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Multiparameter viscoacoustic least-squares reverse time migration with preconditioning using multiparameter Hessian

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Abstract

This study explores the use of a diagonal pseudo-Hessian preconditioner to improve reflectivity estimates in multiparameter viscoacoustic least-squares reverse time migration (M-QLSRTM). We derive a pseudo-Hessian approximation for two parameters: the inverse bulk modulus and the magnitude of the Q factor, and incorporate it into a conjugate gradient least squares (CGLS) optimization framework. Numerical experiments using the BP gas chimney model show that the preconditioning significantly enhances amplitude balancing in the Q-factor reflectivity images, especially beneath the gas cloud and in deeper structures. The results also reveal improved reflector continuity in both parameter images.

Introduction

Seismic imaging in attenuating media requires accurate parameter estimation to compensate for amplitude and phase distortions induced by anelastic effects. Least-squares reverse time migration (LSRTM) improves resolution and reduces migration artifacts by iteratively minimizing the data misfit. Nogueira and Porsani (2024) introduced a multiparameter viscoacoustic LSRTM (M-QLSRTM) approach that simultaneously estimates the reflectivity of the inverse bulk modulus ($\delta\kappa$) and the magnitude of the Q factor ($\delta\tau$). Despite its advantages, M-QLSRTM often exhibits amplitude imbalance, particularly for $\delta\tau$, which is intrinsically less sensitive to seismic data than $\delta\kappa$.

To address this limitation, we adopt the diagonal pseudo-Hessian preconditioning strategy proposed by Shin et al. (2001), which approximates the diagonal elements of the Hessian matrix to effectively scale gradient updates. By neglecting off-diagonal terms and the receiver-side Green's functions, this method improves amplitude balancing while preserving structural information in the image.

In this study, we derive the pseudo-Hessian for M-QLSRTM and assess its impact on the joint reconstruction of $\delta\kappa$ and $\delta\tau$ reflectivity. Numerical experiments using the BP gas chimney model demonstrate that pseudo-Hessian preconditioning significantly enhances $\delta\tau$ imaging, compensating for its lower sensitivity, while maintaining the stability and accuracy of $\delta\kappa$ reconstructions.

Multi-parameter linearized viscoacoustic modeling operator

According to Nogueira and Porsani (2024), the multi-parameter linearized viscoacoustic modeling operator can be expressed as

$$\begin{cases} \kappa_0 \dfrac{\partial^2 \delta p}{\partial t^2} - \nabla \cdot b_0 \nabla \delta p - \tau_0 \nabla \cdot b_0 \nabla \delta p + \delta r = -\delta\kappa \dfrac{\partial^2 p_0}{\partial t^2} + \delta\tau \nabla \cdot b_0 \nabla p_0 - \delta\tau \dfrac{r_0}{\tau_0}, \\ \dfrac{\partial \delta r}{\partial t} = \dfrac{\tau_0}{\tau_{\sigma_0}} \nabla \cdot b_0 \nabla \delta p - \dfrac{\delta r}{\tau_{\sigma_0}}, \end{cases} \tag{1}$$

and the corresponding adjoint equation

$$\begin{cases} \kappa_0 \dfrac{\partial^2 q}{\partial t^2} - \nabla \cdot b_0 \nabla (1 + \tau_0) q - \nabla \cdot b_0 \nabla \dfrac{\tau_0}{\tau_{\sigma_0}} r_q = \Delta d, \\ \dfrac{\partial r_q}{\partial t} - \dfrac{r_q}{\tau_{\sigma_0}} - q = 0. \end{cases} \tag{2}$$

Where q and r_q are the adjoint-state variable of the state variables p and r_p, which p and r_p are wavefields from the non linear forward equation (Nogueira and Porsani, 2024), respectively. δp is the perturbed wavefield, δr is the memory variable perturbed wavefield, Δd is the pressure data residual between the calculated and the observed data. b is the buoyancy parameter, τ_σ is the relaxation time stress. The variables p_0,r_0 and b_0, κ_0, τ_{σ_0}, τ_0 are the background wavefields and background medium parameters, respectively. Based in the work developed by Nogueira and Porsani (2024), the perturbation model $\delta \mathbf{m} = [\delta\kappa \quad \delta\tau]^T$, which is the gradient for M-QLSRTM, is given by

$$\delta \mathbf{m} = [\delta\kappa \quad \delta\tau]^T = \left[-\int_0^T \frac{\partial^2 p_0}{\partial t^2} q \, dt \qquad \int_0^T \left[(\nabla \cdot b_0 \nabla p_0)\, q + \left(\frac{1}{\tau_{\sigma_0}} \nabla \cdot b_0 \nabla p_0 \right) r_q \right] dt \right]^T . \tag{3}$$

