Submission code: 5R9V4875MZ

See this and other abstracts on our website: https://home.sbgf.org.br/Pages/resumos.php

Hybrid Microgravity Inversion with Curvature Correction: The Abdelrahman–Mendon¸ca–Meguid (AMM) Framewor

Ahmed A. Meguid (Geodeep Serviços Geológicos e Geofísicos Ltda)

Hybrid Microgravity Inversion with Curvature Correction: The Abdelrahman–Mendonça–Meguid (AMM) Framework

Abstract

This work introduces the AMM Method (Abdelrahman–Mendonça–Meguid), a hybrid inversion approach designed to enhance the estimation of subsurface void depths from microgravity data. It combines spectral techniques, geometric analysis, and a curvature-based correction factor, producing stable and physically consistent results. The framework naturally incorporates the mathematical constant π as an angular compensator under symmetric geometries. Validation was conducted using both synthetic models and real microgravity data from the Braskem salt mine (Maceió, Brazil).

Introduction

The detection and characterization of subsurface voids is a persistent challenge in applied geophysics, particularly in urban, mining, and karst environments where invasive methods are impractical or unsafe. Among indirect geophysical techniques, microgravity stands out for its cost-effectiveness, portability, and ability to detect mass deficits. However, the low signal-to-noise ratio of gravity data and the inherent non-uniqueness of potential field inversion have historically limited its resolving power.

Traditional methods, such as anomaly fitting, analytic signal processing, and upward continuation, offer quick approximations but often fall short in providing accurate depth estimates — especially in the presence of complex or shallow geometries. Spectral techniques improve stability but tend to underestimate depths due to their inherent averaging over the entire anomaly body. Geometric methods, on the other hand, are more intuitive but lack spectral robustness.

This study addresses these limitations by proposing a unified inversion framework — the AMM Method — that blends spectral estimators with curvature-based geometric corrections. Inspired by potential field theory and the behavior of vector fields near discontinuities, the method introduces a pseudo-magnetic transformation of gravity data, quantifies lateral anomaly width, and applies a curvature ratio to correct for systematic underestimations. The emergence of π as a natural compensator reflects an angular relationship between width and depth under radial symmetry. The result is a more coherent and physically meaningful inversion strategy for high-risk or data-limited contexts.

Methodology

The AMM Method consists of a sequential and integrative approach that merges spectral and geometric principles to estimate the depth of subsurface voids from microgravity data with improved physical consistency.

The process begins by transforming the measured vertical gravity component $g_z(x, y)$ into a pseudo-magnetic vector field $\vec{B}$. This is done by applying Green's identity and calculating the horizontal gradients of g_z, effectively simulating the behavior of a magnetic dipole field. The advantage of this transformation is that it reveals directional properties and geometric symmetry in the gravity anomaly, which are not directly observable in scalar data.

Once the vector field is obtained, the second stage focuses on detecting the lateral boundaries (or "flanks") of the anomaly. These flanks correspond to local maxima and minima in the magnitude of $\vec{B}(x)$, which reflect sharp contrasts in subsurface density. By identifying these flanks, the method calculates the total width λ of the anomaly — a key geometric indicator that is directly tied to the shape and depth of the causative body.

With the anomaly width determined, the method proceeds to estimate the initial (or spectral) depth ζ\zetaζ using a frequency-domain expression. This spectral estimator is weighted by the relative strength and distribution of the anomaly signal, and is mathematically designed to suppress noise and enhance depth stability. However, spectral methods are known to underestimate depth, particularly in cases of sharp lateral gradients or near-surface voids.

To address this limitation, the method introduces a curvature-based correction factor. Specifically, the ratio between the anomaly width λ and the spectral depth ζ is interpreted as a curvature indicator κ. This factor encapsulates how laterally spread an anomaly is relative to its estimated depth. A high curvature (i.e., small $^{\lambda}/_{\zeta}$) suggests a narrow, deep anomaly, while a low curvature points to a broad, shallow structure. The corrected depth is then computed as $z_{real} = {}^{\zeta}/_{\kappa}$, effectively rescaling the depth based on the shape of the anomaly.

A striking outcome of this formulation is the natural emergence of the constant π in the correction step when the anomaly exhibits circular or radial symmetry. In such cases, the depth-to-width ratio tends to approach $^{1}/_{\pi}$, making π\piπ a geometric compensator that bridges angular geometry and potential field behavior.

