSRTM suffers from significant image quality degradation in

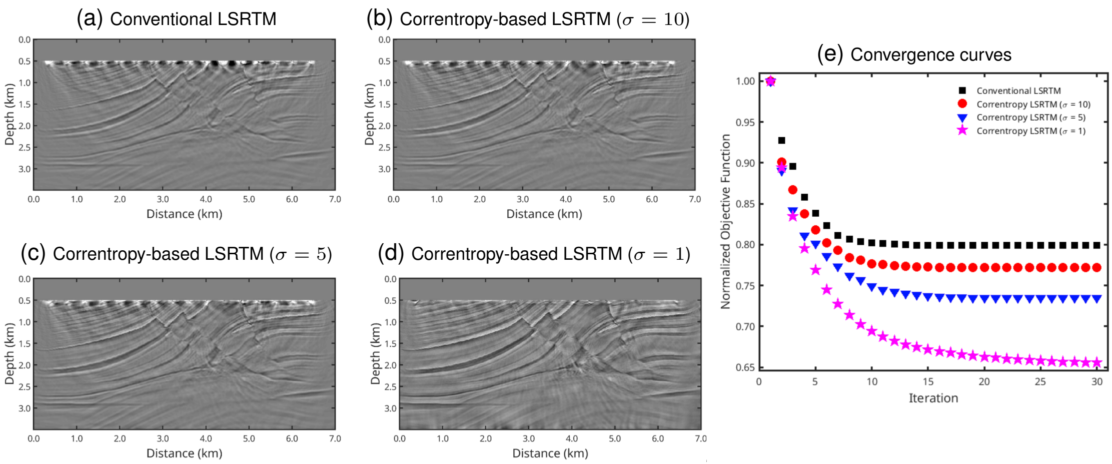

Figure 2: LSRTM results under additive Gaussian noise (SNR = 10 dB). (a) Conventional LSRTM; (b–d) correntropy-based LSRTM; (e) Decay of objective functions by iteration.

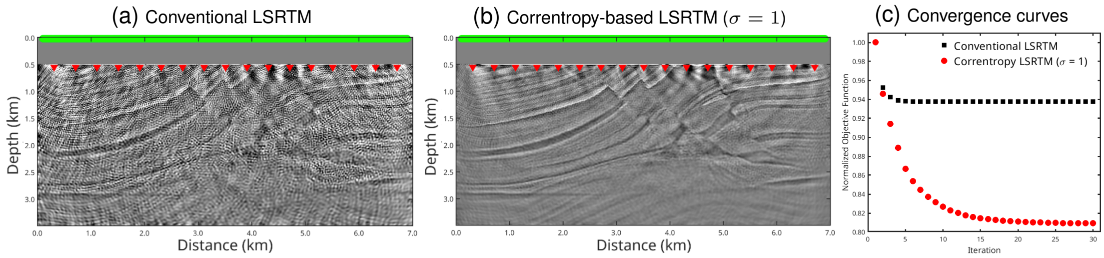

Figure 3: Comparison between (a) conventional LSRTM and (b) correntropy-based LSRTM ($\sigma = 1$) using data contaminated with non-Gaussian noise. (c) Convergence curves of the normalized objective function for both methods.

the presence of non-Gaussian errors, while correntropy-based LSRTM consistently yields higher-resolution models. Overall, we conclude that σ-LSRTM is a promising and robust alternative for seismic inversion that can reduce the need for extensive data preprocessing and is well-suited for integration into automated workflows. Nevertheless, the choice of the hyperparameter σ remains a critical issue. Future research should focus on strategies for optimal selection and adaptive tuning of σ during the inversion process.

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