Preconditioning Using the Multiparameter Hessian

M-QLSRTM implicitly inverts the Hessian operator through CGLS iterations (Nocedal and Wright, 1999) combined with the adjoint-state method. In this framework, the application of the Fréchet derivative and its adjoint is performed on the fly at each CGLS iteration, eliminating the need to explicitly form either the Fréchet derivative or the Hessian matrix. However, the CGLS algorithm may require a relatively large number of iterations to converge to an optimal solution.

Gradient preconditioning plays a crucial role in accelerating CGLS convergence and reducing computational costs. In this study, we investigate the structure of the M-QLSRTM Hessian operator and adopt the diagonal of the pseudo-Hessian for preconditioning, following the approach proposed by Shin et al. (2001). The Hessian operator for the M-QLSRTM problem can be expressed as:

$$\mathbb{H} = \begin{pmatrix} \mathbb{H}^{\kappa\kappa} & \mathbb{H}^{\kappa\tau} \\ \mathbb{H}^{\tau\kappa} & \mathbb{H}^{\tau\tau} \end{pmatrix}, \tag{4}$$

and its corresponding diagonal:

$$\widetilde{\mathbb{H}} = \begin{pmatrix} \widetilde{\mathbb{H}}^{\kappa\kappa} & 0 \\ 0 & \widetilde{\mathbb{H}}^{\tau\tau} \end{pmatrix} = \begin{pmatrix} \int_T \left(\frac{\partial S}{\partial \kappa} w\right) \left(\frac{\partial S}{\partial \kappa} w\right) \, dt & 0 \\ 0 & \int_T \left(\frac{\partial S}{\partial \tau} w\right) \left(\frac{\partial S}{\partial \tau} w\right) \, dt \end{pmatrix} = \begin{pmatrix} \int_T \left(\frac{\partial^2 p_0}{\partial t^2} \frac{\partial^2 p_0}{\partial t^2} \right) dt & 0 \\ 0 & \int_T \left(1 + \frac{1}{\tau_{\theta_0}^2} \right) (\nabla \cdot b_0 \nabla p_0)^2 \, dt \end{pmatrix}. \tag{5}$$

Where S is the forward modeling operator matrix and w is the wavefields vector.

Thus, the model update using the gradient preconditioned by the diagonal of the Hessian matrix in the context of M-QLSRTM is given by:

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \Delta\mathbf{m}_k = \mathbf{m}_k + \alpha\widetilde{\mathbb{H}}^{-1}\delta\mathbf{m}_k, \tag{6}$$

where α is the step length that scales the gradient vector.

Results

To demonstrate the preconditioning by diagonal pseudo-Hessian matrix in M-QLSRTM, we compare the estimated reflectivity model with and without the preconditioning by the diagonal pseudo-Hessian matrix. The numerical experiments was carried out in the 2D BP gas chimney model, showed in Figure 1, which contain 9.925 km $\times$ 4.020 km. We generate the observed data through the true models, which was implemented 25 shots, evenly spaced, and with 400 receivers spaced every 20 m. Also, it was defined 15 iterations during the inversion process.

At Figure 1, we depicted the estimated reflectivity models to compare the results with and without the preconditioning. It is noted that the preconditioning method improved the amplitude of reflectivity images, mainly for $\delta\tau$, working as a normalization for estimated images. There was a highlight of the deeper structural part of $\delta\tau$ images, giving a better amplitude balance. Unlike the $\delta\tau$ parameter, there is no high difference between estimated images for $\delta\kappa$, with and without preconditioning. This is due to inversion robustness of the M-QLSRTM, even without pseudo-Hessian.

These features are more evident in the cropped region, from 3 km to 7 km in horizontal and 2.5 km in depth (Figure 2), which is present the gas cloud. The reflectivity estimated via pseudo-Hessian preconditioning, was capable to recover the region beneath the gas cloud, improving the amplitude in this region, mainly for $\delta\tau$ reflectivity (Figure 2 - (f)).