To refine the anomaly signal before flank detection, the gravity data undergoes preprocessing using a high-order (10th-degree) polynomial residual removal, which isolates local anomalies from regional trends. This is followed by Hilbert filtering to enhance edge clarity and suppress high-frequency noise.

Finally, the method is validated through two applications. First, a synthetic model simulating a spherical void at $500\,m$ depth is used to test the algorithm's performance under controlled conditions. Then, the approach is applied to real-world data from the Braskem salt mine in Maceió (AL), comparing results with borehole-confirmed depths. These validations confirm the method's robustness and physical coherence across synthetic and practical scenarios.

Results

To assess the performance and reliability of the AMM Method, two validation exercises were conducted: one using synthetic data with known model parameters, and the other using real microgravity data acquired in a high-risk urban mining context.

In the synthetic test, the subsurface structure modeled consisted of a single spherical cavity located at $500\,m$ depth. The gravity response was simulated using the GMSIM2D platform, and the resulting anomaly was processed with a 10th-order polynomial residual filter followed by Hilbert transformation. Flank detection yielded a lateral width at $\lambda = 5.00\,m$, and the spectral depth estimation (prior to correction) resulted in a value of approximately at $1637\,m$. This significant overestimation is consistent with known limitations of spectral methods when applied to localized or symmetric anomalies. Upon applying the curvature correction through the ratio at $^{\lambda}/_{\zeta}$, and subsequently incorporating the angular scaling factor π, the estimated depth was refined to approximately $521\,m$. This result represents a relative error of just 4.2% when compared to the modeled value, highlighting the efficacy of the curvature-based correction in reducing systematic spectral bias.

The second application involved real data from the Braskem FL06 dataset, acquired in the salt-mining zone of Maceió, Brazil — a region known for underground void collapses and high geotechnical risk. The observed gravity anomaly exhibited lateral asymmetry and moderate amplitude, consistent with elongated or irregular cavity structures. After applying the same processing pipeline (residual removal, pseudo-magnetic transformation, and flank detection), the spectral depth estimate was found to be significantly deeper than expected. However, the application of the AMM correction strategy yielded a final estimated void depth of $744\ m$, in close agreement with the borehole-confirmed depth of $766\ m$. The absolute error in this case was $22\ m$ (approximately 2.9%), which falls within acceptable limits for engineering decision-making in subsurface risk assessment.

Figure 1 summarizes the complete inversion workflow, illustrating both the synthetic and real-data applications. The left panels display the conceptual model, simulation results, and gravity–magnetic responses for the synthetic case. The right panels show the observed Braskem anomalies, flank separation markers, and the geometric center estimations used in the curvature correction. These visualizations reinforce the interpretability of the method and its adaptability to varying anomaly geometries.

Conclusions

The AMM Method provides a unified and physically consistent framework for estimating the depth of subsurface voids from microgravity data. By coupling spectral inversion with a curvature-based geometric correction, the method overcomes common limitations of both approaches when used in isolation. The introduction of a correction factor based on the width-to-depth ratio, and the natural emergence of π\piπ as an angular compensator under symmetric conditions, reinforce the method's conceptual and mathematical coherence.

In both synthetic and real-world applications, the method yielded depth estimates with errors below 5.0%, even under complex anomaly geometries. These results highlight the AMM framework's ability to reconcile stability, accuracy, and interpretability—key requirements for geophysical diagnostics in high-risk contexts such as salt mining, urban void detection, and subsidence monitoring.

Its modular structure and low data demands make it especially suitable for scenarios where resolution and physical plausibility must be achieved with minimal acquisition effort. As such, the AMM Method represents a practical and theoretically grounded tool for enhancing the role of gravity data in near-surface investigations.

Acknowledgments

The author gratefully acknowledges Prof. Dr. Elsayed M. Abdelrahman (Cairo University) and Prof. Dr. Carlos Alberto Mendonça (University of São Paulo) for their foundational contributions to the art of gravity data inversion and for the conceptual inspiration that led to the development of the method presented in this work.