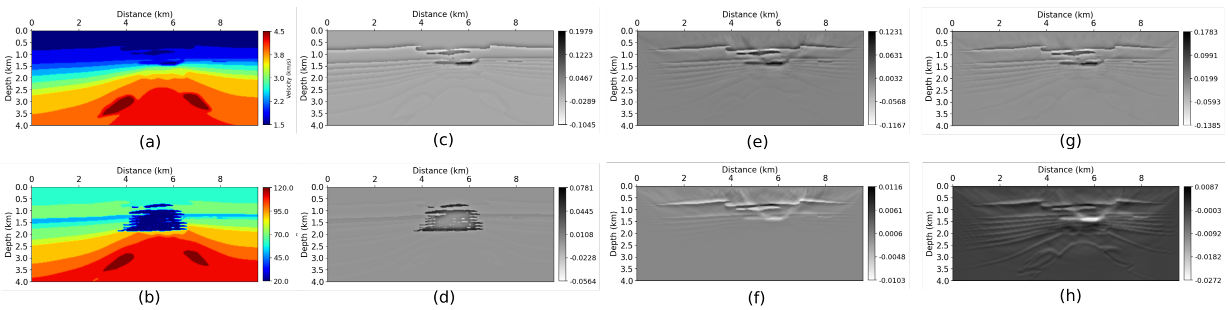

Figure 1: The panel shows true models for compressional wave velocity (a) and Q factor (b) for 2D BP gas chimney model. Also, the true reflectivity models $\delta\kappa$ (c) and $\delta\tau$ (d); and the estimated reflectivity models without (e), (f) and with (g),(h) pre-conditioning by diagonal pseudo-Hessian matrix, respectively.

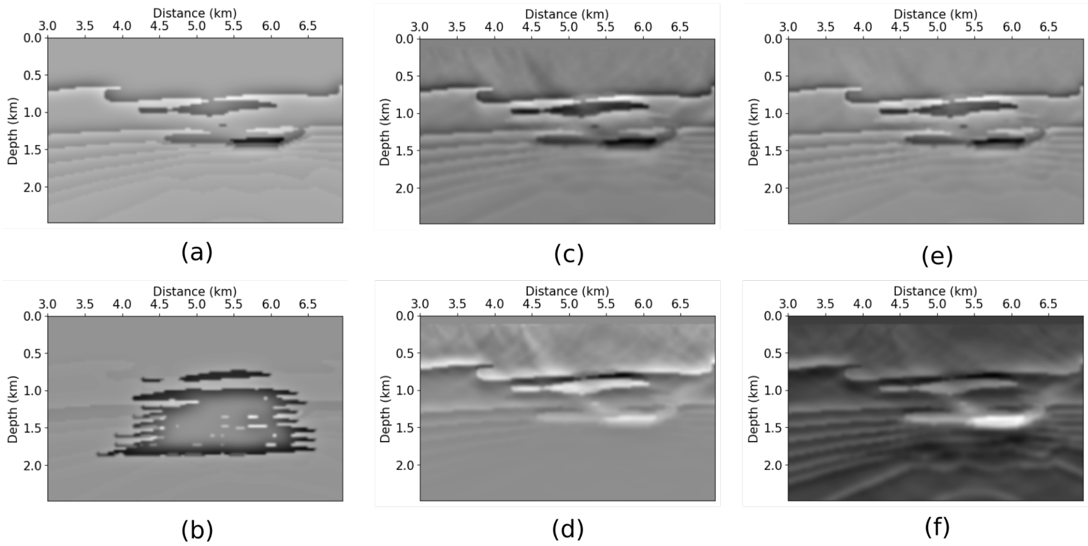

Figure 2: A cropped region, from 3 km to 7 km in horizontal and 2.5 km in depth, on the 2D BP gas chimney model. The panel shows true reflectivity models $\delta\kappa$, (c), and $\delta\tau$, (d); and the estimated reflectivity models without (e), (f) and with (g),(h) pre-conditioning by diagonal pseudo-Hessian matrix, respectively.

Conclusions

The pseudo-Hessian preconditioner effectively enhances amplitude balance of M Q-LSRTM, particularly for attenuation-related reflectivity parameter ($\delta\tau$) recovery, while maintaining the robustness of velocity-related ($\delta\kappa$) reflectivity. Numerical tests confirm that preconditioning mitigates amplitude imbalances in deeper structures, offering a good tool for high-resolution viscoacoustic imaging.

Acknowledgments

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