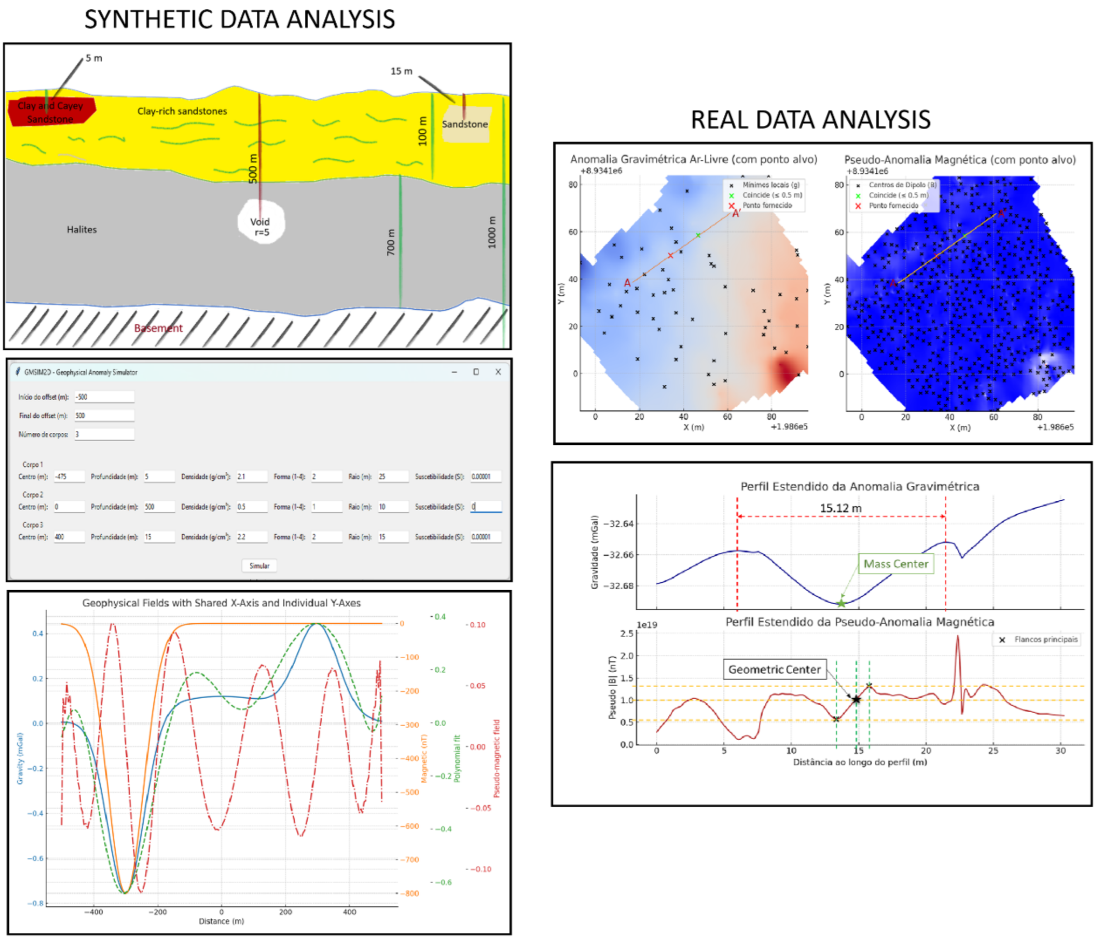

Figure 1: Synthetic and real-case application of the AMM Method. Left: conceptual model, simulation setup, and synthetic field responses. Right: Braskem dataset with gravity and pseudo-magnetic maps, plus profiles used for depth correction.

References

Abdelrahman, E. M., & El-Araby, T. M. (2001). Depth estimation using a least-squares approach applied to magnetic data. *Geophysics*, 66(4), 1119–1126.

Mendonça, C. A., & Meguid, A. A. (2008). Vector transformation of gravity data using Green's identity. *Revista Brasileira de Geofísica*, 26(2), 217–226.

Fedi, M., & Rapolla, A. (1999). Depth estimation via curvature: A new method for potential field data. *Geophysical Prospecting*, 47(4), 507–525.

Blakely, R. J. (1995). *Potential Theory in Gravity and Magnetic Applications*. Cambridge University Press.

Telford, W. M., Geldart, L. P., & Sheriff, R. E. (1990). *Applied Geophysics* (2nd ed.). Cambridge University Press.

Abdelfettah, Y., Schamper, C., & Tabbagh, A. (2014). Microgravity investigation of a subsurface cavity in southern France. *Geophysics*, 79(4), B193–B202.

Barreto, C., et al. (2021). Microgravimetria aplicada à identificação de colapsos em áreas de mineração de sal-gema em Maceió. *Revista Brasileira de Geofísica*, 39(2), 123–136.